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ARTICLE

Coordination frictions and economic growth†

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Abstract

In practice, firms face a number of scarce innovation projects. They choose one towards which to direct their effort, but do not coordinate these choices. This gives rise to coordination frictions. This paper develops an expanding-variety endogenous growth model to study the implications of these frictions for growth and welfare. We find that the coordination failure generates a number of foregone innovations and reduces the economy-wide research intensity. Both effects decrease the growth rate. This creates a general equilibrium effect that endogenously amplifies the fraction of wasteful simultaneous innovation. Furthermore, formalizing the coordination frictions uncovers a novel link between the "stepping on toes" and "standing on shoulders" externalities—their magnitudes are endogenously determined through the ratio of firms to innovation projects. We find that the "stepping on toes" externality is larger for all parameter values.

Keywords: Growth; Frictions; Coordination; Simultaneous Innovation; Search for Ideas

JEL classifications: O30; O31; O32; O33; O40

1. Introduction

Innovators have technological access to many distinct research avenues (ideas).[1] However, it is often the case that several firms engage in an innovation race for the exact same idea, i.e. research avenues are scarce. In particular, Lemley (2011) details anecdotal evidence that virtually every major historical innovation (such as the cotton gin, the steam engine, the computer, and the laser) has been simultaneously innovated by several groups of researchers. Perhaps the most famous example is that of the Alexander Bell and Elisha Gray telephone controversy. On February 14, 1876 Bell filed a patent application for the telephone and only hours later Gray submitted a similar application for the exact same innovation. Furthermore, the same empirical regularity is observed for non-major innovations. Cohen and Ishii (2005) find that a positive fraction of patents for the period between 1988 and 1996 were declared in interference.[2] More recent examples of simultaneous innovation include companies such as Siemens, Philips, Google Inc., Microsoft Corporation, and Yahoo! Inc.[3]

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Furthermore, coordination of research efforts by firms (firm *A* directs its effort towards project 1, firm *B* towards project 2, and so on) is very unlikely in this setting because of two main reasons. First, the size of the "market" for ideas makes coordination very hard to achieve. Second, such coordination requires each firm to know the portfolio of research projects of all of its rivals. This is particularly implausible in the current context given that firms actively employ secrecy as an intellectual property protection mechanism. For example, Hall et al. (2014) report that 67% of all R&D-doing firms rate trade secrets to be somewhat important or highly important to the company and Cohen et al. (2000) find that R&D managers rate secrecy as an effective intellectual property protection mechanism for over 50% of innovations. For a survey of the evidence see, for example, Hall et al. (2014).

Motivated by these observations, we develop an expanding-variety endogenous growth model that features scarce research avenues and lack of research effort coordination in order to study the implications of coordination frictions for growth and welfare. We find that the frictions generate a number of foregone innovations and reduce the economy-wide research intensity. Both effects decrease the growth rate and, as a consequence, endogenously amplify the fraction of duplicative simultaneous innovation. Formalizing the coordination frictions uncovers a novel link between the "stepping on toes" and "standing on shoulders" externalities—their sizes are endogenously determined by the ratio of firms to ideas.[4] The former externality dominates the latter one for all parameter values, thus, decentralizing the planner's allocation requires imposing a tax on R&D investment.

In our model, R&D firms direct their research efforts towards a particular project out of an endogenously determined number of ideas (which we also refer to as research avenues or projects). If innovated, each idea is transformed into one new variety. Firms which secure a patent over a variety produce. We focus on the symmetric equilibrium where firms use identical mixed strategies when directing their R&D efforts.[5] The random realization of these mixed strategies implies that some ideas are simultaneously innovated by many firms while others are not innovated at all. In equilibrium, the number of firms that innovate the same idea simultaneously follows a Poisson distribution with mean equal to the tightness in the market for ideas (the ratio of firms to ideas). The market tightness captures the level of congestion in our economy. When it is high, there are relatively more firms that apply for the same patent, on average, and so the fraction of duplicative innovations is higher, i.e. firms have a higher chance to "step on toes". Knowledge is cumulative—each innovated idea allows firms to "stand on the shoulders of giants" and gain technological access to a number of new research projects. This intertemporal spillover effect is the ultimate source of growth in our economy—an expanding number of ideas permanently alleviates future congestion problems, thus reducing the cost of discovering new varieties. Along the balanced growth path (BGP henceforth), the growth rate of the economy is determined by the growth rate of the number of ideas, which is in turn endogenously determined by the market tightness and the coordination problems.

Because of simultaneous innovation, some of the R&D effort in the economy is wasteful—when several firms innovate the same idea simultaneously only one secures the patent and the rest make a duplicative innovation.[6] At the same time the frictions imply that some profitable ideas will be left uninnovated because, by chance, no firm will choose to direct its research effort towards them. This latter possibility generates a number of foregone innovations. As a result, the growth rate of the decentralized frictional economy (DE henceforth) is lower, as compared to a hypothetical economy in which firms can coordinate their efforts (CE henceforth).[7]

The frictions also amplify the fraction of duplicative simultaneous innovation. In the CE firms know exactly which research avenues their competitors are working on and, as a consequence, all duplicative innovation is the result of intentional competition for patents. In the DE, on the other hand, firms cannot coordinate their efforts so some of the duplication which takes place is unintentional. Nonetheless, the higher fraction of duplicative simultaneous innovation in the DE is not a mechanical consequence of the frictions. Instead, it is the result of a general equilibrium effect. Intuitively, in both the DE and the CE firms enter the R&D sector until the cost of innovation equals the expected return which is given by the probability of receiving a patent times the net present value of monopoly profits. Since the probability of securing a patent is negatively related to the fraction of simultaneous innovation, this free entry condition determines the total fraction of wasteful simultaneous innovation whether some of it is intentional or not. Due to the lower growth rate, in the DE firms discount future profit streams at a lower rate. This increases the value of holding a patent as compared to the CE. Because of this, firms in the DE have a stronger incentive to engage in R&D which leads to higher congestion in equilibrium. As a result, the fraction of wasteful duplication of effort is amplified.

The coordination frictions also reduce the aggregate R&D intensity (as captured by the market tightness), even though firms have greater incentives to engage in innovation activities as compared to the CE. This is the case because for any market tightness, the coordination frictions reduce firms' probability of securing a monopoly position. Given a tightness, the ratio of innovations to ideas is the same for both the DE and the CE. In the DE, however, there is a number of foregone innovations. Hence, a lower fraction of these innovations are distinct which leads to a lower number of patents to be distributed among firms. This reduced probability of securing a patent induces firms to decrease their entry into the R&D sector, and so the DE is left with a lower R&D intensity.

