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SEMI-F-SPACES

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ABSTRACT. Semi-F-spaces are spaces such that given any pair of disjoint cozero sets, every countable subset of one is completely separated from the other. This generalizes the notion of an F-space and is stronger than the property that every countable subset is C^*-embedded. Semi-F-spaces are studied and several examples are given.

0. **Introduction.** An F-space is a space with the property that any two disjoint cozero sets are completely separated. A space has the property that every countable subset is C^*-embedded if and only if given any two disjoint cozero sets, every countable subset of the first is completely separated from every countable subset of the other. In this paper, we look at a property which lies between these two properties. We call a space a semi-F-space if given any two disjoint cozero sets, every countable subset of one is completely separated from the other. Among compact spaces, the property of being a semi-F-space takes a particularly nice form – a compact space is a semi-F-space if and only if every zero set has ω-bounded interior. For normal spaces, the property of being a semi-F-space is closed-hereditary, but in general the property is preserved by neither closed nor open subsets. We will give examples to show this. We will also give a very simple example of an open subset of an F-space which is not an F-space.

1. **Preliminaries.** All given spaces are assumed to be completely regular and Hausdorff. If X is a space, βX denotes the Stone-Čech compactification of X and X^* denotes $\beta X \setminus X$. An ordinal number is the set of its predecessors and a cardinal number is an initial ordinal. The countably infinite cardinal ω_0 is denoted by ω. Unless we specify otherwise, ordinals will be assumed to have the order topology. If D is a discrete space, uD denotes the set of uniform ultrafilters on D, that is, uD is the set of elements of βD which are not in the closure of any subset of D having cardinal less than that of D. If X is a space, X_δ denotes X endowed with the G_δ-topology, that is, the topology on X obtained

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from the given topology by declaring that all G_δ-sets of the original topology are open. In particular, if λ is an ordinal, λ_δ is the order topology on λ strengthened so that all ordinals of countable cofinality are isolated.

An *F-space* is a space such that every two disjoint cozero sets are completely separated. Equivalently, an F-space is a space such that every cozero set is C^*-embedded. A *semi-F-space* is a space such that given disjoint cozero sets, every countable subset of one is completely separated from the other. Clearly every F-space is a semi-F-space. We will see below that not every semi-F-space is an F-space.

A space X is *ω-bounded* if the closure in X of every countable subset of X is compact. In particular, when we say that a subset S of a space is ω-bounded, we mean that S is ω-bounded as a subspace, that is, every countable subset of S has compact closure in S. Obviously every ω-bounded space is countably compact.

A point p of a space X is a *weak P-point* of X if p is not a limit point of any countable subset of X. It is known that ω^* has weak P-points and that uD has weak P-points, where D is the discrete space of cardinal ω_1. (See, for example, [3].)

For definitions and terminology not given here, see [1], [4], or [7].

2. **Basic Properties.** In this section, we give some of the basic properties of semi-F-spaces. We also give some easy examples.

2.1. Lemma. *Every countable subset of a space X is C^*-embedded in X if and only if given any two disjoint cozero sets (or equivalently any two disjoint open sets) U and V of X, every countable subset of U is completely separated from every countable subset of V.*

Proof. Suppose first that every countable subset of X is C^*-embedded in X and let A and B be disjoint open subsets of X. Let C_A and C_B be countable subsets of A and B respectively. Since $C_A \cup C_B$ is countable, it is C^*-embedded in X, so the bounded continuous function $f:(C_A \cup C_B) \to \mathbf{R}$ given by $f|C_A \equiv 0$, $f|C_B \equiv 1$ extends to a bounded continuous function on X. Therefore, C_A and C_B are completely separated.

