Appendix to Comments on "Appropriate Discounting for Benefit-Cost Analysis"

The purpose of this Appendix is to show through simple numerical examples how using the STP, SOC and appropriate discounting methods differ in appraising two hypothetical projects. All values, including the discount rates used, are hypothetical. The examples will be used to illustrate three points:

1. That discounting accounts for the opportunity cost of capital for the public sector and that the STP and SOC discount rates do so incorrectly.
2. That the time value of money is a function of the opportunity cost of capital of the public sector.
3. That the correct application of the shadow price of capital is a required element of appropriate discounting.

1. Discounting accounts for the opportunity cost of capital

The following table shows the net flows of the two projects, their NPVs at the STP, SOC and the market opportunity costs of capital (MOC) discount rate. These are assumed to be 1%, 3% and 7.5% respectively. Notice that the MOC rate is not by itself enough for appropriate discounting. The necessary shadow price of capital adjustment will be introduced later. The IRR of the flows is also shown.

Project A – original net flows					
NPVs at discount rates shown				Net flows for years shown	
1%	3%	7.5%	IRR	0	1-20
$8.27	-$10.74	-$38.83	1.8%	-$100	$6

Project B – original net flows					
NPVs at discount rates shown				Net flows for years shown	
1%	3%	7.5%	IRR	0	1-20
$62.41	$33.90	-$8.25	6.4%	-$100	$9

We can see that discounting at the STP rate would result in accepting both projects, discounting using the SOC rate would result in rejecting , while using the MOC rate would result in rejecting project A and accepting project B. The different conclusions result from the different opportunity costs of capital that are implicit in discounting at the different rates, which leaves net surpluses of differing signs.

The best way of showing that this is indeed responsible for a good part of the differences is to make the opportunity cost of capital explicit. This can be done by assuming that the public sector issues a bond for $100, which it services by paying $3 in interest every year and repays in full at the end of year 20. This flow then becomes a flow of financing that has to be added to the original project

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net flows, resulting in the following.

Project A – financing flow added						
NPVs at discount rates shown				Net flows for years shown		
1%	3%	7.5%	IRR	0	1-19	20
-$27.82	-$10.74	$7.04	5.1%	$0	$3	-$97

Project B – financing flow added						
NPVs at discount rates shown				Net flows for years shown		
1%	3%	7.5%	IRR	0	1-19	20
$26.32	$33.90	$37.63	-2.0%	-$0	$6	-$94

What we have done in the above two flows is to force the opportunity cost of capital to be taken into account at the MOC rate. Having accounted for that, the remaining net flows give the surpluses that the different discounting approaches will discount at their own rates. Looking at the new NPV results for project A we see that is no longer acceptable with the STP rate, the right opportunity cost of capital having been taken out of the flow. The conclusion derived using the SOC rate has also changed. Its overstatement of the opportunity cost of capital has been corrected, and the flow is now left with a surplus that is evaluated to be positive at the SOC rate. Finally note that the NPV with the MOC rate has not changed at all, for either project. This is because it had correctly measured the opportunity cost of capital all along. Mechanically this happens because the NPV of the financing flow computed at the MOC rate is $0.

This illustrates the fact that the process of discounting implicitly subtracts the opportunity cost of capital and, therefore, to measure it correctly, the MOC rate should be used in discounting.

2. The opportunity cost of capital defines the time value of money

The following table gives the present value of one dollar of year 20 at the discount rates used in the examples:

	Value of $1 in year 20
1%	$0.82
3%	$0.55
7.5%	$0.24

Someone who values future income at either a 1% or a 7.5% implicit discount rate while having access to a market in which funds yield 3% becomes a money pump. The former would surely give $0.70 for $1 due in 20 years that he values at $0.82. There will be plenty of suppliers who will get the required $1 due

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in 20 years for $0.55 in the market. Similarly, there would be plenty of takers for $1 in 20 years' time sold for $0.30.

If the net flows of the projects have been converted with conversion factors and distribution weights (which can include inter-temporal ones) into the public sector income numeraire, and the public sector can move funds from the present into the future at the MOC rate, then transitivity of preferences requires the time value of money be defined by the MOC rate. This will therefore be the relative time value of everything that the benefit-cost analysis has managed to quantify and convert into the numeraire. Both the STP and the SOC discount rates give inconsistent time values.

3. Appropriate discounting

While the right discount rate to use is the MOC rate, discounting by it alone is not enough. The welfare cost of displacing private sector investments and of attracting additional savings must be taken unto account to achieve fully the objective of benefit-cost analysis, namely to measure the full welfare impact of a proposed investment in a second best setting. The following table shows the calculation of the shadow price of capital adjustment value to be used.

The table assumes that the interest rate at which the private sector borrows is 8%, which generates an annual willingness to pay for each dollar borrowed of $0.08. If we assume that this includes payment of a 20% tax per dollar borrowed, then the cost of providing funds in this market by the suppliers was $0.064 p.a. If as a result of the public sector's investment one dollar is withdrawn from the market, the net welfare loss is equal to $0.016, the difference between gross willingness to pay and the cost of supply, both of which will be foregone. This is equal to the taxes formerly paid.

Borrowing rate		8%
Annuity per $ borrowed		$0.080
Tax	20%	$0.016
Annual welfare loss per $ displaced		$0.016
Weight	90%	
Lending rate		3%
Annuity per $ lent		$0.030
Tax	20%	$0.006
Annual welfare cost per $ borrowed		$0.024
Annual welfare cost per $ borrowed		-$0.006
Weight	10%	
Annual welfare costs of acquired funds		$0.0138
For an investment of $100		$1.38

In a similar vein, if the interest rate that the savers get in the market is 3% and the tax due on it is 20%, then the opportunity cost of funds that compensates

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savers for foregone consumption is $0.024 per dollar lent. Therefore the market rate of 3% overstates the welfare cost in an amount of $0.012, requiring a welfare adjustment of -$0.012 annually.

Assuming that 90% of the funds originate from displacing private sector investments and 10% from incremental savings, the incremental welfare cost of acquiring one dollar for the public sector investment is $0.012, the weighted average of the two tax-induced adjustments, or $1.12 per year for the $100 to be invested. Notice that this is additional to the opportunity cost of the funds raised in the market, the opportunity cost of which will be taken care of by discounting at the MOC rate. The welfare adjustment will therefore be added to the net flows of the project before discounting takes place. This results in the following flows for the two projects, and shows the NPVs calculated as appropriate for benefit cost analysis, showing the effects of the shadow pricing of capital.

Project A – adjusted for shadow price of capital			
NPV at MOC rate		**Net flows for years shown**	
3%	**IRR**	**0**	**1-20**
-$31.27	-0.7%	-$100	$4.62

Project B – adjusted for shadow price of capital			
NPV at MOC rate		**Net flows for years shown**	
3%	**IRR**	**0**	**1-20**
$13.37	4.4%	-$100	$7.62

Project A should be rejected, while discounting it at the SOC rate resulted in its acceptance. Project B should be accepted, while discounting it at the SOC rate[1] resulted in its rejection.

These are not the conclusion derived discounting the project flows by either the STP or the SOC rates. Therefore neither is appropriate for benefit-cost analysis.

[1] The SOC rate was computed as a weighted average of the lending and borrowing rates using the same weights as were used to compute the shadow price of capital adjustment value.

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