We also examine the implications of coordination frictions for the constrained planing allocation—the planner can choose the number of R&D entrants, but she cannot assign firms to projects. Aside from a monopoly pricing externality, the model also features the usual "stepping on toes" (ST henceforth) and "standing on shoulders" (SS henceforth) externalities. Formalizing the coordination frictions that take place in the economy uncovers a novel link between these two externalities—their relative sizes are endogenously determined by the tightness. The ST one arises due to the possibility of simultaneous innovation in our model. The planner finds the marginal R&D firm's entry beneficial only if it is the sole inventor of its chosen project. The entrant, however, receives a positive payoff even if she faces competition for the patent as long as she wins the race. This creates incentives for firms to over-invest in R&D as compared to the socially optimal level. How large the incentives to over-invest are depended on the chance a firm secures a patent and the chance it is the sole inventor, both of which are endogenously determined by the market tightness. When it is either low or high, the incentives are small, whereas for intermediate values of the tightness they are large. The SS externality arises because firms cannot appropriate the benefits generated by any new ideas that come about from their innovation. As a result they do not value those new ideas. The planner, on the other hand, does because an increase in the pool of ideas permanently alleviates future coordination problems. Thus, the SS externality induces firms to under-invest in R&D as compared to the socially optimal level. When the market tightness is high, the chance the marginal entrant will be the sole inventor is low, hence, it is unlikely that new ideas will be generated from her innovation. This tends to decrease the size of the SS externality. At the same time, a high tightness implies a high level of congestion in the market. Thus, ideas are relatively scarce and, as a result, more valuable. This tends to increase the size of SS. It turns out this latter effect dominates so the SS externality increases in magnitude as the tightness goes up. Hence, the planner cares more about reducing congestion than foregone innovation: the size of the SS externality is primarily governed by how valuable ideas are to the planner, not by how many of them remain uninnovated. Consequently the planner chooses a relatively low market tightness because this delivers relatively low congestion. As a result, the ST externality is larger than the SS externality for all parameter values and implementing the planner's allocation requires imposing a tax on R&D investment.

1.1. Relationship to the Literature

Our paper models firms' choice of direction for their R&D efforts and the coordination problems inherent in this decision. As such, it is related to a recent literature on economic growth which emphasizes matching and other frictions in the innovation process [see, for example, Perla and Tonetti (2014), Lucas and Moll (2014), Benhabib et al. (2014), Chiu et al. (2017), Akcigit and Liu (2016), and Akcigit et al. (2016)]. For example, Akcigit and Liu (2016) emphasizes the importance of informational frictions for growth and welfare. In their model firms have access to two research avenues—a lucrative but risky one and a less profitable but safe one. In the process of R&D firms learn about the nature of the risky project (whether it is a dead-end or not) but have incentives to keep this information secret from their rivals. This creates a disclosure problem as firms may spend resources replicating the dead-end results of their rivals. The authors analyze firms' behavior in a pure strategy equilibrium and investigate the welfare implications.

The work here complements the literature by examining a different source of friction. As Akcigit and Liu (2016) we analyze an information issue—firms keep their R&D activities secret. In contrast to that paper, we emphasize the coordination problems inherent with secrecy. In our economy, firms are unsure which research projects their rivals are working on, whereas in Akcigit and Liu (2016) firms are unsure what their rivals have learned in the process of R&D. Thus, we emphasize the importance of information even when learning is absent and research avenues have certain payoffs. In particular, to the best of my knowledge, this is the first growth paper to emphasize search frictions in the market for ideas which take the form of a coordination failure.[8]

Our economy features the possibility of wasteful simultaneous innovation. In this regard the current paper relates to a literature which studies the implications of duplicative R&D investment for growth and welfare.[9] Papers within that literature include Segerstrom et al. (1990), Corriveau (1994), Corriveau (1998), and Zeira (2011) which focus on duplication of effort in patent races; Aghion et al. (2005) and Acemoglu and Akcigit (2012) which analyze economies where firms duplicate the previous successful innovations of their rivals; Akcigit and Liu (2016) which analyzes a model where firms may unknowingly spend R&D resources to duplicate the dead-end results of their rivals.[10] Jones and Williams (2000) quantify the size of the ST externality which stems from duplication of R&D effort and investigate whether or not the R&D spending in the U.S. economy is sub-optimally high/low; Strulik (2007) tackles the same research question in a model with human capital and impure altruism; Chen et al. (2021) study the level of the optimal capital tax rate when the ST externality is present. In contrast to these studies, we focus on the link between the ST and the SS externalities that arises endogenously from the coordination frictions in the economy.

Our paper is also related to the literature on coordination frictions. We study the coordination problems inherent in firms' choice of R&D projects and in this way contribute to the existing literature which has traditionally examined coordination failure in other contexts, such as the labor and goods markets: Julien et al. (2000), Burdett et al. (2001), and Shimer (2005). In these directed search studies firms/sellers post wages/prices and workers/buyers choose which firm to send an application to/which seller to visit.[11] In equilibrium it is possible that, due to coordination frictions, many workers apply for the same position/many buyers queue at the same seller. This generates congestion in the market. Similarly, in our setting firms choose towards which idea to direct their innovative activity and congestion arises because of the possibility that several firms innovate the same idea simultaneously. An important difference between our model and these aforementioned papers is that in ours congestion has an intertemporal aspect. The R&D firms of today make innovations which decrease the congestion faced by the R&D firms of tomorrow, an effect which the R&D firms of today do not internalize. Thus, the R&D firms of tomorrow play an analogous role to that of workers in the traditional search and matching framework. Furthermore, as we discuss in Section 4, the literature typically finds the decentralized equilibrium to be efficient whereas, in our study, we do not partially because of this dynamic aspect of congestion. Within the macroeconomic literature the closest paper is Gabrovski (2020). In the Gabrovski (2020) economy firms are randomly matched with ideas according to the same urn-ball matching function that arises endogenously in our model. In contrast to our current paper, Gabrovski (2020) does not focus on the coordination problems in the economy and the resulting implications for growth and welfare. Instead, the focus there is on the cyclical properties of R&D investment when innovation quality is endogenous. Within the literature on industrial organization the two closest papers are Kultti et al. (2007) and Kultti and Takalo (2008) which also feature search frictions in the market for ideas. In these papers there is the possibility of simultaneous innovation due to a matching technology which is the same as the equilibrium one in our paper. Kultti et al. (2007) and Kultti and Takalo (2008) focus on intellectual property rights in a partial equilibrium framework with a fixed number of ideas and without free entry into the innovation sector. In contrast, our model focuses on a general equilibrium framework with growth, an endogenously determined number of ideas, and an endogenously determined market tightness through free entry in the R&D sector.

We begin by introducing the environment. We then examine the decentralized equilibrium and highlight the impact of coordination frictions. Next, we study the implications of the frictions for the planner's (constrained) efficient allocation. The last section concludes.

2. The economy

The environment is an augmented version of the textbook model in Barro and Sala-i-Martin (2003) Chapter 6 (BSM henceforth).[12] We set the model in discrete time, so as to allow for the possibility of simultaneous innovation. There are three types of agents—a final good producer, R&D firms, and a unit measure of households. The only point of departure from BSM is in the R&D sector, so as to emphasize the novel features of the model. In particular, R&D projects are scarce and R&D entrants can direct their efforts towards a specific project, but they cannot coordinate their research activities.

2.1. Final Good Sector

The final good is produced by a single price taker, using the following technology

$$Y_t = AL^{1-\lambda} \sum_{n=0}^{N_t} X_{t,n}^{\lambda}, \quad 0 < \lambda < 1, \tag{1}$$

where Y_t is output, L is the fixed labor supply of households, N_t is the number of intermediate varieties, and $X_{t,n}$ is the amount of a particular variety n employed in production. The price of the final good is normalized to unity. The final good firm faces a competitive market for labor, which is hired at the wage w_t, and a monopolistically competitive market for varieties, where a unit of each variety n is bought at the price $P_{t,n}$.