For the converse, suppose that given disjoint cozero sets U and V of X, every countable subset of U is completely separated from every countable subset of V. Let C be any countable subset of X. To show that C is C^*-embedded in X, by the Urysohn Extension Theorem it suffices to show that any two sets which are completely separated in C are also completely separated in X. Using the usual "shoestring" method, if A and B are contained in disjoint closed subsets of C, we can find disjoint open sets $\hat{U}$ and $\hat{V}$ of X such that $A \subseteq \hat{U}$ and $B \subseteq \hat{V}$. Since the cozero sets form a base for the topology of X, and since the union of countably many cozero sets is again a cozero set, we can find cozero sets U and

V of X satisfying $A \subseteq U \subseteq \hat{U}$ and $B \subseteq V \subseteq \hat{V}$. By the hypothesis, A and B are completely separated in X. □

2.2. COROLLARY. *If X is a semi-F-space, then every countable subset of X is C^*-embedded in X.*

2.3. PROPOSITION. *Suppose X is compact. Then X is a semi-F-space if and only if every zero set of X has ω-bounded interior.*

PROOF. Let X be a compact space. Suppose that X is a semi-F-space and Z is a zero set of X. Let $U = \text{Int}_X Z$. We must show that if C is a countable subset of U, then $\text{Cl}_U C$ is compact. As in 2.1, we can find a cozero set W of X such that $C \subseteq W \subseteq U$. By the assumption that X is a semi-F-space, there exists a bounded continuous function $f:X \to \mathbf{R}$ such that $f|X\setminus Z \equiv 1$ and $f|C \equiv 0$. The zero set of f is compact and is contained in U, so $\text{Cl}_U C$ is a closed subset of a compact set and is therefore compact.

For the converse, suppose that X is compact and that every zero set of X has ω-bounded interior. Let U and V be disjoint cozero sets of X and let C be a countable subset of V. We will show that U is completely separated from C. Let Z be the zero set $X\setminus U$. Then C is a countable subset of the ω-bounded set $W = \text{Int}_X Z$, so $\text{Cl}_W C$ is compact. Therefore, $\text{Cl}_W C$ and $X\setminus W$ are disjoint compact subsets of X the first of which contains C and the second of which contains U. Hence, C and U are completely separated. □

2.4. COROLLARY. *If X is a compact F-space, then the interior of any zero set of X is ω-bounded.*

The referee has pointed out that the proof of Proposition 2.3 essentially shows that a normal space X is a semi-F-space if and only if every zero set Z of X has the following property: If C is a countable subset of $\text{Int}_X Z$, then $\text{Cl}_X C \subseteq \text{Int}_X Z$. The referee has also observed that another generalization of Proposition 2.3 is inherent in the fact that the proof of sufficiency does not use compactness of X, only the ω-boundedness of $\text{Int}_X Z$ since in a Tychonoff space, a compact set and a closed set are completely separated. (See [4], 3.11a.)

Since every F-space is a semi-F-space and since every semi-F-space has the property that every countable subset is C^*-embedded, it is natural to ask whether any of these properties coincide. The following examples show that they do not, even in the setting of compact spaces.

2.5. EXAMPLE. There exists a compact semi-F-space which is not an F-space.

CONSTRUCTION. Let D be the discrete space of cardinal ω_1. Let p be a weak P-point of uD. Obtain X from the product $2 \times \beta D$ by identifying $(0, p)$ and $(1, p)$ and let $q:(2 \times \beta D) \to X$ be the quotient map. Denote by $\hat{p}$ the identified point $q((0, p))$. Notice that $\hat{p}$ is a weak P-point of X. Obviously X is compact.

