THE POSSIBLE ORBITAL EVOLUTION OF THE NEAR-EARTH ASTEROIDS

E.I.TIMOSHKOVA
Institute of Theoretical Astronomy,
Leningrad 191187, USSR

Abstract. The subject of this paper is a study of a possible orbital evolution for near-Earth asteroids. The investigation is fulfilled in the frame of the restricted circular three body problem. It is based on the calculations of the Jacobi constant. The osculating elements of some real Apollo-Amor-Aten asteroids are used as the starting parameters. The comparison with the results of other authors is given.

1. Introduction

In the last few years there is large interest in the study of the near-Earth asteroids. The orbital evolution of these bodies has been considered by many authors (see, for example, Kozai, 1980; Vashkovjak, 1980). Here, the most important problem is the stability of the asteroid motions, because the understanding of many cosmogonical and geophysical problems has a connection with the orbital stability. It is known that large computer calculations are required in most of the methods of the evolution investigation.

The discoveries of new planet-crossing asteroids significantly increase the population of near-Earth asteroids every year and it is desirable to have some simple criterion in order to distinguish quickly the asteroids with the irregular behavior of orbits.

The Jacobi integral may be used as such criterion if the model of the circular restricted three-body problem is considered (Szebehely, 1979). It is known that the values of Jacobi constant C for the stationary solutions of this problem are bifurcate and the behavior of the trajectories can be different for one and the same C. The 'measure of a motion stability' is defined as the derivation of the value of C from the critical value $C(L_i)$ at the libration point. The first such approach has been used by G.Hill in the investigation of the Moon orbital stability.

2. Description of calculations

In this paper the Jacobi constant was calculated for 135 near-Earth asteroids with perihelion distance up to 1.33 A.U. Here we include 64 numbered asteroids from "Ephemerides of Minor Planets for 1991" and a large part of unnumbered asteroids. From these 135 objects we have 6 Aten , 70 Apollo and 59 Amor asteroids.

In the sidereal system of osculating elements the Jacobi constant will be defined by the Tisserand's criterion:

$$J = Ck^{-2} = \frac{1}{a} + \frac{2}{a_j}\sqrt{\frac{a(1-e^2)}{a_j(1-\mu)}} + \frac{\mu}{1-\mu}\delta \qquad (1)$$

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 175–178.

and

$$\delta = \frac{2}{\rho} + \frac{\rho^2 - r^2 - {a_j}^2}{{a_j}^3} + \frac{\mu}{a_j} \tag{2}$$

There a, e, i are the semi-major axis, eccentricity and inclination of the asteroid, a_j is the semi-major axis of the perturbing planet, r is the Sun-asteroid distance, $\mu = m_j/(1+m_j)$, m_j is the mass of perturbing planet and k is the Gauss constant. The solar mass is adopted for unit mass; the unit of distance is 1 A.U.

The formulae for the calculations of Jacobi constants $J(L_i)$ for five libration points L_i are very well known and here we do not give them. The following disposition of libration points is used: L_3 is to the left of the Sun, L_1 is between the Sun and the perturbing planet and L_2 is to the right of planet.

The values of J are calculated by formulae (1) and (2) for each asteroid, in the frame of the Sun-planet-asteroid problem, with the six perturbing planets from Mercury ($j = 1$) to Saturn ($j = 6$). The osculating orbital elements were taken as the initial data. For each perturbing planet, the values of $J(L_{ij})$ are also determined. Then, we distinguish those asteroids for which $|J(N) - J(L_{ij})| < 10^{-1}$.

The results of other authors on the orbital evolution of some planet-crossing asteroids were analyzed for the comparison with our calculations. This is important to decide the question about the use of the difference $J(N) - J(L_i)$ for the prediction of unstable motions.

3. Results

Here we have no opportunity to produce all our results. Only some of these are summarized in Tables 1 and 2. In the case of Saturn, the values of Jacobi constant for most of the asteroids differ very much from J at the Saturnian libration points. So, we do not include this planet in Tables 1 and 2.

From results given in Table 1 it follows that for some asteroids the Jacobi constant is very close to the corresponding value at the libration point at once for two or three planets. Apparently it means very unstable asteroid motion. Our conclusions closely correlate with the results of Vashkovjak (1980) and Steel and Baggaley (1985). We may expect very interesting trajectories for the asteroids from the lower part of Table 1. For example, the asteroid 1986 DA has the value of Jacobi constant near equal to the value of J at the libration point L_1 for Jupiter. For a few asteroids, the value $J(N)$ is in the interval of values between the libration points L_2, L_3 or L_2, L_1 for Jupiter. It is interesting that four of these objects have a 5:2 mean motion resonance with Jupiter. The transitions to typical cometary orbits were discovered for the asteroids 1985 WA and 1986 JK by Hahn et al. (1989). They found that the asteroid 1986 DA shows increasing libration amplitude and the transition from the Alinda-class to the Oljato-class of the orbits. Also, the asteroid 1987 QB 'jumped' from 5:2 to 8:3 resonance.