2.2. R&D Sector

The novel features of our model are contained within the R&D sector of the economy. The innovation process has three stages and makes a distinction between potential innovations (ideas) and actual innovations (new varieties). At stage one, firms enter the R&D sector at a cost $\eta > 0$ units of the final good. The number of R&D entrants is denoted by μ_t and is to be determined in equilibrium. At stage two firms direct their innovative effort towards a particular R&D project from a pool of ideas. The number of ideas in the pool is denoted by ν_t. The choice is private knowledge and firms cannot coordinate their efforts. To capture this coordination failure, we follow the existing literature and focus on a symmetric equilibrium where firms use identical mixed strategies.[13] Ideas are identical and if a firm innovates an idea, it transforms into exactly one new variety. Innovation takes one period—a firm which enters at time t innovates the chosen project at time $t+1$. Thus, the only source of uncertainty in our model is the random realization of firms' equilibrium mixed strategies—some ideas may be innovated by many firms simultaneously, while others may not be innovated at all. We should note that in an economy with a small number of ideas this idiosyncratic uncertainty translates into aggregate uncertainty. However, due to positive long-run

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growth, as we will see later, the pool of ideas eventually becomes large, i.e. $\nu_t \to \infty$, regardless of the initial number of ideas, ν_0. As a result, there is no aggregate uncertainty along the BGP. Since our analysis focuses precisely on the BGP, we restrict our attention to the limit economy, where the number of participants in the coordination game is large. This is a standard approach in the literature on coordination frictions—see, for example, Shi (2001), Shi (2005), Julien et al. (2000), Julien et al. (2018), Gomis-Porqueras et al. (2017), Shimer (2005), Julien et al. (2014). Innovators apply for a patent which grants perpetual monopoly rights over the variety. Each innovation is protected by exactly one patent—if several firms simultaneously apply for the same patent, then each has an equal chance of receiving it. Stage three is as in BSM. Patent holders supply their variety in a monopolistically competitive market. Both the average and marginal costs of production are normalized to unity so profits are given by $\pi_{t,n} = (P_{t,n} - 1)X_{t,n}$. Furthermore, the value of holding a monopoly over a variety n at time t is given by $V_{t,n} \equiv \sum_{i=t+1}^{\infty} d_{it}\pi_i(n)$, where d_{it} is the factor with which firms discount future profits.

A necessary condition for positive long term growth in our model is that the number of ideas, ν_t, grows at a positive rate. We follow Kortum (1997) and Romer (1990), among others, and assume that knowledge is cumulative. Patenting an idea at time t allows firms to "stand on the shoulders of giants" and gain access to $M > 1$ new research avenues at $t + 1$.[14] Once an idea is innovated, it is no longer a potential R&D project and so it is removed from the pool.[15] Thus, the net increase in the pool of ideas from innovating one new variety is $M - 1$. Due to the frictions in our model, there is a chance that an idea is not innovated, i.e. no firm directs its research efforts towards the idea in question. Let us denote the fraction of uninnovated ideas by ϕ_t, then the law of motion for ideas is given by

$$\nu_{t+1} = \nu_t + (1 - \phi_t)(M - 1)\nu_t. \tag{2}$$

As each innovated idea is transformed into a new variety, it follows that

$$N_{t+1} = N_t + (1 - \phi_t)\nu_t. \tag{3}$$

2.3. Households

Households are endowed with a discount factor β and a per-period utility function $U(C_t) = \ln C_t$. They save by accumulating assets, which in this economy are claims on intermediate firms' profits. In particular, households have access to a mutual fund that covers all intermediate good firms. Let a_t denote the amount of shares held by the representative household at the beginning of period t. Each period all profits are redistributed as dividends, thus the total assets of the household entering period t are $a_t \sum_{n=0}^{N_t}(\pi_{t,n} + V_{t,n})$. At time t households decide on the shares they would like to hold at $t + 1$, a_{t+1}. The mutual fund at that time covers all firms which exist at time $t + 1$, N_{t+1}. Hence, the household's budget constraint is given by

$$a_{t+1} \sum_{n=0}^{N_{t+1}} V_{t,n} = a_t \sum_{n=0}^{N_t} (\pi_{t,n} + V_{t,n}) + w_t L - C_t. \tag{4}$$

The household's first order conditions imply the Euler equation below

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left(\sum_{n=0}^{N_{t+1}} (\pi_{t+1}(n) + V_{t+1}(n)) \right) \left(\sum_{n=0}^{N_{t+1}} V_{t,n} \right)^{-1}. \tag{5}$$

The intuition is standard—consumers equate the marginal utility at time t with the discounted marginal utility at time $t + 1$, times the gross rate of return on their assets.

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2.4. Balanced Growth Path Equilibrium

The usual profit maximization of intermediate and final good firms implies that $P_{t,n} = 1/\lambda$ and $X := X_{t,n} = (\lambda^2 A)^{1/(1-\lambda)} L$. Thus, every intermediate good firm receives the same per period profits of $\pi := \pi_{t,n} = X(1-\lambda)/\lambda$. This implies that $V_t := V_{t,n} = \sum_{i=t+1}^{\infty} d_{it}\pi$—every firm is equally valuable. Since each variety carries the same amount of profits, the stage two equilibrium strategy of firms is to direct their R&D effort towards each idea with equal probability. This implies the following equilibrium outcome.

Proposition 1. *The number of firms which direct their R&D effort towards a particular idea follows a Poisson distribution with mean θ_t, where $\theta_t \equiv \mu_t/\nu_t$.*

A proof is in the appendix. The random realization of firms' equilibrium strategies gives rise to the standard urn-ball matching technology.[16] In equilibrium some firms find themselves competing with many rivals for the same patent while others may not face any competition at all. The average number of firms that compete for the same patent is given by the ratio of firms to ideas, θ_t. This is the tightness in the market for ideas and is analogous to the queue length in models of directed search in the labor market.[17] In the labor market, a high θ_t implies each firm receives on average more applications from workers and so the market is more congested. In the current context, a higher θ_t implies more firms, on average, find themselves competing for the same innovation and so the market is more congested. In particular, an R&D firm becomes a monopolist with probability $\sum_{m=0}^{\infty} Pr(\text{exactly } m \text{ rival firms direct their research effort towards the particular idea})/(m+1) = \sum_{m=0}^{\infty} e^{-\theta_t}\theta_t^m/(m+1)! = \left(1-e^{-\theta_t}\right)/\theta_t$. Since only one firm gets the patent, the simultaneous innovations made by all other rivals represent wasteful duplication of effort. Thus, a fraction $\omega \equiv 1-(1-e^{-\theta_t})/\theta_t$ of all firms make a wasteful simultaneous innovation.[18] This duplication of effort is analogous to the congestion in the labor market where the firm can hire only one worker and the applications of all other workers who apply for the same job are turned down. The market tightness also dictates the fraction of foregone innovation $\phi_t = Pr(\text{nofirmdirectsitsresearchefforttowardsaparticularidea}) = e^{-\theta_t}$. As θ_t increases the chance that no firm chooses to work on any particular project is lower and so the fraction of foregone innovation decreases as well.[19] Hence, the laws of motion for ideas and varieties transform to

$$\nu_{t+1} = \nu_t + \left(1-e^{-\theta_t}\right)(M-1)\nu_t, \tag{6}$$

$$N_{t+1} = N_t + \left(1-e^{-\theta_t}\right)\nu_t. \tag{7}$$

Given free entry, it follows that[20]

$$\eta = \frac{1-e^{-\theta_t}}{\theta_t} V_t. \tag{8}$$

The equilibrium tightness is pinned down by the net present value of profits and the entry cost. As a consequence the fractions of wasteful duplication of effort, ω_t, and foregone innovation, ϕ_t, are determined by free entry as well. Higher profits (or lower costs) induce firms to tolerate a lower chance of securing a monopoly position which results in a higher equilibrium tightness. This implies more duplication of effort and lower foregone innovation. It is worth noting that due to the usual "over-grazing" problem firms have an incentive to compete for patents so some of the duplication which takes place in the economy is intentional. At the same time, due to the coordination problems firms may end up in a patent race with too many rivals. Hence, some of the simultaneous innovation which takes place is unintentional. R&D entrants foresee this possibility and so in equilibrium the free entry condition pins down the aggregate level of wasteful innovation whether it is intentional or not.