Let U and V be disjoint cozero sets of X and let C be a countable subset of V. In order to prove that X is a semi-F-space, it suffices to show that $\mathrm{Cl}_X U \cap \mathrm{Cl}_X C = \phi$. Notice that since $2 \times \beta D$ is an F-space the sets $q^{\leftarrow}(U)$ and $q^{\leftarrow}(V)$ have disjoint closures in $2 \times \beta D$. Therefore, $q^{\leftarrow}(U)$ and $q^{\leftarrow}(C)$ have disjoint closures in $2 \times \beta D$. Therefore, either $\mathrm{Cl}_X U \cap \mathrm{Cl}_X C$ is empty or $\mathrm{Cl}_X U \cap \mathrm{Cl}_X C = \{\hat{p}\}$. But since $\hat{p}$ is a weak P-point, the only way that it can be in $\mathrm{Cl}_X C$ is if $\hat{p} \in C$, that is, if $q^{\leftarrow}(C) \supseteq 2 \times \{p\}$. But the closure in $2 \times \beta D$ of $q^{\leftarrow}(U)$ does not intersect $q^{\leftarrow}(V)$ so it cannot intersect $2 \times \{p\}$. Therefore, $\mathrm{Cl}_X(U) \cap \mathrm{Cl}_X(C)$ is empty. It follows that X is a semi-F-space.

To see that X is not an F-space, let $f:\beta D \to \mathbf{R}$ be a continuous function whose zero set Z is contained in D^*. Then $U = q^{\rightarrow}(\{0\} \times (X \backslash Z))$ and $V = q^{\rightarrow}(\{1\} \times (X \backslash Z))$ are disjoint cozero sets of X each of whose closures contain $\hat{p}$. □

2.6. Example. There exists a compact space X such that every countable subset of X is C^*-embedded in X but X is not a semi-F-space.

Construction. Let p_0 be a weak P-point of ω^* such that p_0 is not a P-point of ω^*. Such points exist – see [6]. Obtain X from $(2 \times \beta\omega) \backslash (\{0\} \times \omega)$ by identifying $2 \times \{p_0\}$ to a point. Let q be the quotient map. Then using the fact that p_0 is a weak P-point, it is easy to show that every countable subset of X is C^*-embedded in X. To show that X is not a semi-F-space, let $Z = Z(f)$ be a zero set of ω^* whose boundary in ω^* contains p_0. We can do this because p_0 is not a P-point. Let $g:X \to \mathbf{R}$ be defined by $g(x) = f(z)$ provided that $q(z) = x$ and $z \in \{0\} \times \omega^*$, $g(x) = 0$ otherwise. It is routine to show that g is well-defined and continuous. But the interior of the zero set of g contains the countable set $q^{\rightarrow}(\{1\} \times \omega)$ which has a limit point $q(p_0)$ which is not in the interior. Therefore, by 2.3, X is not a semi-F-space. □

3. **Subspaces of semi-F-spaces.** It is clear from the definition that a C^*-embedded subset of a semi-F-space is again a semi-F-space. Therefore, every closed subset of a normal semi-F-space is again a semi-F-space. It is also easy to see that a cozero subset of a semi-F-space is a semi-F-space. In this section, we show that the property of being a semi-F-space is neither closed hereditary nor open hereditary.

3.1. Example. There exists an F-space and a closed subset which is not a semi-F-space.

Construction. First notice that if X is a space satisfying $\omega \subseteq X \subseteq \beta\omega$, then X is extremally disconnected and hence an F-space. Therefore, if we can find any subset S of ω^* which is not a semi-F-space, the subspace $X = S \cup \omega$ of $\beta\omega$ would be an F-space whose closed subset S is not a semi-F-space. So we need only find a subset S of ω^* such that S is not a semi-F-space.