In all cases it is the characteristic feature of the chaotic orbital evolution. The asteroid 1990 HA shows the similarity of its parameters to the corresponding 1987 QB parameters, with the exception of the semi-major axis. Therefore, it is required

TABLE I
The parameters of some selected Apollo-Amor asteroids

Nr	Asteroid	$J - J(L_{ij})$	L_{ij}	a	e	i^o
433	Eros	-0.0017	L_{43}	1.45832	0.22287	10.826
		-0.0536	L_{44}			
2202	Pele	0.00336	L_{13}	2.28941	0.51280	8.789
3361	Orpheus	-0.09363	L_{43}	1.20925	0.32265	2.688
		0.05853	L_{12}			
		-0.03638	L_{44}			
1943	Anteros	-0.01519	L_{43}	1.43036	0.25589	8.704
		-0.05552	L_{44}			
4015	1979 VA	-0.08131	L_{43}	2.64109	0.62275	2.785
		-0.00844	L_{15}			
	1982 DB	-0.05247	L_{43}	1.48954	0.36043	1.421
3908	1980 PA	-0.01546	L_{43}	1.92504	0.45794	2.168
1221	Amor	-0.0380	L_{43}	1.91973	0.43535	11.898
1620	Geographos	-0.01896	L_{42}	1.24473	0.33538	13.320
2063	Bacchus	-0.10002	L_{42}	1.07754	0.34940	9.419
		-0.02058	L_{44}			
	1980 WF	-0.00551	L_{43}	2.23077	0.51407	6.412
3288	Seleucus	0.01340	L_{13}	2.03271	0.45723	5.932
		-0.00640	L_{25}			
		0.00062	L_{35}			
1036	Ganymed	-0.00073	L_{25}	2.66567	0.53710	26.482
		0.00628	L_{35}			
	1985 WA	-0.00114	L_{45}	2.84596	0.60143	9.743
		0.00715	L_{13}			
	1986 JK	-0.01283	L_{45}	2.80173	0.67972	2.140
	1986 DA	0.00002	L_{15}	2.81129	0.58532	4.296
	1987 QB	-0.00050	L_{25}	2.80255	0.59368	3.462
		0.00652	L_{35}			
	1983 TF2	-0.00158	L_{25}	2.43893	0.73633	14.704
		0.00543	L_{35}			
	1987 PA	0.00451	L_{13}	2.73623	0.55681	16.121
		0.00397	L_{15}			
	1990 HA	-0.00004	L_{15}	2.56692	0.69212	3.882
		0.00020	L_{25}			
		-0.00699	L_{42}			

TABLE II
The values of Jacobi constant J for 6 Aten-asteroids

Nr	Asteroid	q	Mercury	Venus	Earth	Mars	Jupiter
2340	Hathor	0.4643	7.9697	3.8384	2.8173	2.0529	1.3228
2100	Ra-Shalom	0.4689	7.7608	3.7696	2.7815	2.0418	1.3353
2062	Aten	0.7900	8.6269	4.0070	2.8632	2.0070	1.1892
	1954 XA	0.5088	8.1408	3.9701	2.9374	2.1644	1.4261
	1989 UQ	0.6711	8.7473	4.0895	2.9362	2.0730	1.2484
	1989 VA	0.2951	6.3643	3.3268	2.5748	2.0118	1.4742
	$J(L_1)$		7.7503	4.1486	3.0009	1.9692	0.5846
	$J(L_4)$		7.7500	4.1475	3.0000	1.9690	0.5769

to study the long time orbit evolution of 1990 HA by numerical integration.

From Table 2 it is seen that for all Aten-asteroids in problem with Mars, Jupiter and also Saturn (not shown in the table) we have $J(N) > J(L_1)$. It is true also for Mercury, except for asteroid 1989 VA . This result means that the movement of these asteroids is Hill-stable and they can not catch in the nearest neighborhood of the corresponding perturbing planet. On the contrary, in the cases of Earth and Venus we have $J(N) < J(L_4)$ and the Hill-stability of the asteroid orbits is absent.

The results presented here should be considered as preliminary. Nevertheless, it seems that the Jacobi constant can be used to distinguish the asteroids with possible irregular motion.

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