It is straightforward to establish that output, varieties, consumption, entry into R&D, and the stock of ideas all grow at the same rate, g, along the BGP. Namely, $g = (1 - e^{-\theta})(M - 1)$, where θ is the value of the market tightness along the BGP.[21] As in BSM Y_t, C_t, N_t, and μ_t all grow at the same rate. In our model, the number of ideas, ν_t, also grows at this rate. In fact, the expansion of ν_t is the ultimate source of growth in the economy. Because knowledge is cumulative, innovation today increases the number of ideas in the future. This permanently reduces the severity of the coordination problems and subsequently the cost of securing a monopoly position.[22] This lower cost in turn induces higher entry into R&D up to the point where congestion and, as a consequence, the tightness reach their BGP levels.

It is convenient to solve the model by looking at the stable ratios θ, $\frac{\nu}{N}$ and $\frac{C}{N}$. From the law of motion of ideas and varieties, and from $g_N = g_\nu$, it follows that $\frac{\nu}{N} = M - 1$. Next, combining the household budget constraint with the free entry condition and the law of motion for varieties yields $C/N = (1 + \lambda)\pi/\lambda - \eta\theta(M - 1)$. Lastly, we can use the fact that $g_C = g_\nu$, the Euler equation, the law of motion for ν_t, and the free entry condition to find an implicit solution for the market tightness.

$$\eta = \left(\frac{1 - e^{-\theta}}{\theta}\right) \frac{\beta\pi}{1 + (1 - e^{-\theta})(M - 1) - \beta}. \tag{9}$$

Even though we cannot explicitly solve for θ, it is straightforward to establish that the solution is unique. Intuitively, as θ increases the market for ideas gets more congested and each firm's chance of becoming a monopolist decreases. At the same time, higher market tightness implies a higher growth rate. This, in turn, increases the rate with which firms discount future profit streams and, as a consequence, decreases the value of holding a patent. Both of these effects decrease the incentives to enter the R&D sector when the market tightness is high and vice versa.

Proposition 2. *The equilibrium market tightness, θ, the growth rate, g, and the fraction of wasteful innovation, ω, are:*

- *increasing in A and β*
- *decreasing in η and M*

The proof, derived by taking the total derivative of (9), appears in the appendix. Intuitively, an increase in productivity leads to higher profits. High profits, in turn, raise the value of being a monopolist, V_t. This increases firms' incentives to innovate, which leads to a higher number of R&D entrants and subsequently to a higher market tightness. This higher tightness implies the market is more congested and so a higher fraction of firms make a duplicative simultaneous innovation, i.e. ω increases. At the same time a higher θ reduces the fraction of foregone innovation, which ultimately increases the growth rate. Similarly, a higher entry cost, η, discourages entry into R&D, which decreases θ, ω, and g. An increase in β or a decrease in M both lead to an increase in the effective discount factor, $\beta C_t/C_{t+1}$, along the BGP. Thus, firms value future profits more, which increases the value of a patent, V_t, and ultimately the market tightness, the fraction of wasteful duplication of effort, and the growth rate.

3. The impact of coordination frictions

The goal of our analysis is to study the impact of coordination frictions in the economy. To this end we compare the DE's BGP to the BGP of a hypothetical CE. The only difference between the latter economy and the DE is that firms can coordinate their research efforts at stage two of the innovation process. Let superscript c denote the value of any variable in the CE along the BGP.[23] Since firms can coordinate their R&D investments all projects are innovated by the exact same number of firms simultaneously. Moreover, all research avenues are undertaken, and

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subsequently, all ideas are innovated. Given free entry, however, the CE features a positive fraction of wasteful duplication of effort due to the usual "over-grazing" problem, i.e. $\mu_t^c > \nu_t^c$. Firms intentionally choose to compete for patents and so their chance of securing a monopoly position is $1/\theta^c$, which is less than unity in equilibrium. Nonetheless, the resulting waste, $\omega^c = 1 - 1/\theta^c$, is smaller than the one in the DE.

Proposition 3. *In the coordination economy all ideas are innovated and the growth rate equals $M - 1$. Furthermore,*

$$\omega - \omega^c = \frac{e^{-\theta}(M-1)\eta}{\beta\pi} > 0. \tag{10}$$

A proof is in the appendix. The coordination frictions amplify the fraction of wasteful simultaneous innovation. This, however, is not a mechanical consequence of the frictions, but rather the result of an endogenous general equilibrium effect—free entry dictates the level of congestion in both economies, and so the fraction of wasteful innovation is an endogenous object, whether some of the duplication that takes place is unintentional or not. As a result the relative sizes of ω and ω^c are pinned down by how lucrative it is to hold a monopoly position in each of the two economies. In the CE there is no forgone innovation, $\phi^c = 0$, so the growth rate is higher as compared to the one in the DE. This higher growth rate increases the effective discount factor, which in turn reduces the value of holding a monopoly position. Hence, $V_t > V_t^c$ so firms find holding a patent more lucrative in the DE as compared to the CE. Analytically, the level of amplification, $\omega - \omega^c$, equals the difference in the growth rates, $g^c - g = e^{-\theta}(M-1)$, divided by the discounted normalized profits, $\beta\pi/\eta$.

Proposition 4. *The market tightness in the decentralized economy is lower than that in the coordination economy, i.e. $\theta < \theta^c$.*

A proof is in the appendix. The coordination frictions reduce the economy-wide R&D intensity (as captured by the market tightness), even though the DE features a higher fraction of wasteful simultaneous innovation. This is the case because, for a given market tightness, the frictions reduce an entrant's chance of securing a monopoly position. In particular, the probability of securing a patent in the DE for a given tightness $\tilde{\theta}$, $Pr(\text{patent}|\tilde{\theta}) = (1 - e^{-\tilde{\theta}})/\tilde{\theta}$, is only a fraction $1 - e^{-\tilde{\theta}}$ of the one in the CE, $Pr(\text{patent}|\tilde{\theta})^c = 1/\tilde{\theta}$. As firms cannot coordinate their efforts, in the DE only a fraction $1 - e^{-\tilde{\theta}}$ of ideas are patented. Thus, even though the number of patent applications per idea, $\tilde{\theta}$, is the same in both economies, in the DE there are relatively less patents to be distributed among innovators. This decreases each entrant's chance of securing a monopoly position and subsequently reduces the incentives to enter the R&D sector. This is true even though the DE features a higher value of holding a patent. In other words, the effect of a lower probability of securing a patent dominates the effect of an increase in the net present value of profits and ultimately reduces the incentives to enter the R&D sector and decreases the market tightness. Furthermore, the decrease in the tightness provides an indirect channel through which the presence of foregone innovation reduces the growth rate in the DE—a lower tightness decreases each idea's chance of being innovated which results in a lower aggregate number of innovations.

The economic impact of coordination frictions is higher when profits are low, households are more inpatient, and the entry cost is higher. The next proposition states the result.