Let $\{A_k: k \in \omega\}$ be a family of pairwise disjoint non-empty clopen subsets of ω^*. For each $k \in \omega$, let $\{C_n^k: n \in \omega\}$ be a pairwise disjoint family of non-empty clopen subsets of A_k. Choose $x_n^k \in C_n^k$ and for each k, choose

$$p_k \in \mathrm{Cl}_{\beta\omega}\{x_n^k: n \in \omega\}\backslash\{x_n^k: n \in \omega\}.$$

Let $C^k = \cup_{n\in\omega} C_n^k$. Finally, let

$$S = \bigcup_{k\in\omega} [A_k\backslash Bdry_{\omega^*}C^k] \cup [\mathrm{Cl}_{\omega^*}\{p_k: k \in \omega\}\backslash\{p_k: k \in \omega\}].$$

To show that S is not a semi-F-space, put $U = \cup_{k\in\omega} C^k$ and $V = \cup_{k\in\omega}(A_k\backslash C^k) \cap S$. Since each of the sets C^k is a union of the countably many clopen subsets C_n^k, $n \in \omega$, of S – in fact, of ω^* – each set C^k is a cozero set of S. Therefore, U, being a union of countably many cozero sets of S is itself a cozero set of S. Each set of the form $C^k \cap S$ is clopen in S (although such sets are not closed in ω^*), so each set $(A_k\backslash C^k) \cap S$ is clopen in S. Therefore, V is also a cozero set of S. The cozero sets U and V are obviously disjoint. Let $D = \{x_n^k:n,\ k \in \omega\}$. Then D is a countable subset of U. But D and V are not completely separated in S: If $f|D \equiv 0$ and $f|V \equiv 1$, then for $p \in \mathrm{Cl}_{\omega^*}\{p_k: k \in \omega\}\backslash\{p_k: k \in \omega\}$, f assumes the values 0 and 1 on every neighborhood of p so f does not extend continuously to p. □

The next example serves two purposes. First, it gives a very easily described example of an open subset of an F-space which is not an F-space, and secondly, it will help in the example below of an open subset of an F-space which is not a semi-F-space. We note that a more complicated example of an open subset of an F-space which is not an F-space is given by Dow in [2]. Examples 3.2 and 3.4 use the following fact: If D is a dense subset of X and $f:D \to \mathbf{R}$ is a continuous function such that for each $p \in X$, f extends to a continuous real-valued function $D \cup \{p\}$, then f extends to a continuous real-valued function on X. (See [4], 6H.)

3.2. Example. There exists an F-space X and an open subset U such that U is not an F-space.

Construction. Let p be any element of ω^*. Let $\hat{X} = A \times B$ where $A = [(\omega_1 + 1) \times (\omega_2 + 1)]_\delta$ and $B = \omega \cup \{p\}$. Let $X = \hat{X}\backslash(\omega_1 \times \omega_2 \times \omega)$. We can think of $\hat{X}$ as being a corner of a room with floor $A \times \{p\}$ and walls $(\omega_1 + 1) \times \omega$ and $(\omega_2 + 1) \times \omega$. Then X is just the floor and the two walls. To show that X is an F-space, it is (more than) enough to show that $\hat{X}$ is a Lindelöf F-space, since X is a closed subset of $\hat{X}$. A is Lindelöf because it has the G_δ-topology on a compact scattered space. (See for example [5].) Since $\hat{X}$ is the union of countably many copies of A, $\hat{X}$ is also Lindelöf. Notice that for each $x \in B$, the subspace $A \times \{x\}$ of $\hat{X}$ is a P-space. Suppose C is a cozero set of $\hat{X}$ and $f: C \to [0, 1]$ is continuous. We must show that f extends continuously to a function $\hat{f}$: $\mathrm{Cl}_{\hat{X}}\, C \to [0, 1]$. Then the fact that $\hat{X}$ is an F-space will follow from normality. Suppose $x \in \mathrm{Cl}_{\hat{X}}\, C \backslash C$. Then $x = (a, p)$ for some $a \in A$, because if $n \in \omega$, $A \times \{n\}$ is clopen in $\hat{X}$ and so is $C \cap (A \times \{n\})$. This is because A is a P-space so its cozero sets are clopen. Furthermore, since $\hat{X} \backslash (A \times \{p\})$ is dense in $\hat{X}$, $x \in \mathrm{Cl}_{\hat{X}}(C \backslash (A \times \{p\}))$. For each $n \in \omega$, either $(a, n) \notin C$ or there exists a neighborhood U_n of a in A such that $U_n \times \{n\} \subseteq C$ and $f | U_n \times \{n\}$ is constant, say $f | U_n \times \{n\} \equiv r_n$. Define $g: \omega \to [0, 1]$ by

$$g(n) = \begin{cases} r_n & \text{if } U_n \text{ is defined} \\ 0 & \text{otherwise.} \end{cases}$$