Proposition 5. *The level of amplification of wasteful innovation, $\omega - \omega^c$, and the amount by which the tightness is reduced, $\theta^c - \theta$ are*

- *decreasing in A and β*
- *increasing in η*

A proof is in the appendix. The fraction of foregone innovation depends only on the market tightness in the DE. When the tightness is low, the probability that an idea is not matched with any firm is high, which leads to a high fraction of foregone innovation and vice versa. The level of the amplification of wasteful innovation moves in the same direction as ϕ. This is because firms in the DE are willing to tolerate lower probability of securing a monopoly position only due to the higher value of holding a patent induced by $g < g^c$. As the fraction of foregone innovation decreases, the difference in the growth rates decreases as well. This reduces the incentives for firms to tolerate extra congestion, which decreases the amplification.

When ϕ is low, the incentives for firms in the DE to over-invest (as compared to the CE) induced by the difference in the growth rates is low as well. This generates an upward pressure on the difference in research intensities, $\theta^c - \theta$. At the same time, a smaller fraction of forgone innovation implies that, for a given market tightness, there are relatively more patents to be distributed among firms in the DE. Hence, $Pr(\text{patent}|\tilde{\theta})^c - Pr(\text{patent}|\tilde{\theta})$ decreases, which in turn increases the incentives for firms in the DE (relative to CE) to enter the R&D sector. Consequently, this generates a downward pressure on $\theta^c - \theta$. For a decrease in the fraction of foregone innovation induced by changes in A, β, or η the latter effect dominates the former.

4. The constrained planning problem

Next we turn to the planner's constrained efficient allocation—she chooses the optimal BGP allocation subject to the coordination frictions in the market for ideas. Without loss of generality, we impose symmetry in the intermediate varieties, i.e. $X_{t,n} = X_t(n')$ for any varieties n and n'. Thus, the planner faces the problem of choosing production of varieties, consumption, a number of varieties, a number of ideas, and a market tightness in order to maximize welfare subject to the resource constraint, the laws of motion for ideas and varieties, and the coordination frictions. The planner's problem, using a recursive formulation, is summarized below:

$$V^P(N_t, \nu_t) = \max_{C_t, X_t, \theta_t, N_{t+1}, \nu_{t+1}} \ln C_t + \beta V^P(N_{t+1}, \nu_{t+1})$$

$$\text{s.t.} \quad AL^{1-\lambda} N_t X_t^{\lambda} = N_t X_t + C_t + \eta \theta_t \nu_t, \tag{11}$$

$$N_{t+1} = N_t + \left(1 - e^{-\theta_t}\right) \nu_t, \tag{12}$$

$$\nu_{t+1} = \nu_t + (1 - e^{-\theta_t})(M-1)\nu_t. \tag{13}$$

Maximizing with respect to X_t yields the usual solution for varieties $X^* := X_t = (\lambda A)^{1/(1-\lambda)} L$. As in BSM the difference between the planner's solution and the decentralized outcome comes from the monopoly pricing of intermediate goods. Let $\pi^* = X^*(1-\lambda)/\lambda$ denote the implied per period monopoly profits at the efficient level of intermediate varieties. Then, it is straightforward to establish that the rest of the first order conditions can be reduced to the following system of equations:

$$\frac{h_t}{\phi_t} = \beta \frac{C_t}{C_{t+1}} \left(\pi^* + \frac{h_{t+1}}{\phi_{t+1}} \right), \tag{14}$$

$$\frac{\psi_t}{\phi_t} = \beta \frac{C_t}{C_{t+1}} \left(-\eta \theta_{t+1} + \left(1 - e^{-\theta_{t+1}}\right) \left(\frac{h_{t+1}}{\phi_{t+1}} + \frac{\psi_{t+1}}{\phi_{t+1}} (M-1) \right) + \frac{\psi_{t+1}}{\phi_{t+1}} \right), \tag{15}$$

$$\eta = e^{-\theta_t} \left(\frac{h_t}{\phi_t} + \frac{\psi_t}{\phi_t} (M-1) \right), \tag{16}$$

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where ϕ_t, h_t, ψ_t are the multipliers associated with (11), (12), and (13), respectively. The first equation above, (14), characterizes the planner's valuation of varieties: the value of a variety equals the discounted sum of per period profits, π^*, and the continuation value h_{t+1}/ϕ_{t+1}. There are only two differences as compared to the DE [equation (5)]—the level of profits is higher and the planner chooses a different tightness. The second equation, (15), characterizes how the planner values an idea: it is the discounted sum of several terms. The first term is the dividend, $-\eta\theta_{t+1}$, which represents the average R&D effort spent per idea. It captures the intuition that unlike other assets, which carry positive returns, an idea is only valuable if it is innovated. Hence, the planner finds it costly to keep a stock of ideas because it diverts resources away from consumption and into R&D. The second term represents the capital gain from innovation—the probability an idea is innovated, $(1 - e^{-\theta_{t+1}})$, times the social benefit from innovating. This benefit is the value of the extra variety, h_{t+1}/ϕ_{t+1}, plus the value of the extra ideas that would be added to the pool because of innovation, $\psi_{t+1}/\phi_{t+1}(M - 1)$. Lastly, the idea carries its continuation value ψ_{t+1}/ϕ_{t+1}. The last equation, (16), depicts the entry decision of the planner. She finds it optimal for firms to enter up until the expected social benefit from the marginal entrant is equal to the cost of innovating, η. The benefit that the entrant generates is the value of the extra variety and the extra ideas which come about from her innovation, times the probability she is the sole inventor, $e^{-\theta_t}$.[24]

Apart from the monopoly pricing of intermediate goods, there are two additional externalities in the model which are illustrated in equation (16). First, the ST externality manifests through the difference in the fraction of socially and privately beneficial innovations. The planner finds the marginal entry beneficial only if the firm is the sole inventor. Firms, on the other hand, value entry even if they duplicate an innovation, as long as they receive the patent for it. In particular, due to this "stepping-on-toes" effect, the probability of a privately beneficial innovation is $(1 - e^{-\theta_t})/\theta_t > e^{-\theta_t}$. Hence, the ST externality induces firms to over-invest in R&D as compared to the efficient allocation. Second, there is the SS externality—firms cannot appropriate the benefit of any ideas that come about from their innovations, so they do not value them. The planner, on the other hand, does because they permanently alleviate future coordination problems. Specifically, more innovation today increases the amount of future research avenues, which allows the economy to innovate more varieties without increasing the congestion problems. Thus, the extra ideas permanently reduce the cost of discovering new varieties.[25] As a result, the SS externality creates incentives for firms to under-invest as compared to the efficient allocation.

The ST and SS externalities in our model are tightly linked to the coordination frictions in the economy. Thus, it is instructive to compare our externalities to those studied in the labor search literature. To begin with, we can draw an analogy between the ST externality in our model and the congestion externality present in the baseline random search model of the labor market [Pissarides (2000), Chapter 8].[26] In both models, firms do not take into account that their entry increases congestion in the market, which makes it harder for other firms to get matched. In the labor market, the congestion externality induces entry to be either too high or too low depending on the way the firm and the worker share the surplus of the match. If the firm extracts too much of the surplus—there is too much entry in equilibrium, if it extracts too little—entry is too low.[27] When firms (or workers) direct their search, however, competition drives surplus shares for the firm and the worker to their efficient levels, so the congestion externality disappears [Julien et al. (2000); Shimer (2005); Shi (2001)]. In our model, on the other hand, the ST externality is present even though search is directed. This is because, in our setting, ideas are not active economic actors, so firms always extract the full amount of the surplus. In addition, because firms extract the full surplus of the firm-idea match, the ST externality always induces over-entry in equilibrium. Next, the SS externality in our model can be viewed as analogous to the thick-market externality studied in the labor search literature [see, for example, Diamond (1982) and Howitt and McAfee (1987)]. Both externalities have a similar flavor—if more agents participate in the market congestion is lower and therefore it is easier for matches to occur.[28] This thick-market externality operates through a matching function that has increasing returns to scale, so a larger market today leads

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to lower congestion today. In contrast, the SS externality is intertemporal—more R&D entrants today leads to higher levels of innovation, which reduces congestion tomorrow.

To study the magnitude of the externalities, we can decompose the difference between the planner's valuation of the benefit of entry at the efficient allocation, η, and the firm's valuation of this benefit, V times the probability of securing a patent. Using $\mathcal{M}$, $\mathcal{SS}$, and $\mathcal{ST}$ to denote the magnitude of the monopoly pricing, SS, and ST externalities, respectively, this decomposition is given by

$$\eta - \left(\frac{1-e^{-\theta}}{\theta}\right) V = \underbrace{\left(\frac{h}{\phi} - V\right)\left(\frac{1-e^{-\theta}}{\theta}\right)}_{\mathcal{M}} + \underbrace{e^{-\theta}(M-1)\frac{\psi}{\phi}}_{\mathcal{SS}} - \underbrace{\left(\frac{1-e^{-\theta}}{\theta} - e^{-\theta}\right)\frac{h}{\phi}}_{\mathcal{ST}}. \tag{17}$$

Conceptually, each of $\mathcal{M}/\mathcal{SS}/\mathcal{ST}$ measures how much more would the planner value the marginal entrant if monopoly pricing/SS/ST was the only externality in the model. In the case of $\mathcal{M}$, this quantity is given by the difference of the planner's valuation of the extra variety minus the firm's valuation, multiplied by the chance the entrant secures a monopoly position; in the case of $\mathcal{SS}$, it is the chance the entrant is the sole inventor times the number of extra ideas that are generated by this sole inventor times the value of each idea; in the case of $\mathcal{ST}$, it is given by the planner's valuation of the extra variety times the difference between the chance an entrant receives a patent and the chance she is the sole inventor.

The probability an entrant is the sole inventor and the probability she secures a patent are both endogenous objects determined by the coordination frictions. Consequently, since these probabilities determine $\mathcal{ST}$ and $\mathcal{SS}$, the frictions in our economy link the ST and SS externalities. To shed light on this novel link, which operates through the market tightness, use equations (16) and (14) to derive expressions for $\mathcal{SS}$ and $\mathcal{ST}$ that are explicit in θ:

$$\mathcal{SS} = \eta - \frac{e^{-\theta}\beta\pi^*}{1+(1-e^{-\theta})(M-1)-\beta}, \tag{18}$$

$$\mathcal{ST} = \left(\frac{1-e^{-\theta}}{\theta} - e^{-\theta}\right)\frac{\beta\pi^*}{1+(1-e^{-\theta})(M-1)-\beta}. \tag{19}$$

The ST externality is relatively small when the tightness is either small or large. For intermediate values of θ, it is relatively large. Intuitively, when the tightness is low, there is a small chance that the marginal entrant would compete with another firm for the patent. Thus, the probability of securing a monopoly position is essentially the probability of being the sole inventor, both of which are close to 1. When the tightness is large, there is little chance the marginal entrant will be the sole inventor. At the same time, the firm also expects to win the race with a small probability because the market is congested and she is likely to face many rivals. Thus, both probabilities are close to 0. When the tightness is in an intermediate range, on the other hand, the entrant expects to see some competition for the patent but the expected number of rivals is not too high. This leaves her with a high chance of securing a patent even when she is not the sole inventor. This drives a wedge between the two probabilities and consequently increases $\mathcal{ST}$.

A higher market tightness, on the other hand, always leads to an increase in the magnitude of the SS externality. As θ increases two opposing effects impact $\mathcal{SS}$. First, the chance that an entrant would be the sole inventor decreases. Thus, the probability that any new ideas would be added to the pool as a consequence of the firm's entry decreases. This effect tends to reduce $\mathcal{SS}$. Second, a higher tightness implies that ideas are relatively more scarce and so the value of the new ideas generated by the entrant is larger. This effect tends to increase $\mathcal{SS}$. Given the equilibrium urn-ball matching technology, it turns out that the latter effect dominates the former. Hence, the magnitude of the SS externality is increasing in the market tightness.

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Furthermore, we find that the magnitudes of the two externalities are such that $\mathcal{ST} > \mathcal{SS}$ holds for all parameter values. To see this, first observe that along the BGP the planner's allocation is characterized by

$$\left(\frac{\nu}{N}\right)^* = M - 1, \tag{20}$$

$$\left(\frac{C}{N}\right)^* = \pi^* - \eta\theta^*(M-1), \tag{21}$$

$$1 + \left(1 - e^{-\theta^*}\right)(M-1) = \beta\left(1 + \frac{\pi^*}{\eta}e^{-\theta^*} + (1 - e^{-\theta^*} - \theta^* e^{-\theta^*})(M-1)\right). \tag{22}$$

Then, from equations (14) and (22), it follows that $\mathcal{ST} > \mathcal{SS}$ if and only if $\pi^* - \eta\theta^*(M-1) > 0$. But this has to hold, from equation (21), as the efficient allocation must feature $C_t > 0$.

Proposition 6. *The absolute value of the ST externality is larger than that of the SS externality.*

The proof is in the preceding text. Intuitively, congestion in the economy is undesirable because it diverts resources away from consumption and into duplicative simultaneous innovation. Foregone innovation, on the other hand, is undesirable because it leaves some ideas uninnovated and, as a consequence, congestion tomorrow is higher than it could have been otherwise. Thus, reducing congestion is of first-order importance to the planner, whereas reducing foregone innovation only comes second. This intuition is also demonstrated by the behavior of the SS externality as we vary the market tightness. A higher tightness leads to a higher $\mathcal{SS}$ because congestion is high (which makes ideas more valuable), even though foregone innovation (the chance the firm is the sole inventor) is low. Thus, when the planner picks an optimal market tightness she does so by striking a balance between congestion and foregone innovation in favor of low congestion. As a result, the ST externality dominates the SS one. Since the planner chooses a market tightness that delivers low congestion for any set of parameter values, $\mathcal{ST} > \mathcal{SS}$ holds always.

We should note, however, that the results in Proposition 6 rely on our assumptions regarding the nature of competition between firms for innovations. Under alternative assumptions the size of the ST externality may be lower (or even zero), in which case it may be possible for the SS externality to dominate the ST one for some parameter values. First, our analysis focuses exclusively on the rivalrous nature of innovation. Thus, we have not considered any potential positive spillover effects that might be present when several firms simultaneously innovate the same idea. For example, Gabrovski (2020) considers a framework where ideas invented by more firms simultaneously contribute, on average, more new ideas to the future pool of ideas. In that context congestion decreases the cost of future innovation (in the same spirit as the SS externality) and, as a consequence, the size of the ST externality is lower. Second, we have assumed that firms receive patents for their innovations. In reality many firms rely instead on secrecy to protect their intellectual property (see, for example, Hall et al. (2014) and the references therein). Under the alternative assumption that firms protect their innovations through secrecy, the size of the ST externality would depend on the nature of competition between firms. For example, if we assume competition is such that firms which innovate simultaneously share the monopoly profits from the new variety, then the size of the ST externality would be the same as in our baseline setting with patents.[29] On the other hand, if we assume Bertrand competition firms only expect to receive positive profits if they are the sole inventor, i.e. with probability $e^{-\theta}$. Thus, competition among firms reduces the surplus each firm can extract from the innovation, so the ST externality disappears completely.[30]

Since the ST externality dominates the SS one, implementing the efficient allocation in the DE requires the government to impose a tax on R&D investment.[31] In particular, suppose that the government imposes a subsidy on the purchases of intermediate varieties at a rate s and a tax on R&D activities at a rate τ. Furthermore, if the government keeps a balanced budget through the means of lump-sum transfers, then the optimal policy is summarized below.