Extend g to a continuous function $\hat{g}: \beta\omega \to [0, 1]$. Define $\hat{f}(x) = \hat{g}(p)$. Then clearly this defines a continuous extension of f to x. Therefore, we have continuously extended f, so $\hat{X}$ is an F-space. To find an open subset of X which is not an F-space, let $U = X \backslash \{(\omega_1, \omega_2)\} \times B$, that is, U is obtained by removing the meet of the two walls. Then $C_1 = (\omega_1 \times \{\omega_2\} \times \omega) \cap U$ and $C_2 = (\{\omega_1\} \times \omega_2 \times \omega) \cap U$ are disjoint cozero sets of U – they are cozero sets because each is a union of countably many clopen subsets of U. But these sets are not completely separated in U because any continuous function which is identically 0 on one of these sets and identically 1 on the other would have to completely separate the top and right edges of $(\omega_1 + 1)_\delta \times (\omega_2 + 1)_\delta \times \{p\} \backslash \{(\omega_1, \omega_2, p)\}$. It is clear that no such function exists. Therefore, U is not an F-space. □

We remark that since X is a closed subset of the Lindelöf space $\hat{X}$, the space X is also Lindelöf. This observation will be used in 3.4.

In order to get an F-space with an open subspace which is not a semi-F-space, we will start with the space X from the previous example and add countably many points in such a way that the wall $(\omega_1 + 1) \times \{\omega_2\} \times B$ is in the closure of a countable discrete set. In order to have any hope of getting an F-space when we do this, we will need to add the points in such a way that the countable set we add is C^*-embedded, so we will need to know that $(\omega + 1)_\delta \times B \subseteq \omega^*$. This is the content of the next lemma.

3.3. LEMMA. *Suppose $p \in \omega^*$ and $B = \omega \cup \{p\}$. Then $(\omega_1 + 1)_\delta \times B$ is homeomorphic to a subset of ω^*.*

PROOF. It is known that $(\omega_1 + 1)_\delta$ embeds in ω^*. (This is due to van Douwen. See [6].) Let $\{A_n: n \in \omega\}$ be a family of non-empty pairwise disjoint clopen subsets of ω^* and for each $n \in \omega$, let $W_n = \{\alpha^n_\lambda: \lambda \leqq \omega_1\}$ be a faithfully indexed copy of $(\omega_1 + 1)_\delta$ contained in A_n. For each $\lambda \leqq \omega_1$, the set $I_\lambda = \{\alpha^n_\lambda: n \in \omega\}$ is

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a countably infinite discrete set, that is, each I_λ is homeomorphic to ω. Since countable subsets of ω^* are C^*-embedded, for each $\lambda \leqq \omega_1$, we can find a point $p_\lambda \in \mathrm{Cl}_{\omega^*} I_\lambda \backslash I_\lambda$ such that for $S \subseteq \omega$, $p_\lambda \in \mathrm{Cl}_{\omega^*}\{a^n_\lambda : n \in S\}$ if and only if $p \in \mathrm{Cl}_{\beta\omega} S$. Define

$$h:(\omega_1 + 1)_\delta \times B \rightarrow \bigcup_{\lambda \leqq \omega_1} (I_\lambda \cup \{p_\lambda\}) \text{ by } h(\lambda, n) = \alpha^n_\lambda \text{ and } h(\lambda, p) = p_\lambda.$$

We will show that h is a homeomorphism.