Proposition 7. *The optimal subsidy on the purchase of intermediate varieties is given by $s^* = 1 - \lambda$. The optimal tax rate on R&D entry is given by*

$$\tau^* = \frac{\beta\pi^*(1 - e^{-\theta^*})}{\eta\theta^*[1 + (1 - e^{-\theta^*})(M - 1) - \beta]} - 1 > 0. \tag{23}$$

A proof is in the appendix.

5. Conclusion

In practice firms have technological access to a scarce number of research avenues. They choose an avenue towards which to direct their R&D efforts but because of informational issues cannot coordinate their choices. Motivated by these observations, we develop an expanding-variety endogenous growth model to study the implications of coordination frictions for growth and welfare. Our analysis shows the frictions generate a number of foregone innovations and reduce the economy-wide research intensity. Both of these effects reduce the economy's growth rate. This leads to a general equilibrium effect that endogenously amplifies the fraction of wasteful simultaneous innovation as compared to a frictional economy where firms can coordinate their R&D activities. When we turn our attention to welfare we find that formalizing the coordination frictions uncovers a novel link between the "stepping on toes" and "standing on shoulders" externalities. This link, which operates through the market tightness, determines the relative sizes of the two externalities. Either one may dominate the other in size, depending on the level of congestion in the economy. However, we show that at the efficient allocation the ST externality is larger for all parameter values. Thus, implementing the planner's allocation requires imposing a tax on R&D investment.

Notes

1 For example, during 2015 the U.S. Patent and Trademark Office granted more than quarter of a million patents.

2 Patents are declared in interference if two innovators file for the same patent within 3 months of each other (6 months for major innovations).

3 Siemens applied for a patent for a positron emission tomography scanner on April 23, 2013 (application number 13/868,256). Most claims are rejected because Philips (application number 14/009,666 filed on March 29, 2012 and application number 14/378,203 filed on February 25, 2013) had simultaneously made similar innovations. Google Inc. filed a patent application on November 1, 2012 (number 13/666,391) for methods, systems, and apparatus that provide content to multiple linked devices. All twelve claims contained in the application are rejected because of simultaneous innovations made by Yahoo! Inc. (application number 13/282,180 with filing date October 26, 2011), Microsoft Corporation (application number 13/164,681 with filing date June 20, 2011), and Comscore Inc. (application number 13/481,474 with filing date May 25, 2012). The information on the patent applications is taken from the U.S. Patent and Trademark Office Patent Application Information Retrieval.

4 Zeira (2011) provides an alternative way to formalize the "stepping on toes" externality. In that study, the externality arises from the interaction between two assumptions: innovations take time to find and the search cost differs across innovations. In contrast, the current study formalizes this externality through coordination frictions. This allows us to also endogenize the "standing on shoulders" externality as well as uncover a novel link between the two externalities.

5 This is a standard approach the existing literature has used to capture coordination issues. See, for example, Julien et al. (2000), Burdett et al. (2001), Shimer (2005).

6 In practice, it may be socially beneficial to have several firms innovate the same project simultaneously. This could be the case because the process of innovation is uncertain or because of knowledge spillovers. However, our paper focuses the wasteful nature of duplication and so we abstract the analysis from any positive spillovers associated with duplication. For a paper that does explore the economic implications of knowledge spillovers associated with simultaneous innovation see Gabrovski (2020).

7 This effect is similar to the information externality identified in Akcigit and Liu (2016). In their paper firms may abandon a risky research project too early because they lack information on what their rivals have learned in the process of R&D. In our setting, in contrast, some projects are not undertaken at all because firms are unsure which projects their rivals are working on.

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8 Existing growth models have focused instead on a search process which takes the form of arrival rate of innovations [e.g. Aghion and Howitt (1992) and Klette and Kortum (2004)], a McCall-type search for innovations [e.g. Perla and Tonetti (2014), and Lucas and Moll (2014)], or frictions in the market for innovations [e.g. Chiu et al. (2017) and Akcigit et al. (2016)].
9 The literature has also examined extensively issues related to R&D investment that is not necessarily duplicative. For recent examples, see Davis and Hashimoto (2015), Morimoto and Tabata (2020), Yang (2021), and Chu and Wang (2022) among others.
10 For a review of the literature see, for example, Aghion et al. (2014). In a recent study, Hopenhayn and Squintani (2021) also examine the implications of duplicative R&D investment for welfare. Their model features the ST externality as well but, in contrast to our economy, theirs does not feature growth, the number of researchers is fixed, and the number of new ideas available to researchers is exogenously determined.
11 In Julien et al. (2000) it is workers who announce their reservation wages and firms who direct their search towards a specific worker.
12 Alternatively, see Acemoglu, (2009), Chapter 13.
13 See, for example, Julien et al. (2000), Burdett et al. (2001), and Shimer (2005).
14 Our paper focuses on the rivalrous aspect of coordination frictions. To this end, the analysis abstracts from any potential positive spillovers when several firms innovate the same project simultaneously. For a paper that does explore the economic consequences of such positive spillovers see Gabrovski (2020).
15 Each innovation is protected by a patent, so no firm has an incentive to imitate at a late date. Thus, the idea no longer represents a profitable R&D project and as a consequence it is no longer in ν_{t+1}.
16 See, for example, Wolinsky (1988), Lu and McAfee (1996), Julien et al. (2000), Burdett et al. (2001), and Shimer (2005).
17 See, for example, Julien et al. (2000) and Shimer (2005).
18 Only one firm can hold a patent over a certain variety. Hence, whenever $m \geq 1$ firms innovate the same idea, $m-1$ of them make a wasteful duplicative innovation. Each entrant makes an innovation, so the total number of innovations is μ_t. The total number of useful innovations equals the total number of new varieties, $(1-e^{-\theta_t})\nu_t$. Thus, the fraction of innovations which represent wasteful duplication of effort is simply $1-(1-e^{-\theta_t})/\theta_t$.
19 In our analogy of the labor market, this probability represents the chance no worker applies for a specific vacancy (Shimer, 2005).
20 We restrict the analysis to a set of parameter values which ensures that firms have an incentive to enter the R&D sector, i.e. $\eta < (1-\lambda)\beta(\lambda^2 A)^{1/(1-\lambda)}L/[\lambda(M-\beta)]$.
21 A proof is available upon request.
22 The average cost of securing a monopoly position is $\eta/Pr(\text{monopoly}) = \eta\theta/(1-e^{-\theta})$, which is decreasing in ν_t.
23 An explicit characterization of the CE's BGP is contained within the proof of Proposition 3.
24 If the firm is not the sole inventor then the innovation would have been made even if the firm did not enter the R&D sector. Hence, the firm's entry did not generate any social benefit.
25 The average cost of discovering one new variety is $\eta/Pr(\text{soleinventor}) = \eta e^{\theta_t}$, which is decreasing in the number of ideas.
26 The congestion externality is present in models outside of the labor market context. See Gabrovski and Ortego-Marti (2021) for a discussion of the congestion externality in a search model of the housing market.
27 If the firm extracts just the right fraction of the surplus (the firm's bargaining power equals the elasticity of the matching function with respect to vacancies) then entry is at the socially efficient level. This is the well-known Hosios-Mortensen-Pissarides condition (Hosios, 1990; Pissarides, 2000).
28 The literature has also identified a thick-market externality that operates through the quality of the match, i.e. matches are of generally higher quality in larger markets. See Diamond (1981), Petrongolo and Pissarides (2006), Gautier and Teulings (2009), and Ngai and Tenreyro (2014).
29 To see this, observe that in this scenario an entrant's expected profits are still given by the right hand side of equation (8).
30 This is analogous to the reason why in labor models with directed search the equilibrium is constrained efficient: competition among firms drives the surplus that firms can extract from the match to the socially efficient level.
31 Even though it is optimal to impose a tax on R&D spending, it may be the case that the decentralized economy suffers from under-investment, i.e. $\theta < \theta^*$. This is due to the monopoly distortion induced by patents. Whether or not there will be under-investment in equilibrium depends on parameter values.