Clearly h is one-to-one and onto. Obviously, the restriction of h to each of the clopen sets $(\omega_1 + 1)_\delta \times \{n\}$ where $n \in \omega$ is a homeomorphism. Once we show that the restriction of h to $(\omega_1 + 1)_\delta \times \{p\}$ is a homeomorphism, the fact that h is a homeomorphism will follow fairly easily from the choice of the p_λ's. If $\lambda_0 < \omega_1$, we must find a neighborhood N of p_{λ_0} such that $p_\lambda \notin N$ for $\lambda \neq \lambda_0$. We will in fact do a little more. Fix λ_0. For each $n \in \omega$, let V_n be a clopen (in ω^*) neighborhood of $\alpha^n_{\lambda_0}$ such that $\alpha^m_\lambda \notin V_n$ for $m \neq n, \lambda \neq \lambda_0$. Then $V = \cup_{n \in \omega} V_n$ and $W = \cup_{n \in \omega} (A_n \backslash V_n)$ are disjoint cozero sets of ω^* so they are completely separated. Therefore, we can find an ω^*-neighborhood $\hat{V}$ of $\mathrm{Cl}_{\omega^*} V$ which does not intersect $\mathrm{Cl}_{\omega^*} W$. But then $\hat{V}$ is a neighborhood of p_{λ_0} which contains no point of the form p_λ or of the form α^n_λ for $\lambda \neq \lambda_0$.

Next we show that neighborhoods of p_{ω_1} contain all but countably many p_λ's. Suppose that V is a clopen ω^*-neighborhood of p_{ω_1}. For each n such that $\alpha^n_{\omega_1} \in V$, let C_n be a countable subset of ω_1 such that for $\lambda \notin C_n$, $\alpha^n_\lambda \in V$. Let $C = \cup_{n \in \omega} C_n$. Then C is countable and clearly $\{p_\lambda : \lambda \notin C\} \subseteq V$. A similar argument shows that all countable subsets of $\{p_\lambda : \lambda < \omega_1\}$ are clopen in the relative topology of $\{p_\lambda : \lambda \leqq \omega_1\}$.

To complete the proof we must show that V is a neighborhood of (λ, p) if and only if $h^{\rightarrow}(V)$ is a neighborhood of p_λ. If $\lambda < \omega_1$, we may assume that V has the form $\{\lambda\} \times A$ where A is a $\beta\omega$-neighborhood of p in B. But then the choice of p_λ and the second paragraph of the proof assures that $h^{\rightarrow}(V)$ is a neighborhood of p_λ if and only if V is a neighborhood of (λ, p). This leaves the point (ω_1, p). In this case, we may assume that V has the form $U \times W$ where $U \subseteq \omega_1$ and $\omega_1 \backslash U$ is countable, and W is a B-neighborhood of p. If we now use the argument of the preceding paragraph along with the choice of the p_λ's, we get the result. □

In light of 3.3, we will view $(\omega_1 + 1)_\delta \times B$ as being a subspace of ω^*. Then $[(\omega_1 + 1)_\delta \times B] \cup \omega$ is extremally disconnected.

3.4. Example. There exists an F-space and an open subset which is not a semi-F-space.

Construction. Suppose $p \in \omega^*$ and $B = \omega \cup \{p\}$. Let X be as in 3.2 and let $\tilde{X}$ be $[(\omega_1 + 1)_\delta \times B] \cup \omega$ considered as a subspace of $\beta\omega$. Obtain Y from the discrete union of X and $\tilde{X}$ by identifying the point (λ, ω_2, z) of X with the point (λ, z) of $\tilde{X}$. Then X and $\tilde{X}$ are closed subspaces of Y. The identified points, that is, the elements of $X \cap \tilde{X}$, will be labelled as if they were elements of X. We omit the routine proof that Y is Hausdorff and regular. Since X is Lindelöf and Y differs from X by only countably many points, namely, the points of ω, Y is Lindelöf and therefore normal.

To prove that Y is an F-space, we must show that every cozero set is C^*-embedded in its closure. Suppose C is a cozero set of Y and $f:C \to [0, 1]$ is continuous. Let y be a point of $\mathrm{Cl}_Y C \setminus C$. We will find a continuous extension of f to y. If $y \in \tilde{X}$, then since $\tilde{X}$ is extremally disconnected, and therefore an F-space, the restriction $f|(C \cap \tilde{X})$ extends to a continuous function $\tilde{F}:\tilde{X} \cup \{y\} \to [0, 1]$. We claim that $g:C \cup \{y\} \to [0, 1]$ given by $g|C = f$, $g(y) = \tilde{F}(y)$ is continuous. Since $g|[(C \cup \{y\}) \cap \tilde{X}]$ is continuous, we need only check that $g|[(C \cup \{y\}) \cap X]$ is continuous. It follows from the facts that $[(\omega_1 + 1) \times (\omega_2 + 1)]_\delta$ is a P-space and that $y \in \tilde{X}$ that the only way that y can be in the closure of $C \cap X$ is if y is in the closure of $C \cap X \cap \tilde{X}$. The restriction $f|C \cap X$ extends continuously to a function $F:X \to [0, 1]$ because X is an F-space. Since $F|C \cap X \cap \tilde{X} = \tilde{F}|C \cap X \cap \tilde{X}$ and $y \in \mathrm{Cl}_Y(C \cap X \cap \tilde{X})$, $F(y) = \tilde{F}(y) = g(y)$. Therefore, g is continuous on $C \cap X$.

If $y \in Y \setminus \tilde{X}$, then $f|C \cap X$ extends to a continuous function $F:X \to [0, 1]$, because X is an F-space. Define $g:C \cup \{y\} \to [0, 1]$ by $g|C = f$, $g(y) = f(y)$. Then the restriction of g to each element of the open cover $\{C, [C \cup \{y\}] \setminus \tilde{X}\}$ of $C \cup \{y\}$ is continuous, so g is a continuous extension of f.

We still have to show that Y has an open subspace which is not a semi-F-space. The open set V will be Y with the meet of the two walls of X removed. More precisely, $V = Y \setminus \{(\omega_1, \omega_2, z) \in X: z \in B\}$. Then the set $C_1 = [\{\omega_1\} \times \omega_2 \times \omega] \cap V$ is a cozero set of V – the argument is the same as in 3.2 – and the set $C_2 = \omega$ is a cozero set of Y and therefore of V. But C_1 and C_2 are not completely separated in V – as in 3.2, any continuous function which completely separated C_1 and C_2 would have to completely separate the top and right edges of $(\omega_1 + 1)_\delta \times (\omega_2 + 1)_\delta \times \{p\} \setminus \{(\omega_1, \omega_2, p)\}$. Since C_2 is countable, this shows that V is not a semi-F-space. □

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REFERENCES

1. W. W. Comfort and S. Negrepontis, *Theory of ultrafilters*, Springer-Verlag, 1974.

2. A. Dow, *CH and open subspaces of F-spaces*, Proc. Amer. Math. Soc. **89** (1983), pp. 341-345.

3. A. Dow and J. van Mill, *On nowhere dense ccc P-sets*, Proc. Amer. Math. Soc. **80** (1980), pp. 697-700.

4. L. Gillman and M. Jerison, *Rings of continuous functions*, van Nostrand, 1960.

5. R. Levy and M. D. Rice, *Normal P-spaces and the G_δ-topology*, Coll. Math. **44** (1981), pp. 227-240.

6. J. van Mill, *An introduction to $\beta\omega$*, *Handbook of set theoretic topology*, edited by K. Kunen and J. Vaughan, North-Holland, 1985.

7. R. Walker, *The Stone-Čech compactification*, Springer-Verlag, 1974.

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