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APPENDIX: Proofs omitted from the text

Proof of Proposition 1

Proof. We follow the literature on coordination frictions in the labor market and treat our economy with a large number of ideas of ideas as the limiting case of a finite-idea economy [see, for example, Shi (2001), Shi (2005), Julien et al. (2000), Julien et al. (2018), Gomis-Porqueras et al. (2017), Shimer (2005), Julien et al. (2014)]. In particular [as in Julien et al. (2000) and Burdett et al. (2001)], we characterize the equilibrium when the number of ideas and firms is finite and then evaluate the resulting equilibrium outcome in the limit as the number of projects tends to infinity (keeping the market tightness constant).

First, by assumption, the firm's probability of securing a monopoly position given that there are exactly n rivals, $Pr(\text{monopoly}|n) = 1/(n+1)$. In a symmetric equilibrium all firms place the same probability s_i of directing their effort towards a particular idea i. Then, the chance that a firm would face exactly n rivals is

$$Pr(n) = \binom{\mu_t - 1}{n} s_i^n (1 - s_i)^{\mu_t - 1 - n}.$$

Hence, the probability of securing a monopoly position is given by

$$\begin{aligned}
Pr(\text{monopoly}) &= \sum_{n=0}^{\mu_t - 1} Pr(\text{monopoly}|n)P(n) = \sum_{n=0}^{\mu_t - 1} \binom{\mu_t - 1}{n} s_i^n (1 - s_i)^{\mu_t - 1 - n} \frac{1}{n+1} \\
&= \frac{1}{\mu_t} \sum_{n=0}^{\mu_t - 1} \binom{\mu_t}{n+1} s_i^n (1 - s_i)^{\mu_t - 1 - n} = \frac{1}{\mu_t s_i} \left(\sum_{n=0}^{\mu_t} \binom{\mu_t}{n} s_i^n (1 - s_i)^{\mu_t - n} - (1 - s_i)^{\mu_t} \right) \\
&= \frac{1 - (1 - s_i)^{\mu_t}}{\mu_t s_i}.
\end{aligned} \tag{24}$$

Next, we show that $s_k = s_j$ for all $k, j \in \nu_t$. Suppose not. Then, there exists some k, j such that $s_k > s_j$. But for any $i \in \nu_t$, we have that

$$\frac{\partial Pr(\text{monopoly})}{\partial s_i} = \frac{\mu_t^2 s_i (1 - s_i)^{\mu_t - 1} - \mu_t [1 - (1 - s_i)^{\mu_t}]}{(\mu_t s_i)^2}. \tag{25}$$

For any $s_i \in (0, 1)$, it follows that $Pr(\text{monopoly})$ is decreasing in s_i if and only if $(1 - s_i)^{\mu_t - 1} < Pr(\text{monopoly})$ which clearly holds since $\mu_t \geq 2$. Now, for $s_i = 1$, we have that $\partial Pr(\text{monopoly})/\partial s_i = -1/\mu_t < 0$. Furthermore, it is easy to see that

$\lim_{s_i \to 0} \partial Pr(\text{monopoly})/\partial s_i = -(\mu_t - 1)/2 < 0$. Hence, $Pr(\text{monopoly})$ is decreasing in s_i everywhere in its domain. Then, $s_k > s_j$ implies that $Pr_k(\text{monopoly}) < Pr(\text{monopoly})_j$, which then implies that $Pr_k(\text{monopoly})V_{k,t} < Pr_j(\text{monopoly})V_{j,t}$ since all varieties are equally profitable. Thus, $s_k > s_j$ cannot be an equilibrium. Hence, we must have $s_i = s_j = 1/\nu_t$ for all $i, j \in \nu_t$. Finally, it is evident that no firm has an incentive to deviate from that strategy since the expected payoff from innovating an idea is the same for all research avenues.

Then, it follows that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) = \binom{\mu_t}{n}\left(\frac{1}{\nu_t}\right)^n\left(1 - \frac{1}{\nu_t}\right)^{\mu_t - n}. \tag{26}$$

Taking the limit as $\mu_t, \nu_t \to \infty$ (keeping the ratio θ_t constant) we get that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) \to \frac{\theta_t^n e^{-\theta_t}}{n!}. \tag{27}$$

□

Proof of Proposition 2

Proof. Totally differentiating both sides of (9) with respect to π yields

$$\frac{\theta}{\pi} = \frac{\beta}{\eta}\left(\frac{1 - e^{-\theta}}{\theta}\right)\left[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\left(\frac{1 - e^{-\theta} - \theta e^{-\theta}}{\theta^2}\right)\right]^{-1} > 0, \tag{28}$$

which is positive since $1 - e^{-\theta} - \theta e^{-\theta} > 0$. As $\frac{\theta}{A} = \frac{\theta}{\pi} \times \frac{\pi}{A}$ and profits, π, are increasing in productivity, A, it follows that $\frac{\theta}{A} > 0$. Similarly, totally differentiating (9) with respect to β, η, and M yields

$$\frac{\theta}{\beta} = \left[1 + \frac{\pi}{\eta}\left(\frac{1 - e^{-\theta}}{\theta}\right)\right]\left[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\left(\frac{1 - e^{-\theta} - \theta e^{-\theta}}{\theta^2}\right)\right]^{-1} > 0, \tag{29}$$

$$\frac{\theta}{\eta} = -\frac{\beta\pi}{\eta^2}\left(\frac{1 - e^{-\theta}}{\theta}\right)\Bigg]\left[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\left(\frac{1 - e^{-\theta} - \theta e^{-\theta}}{\theta^2}\right)\right]^{-1} < 0, \tag{30}$$

$$\frac{\theta}{M} = -\left(1 - e^{-\theta}\right)\left[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\left(\frac{1 - e^{-\theta} - \theta e^{-\theta}}{\theta^2}\right)\right]^{-1} < 0. \tag{31}$$

□

Proof of Proposition 3

Proof. Let us first explicitly characterize the BGP of the CE. The assumption we have placed on the parameter vales ensures that firms find all research avenues profitable. Hence, in equilibrium, all ideas are innovated, i.e. $\mu_t \geq \nu_t$, and each firm secures a patent with probability $Pr(\text{monopoly}) = 1/\theta_t$. Hence, the laws of motion for ideas and varieties are given by

$$\nu_{t+1} = M\nu_t, \tag{32}$$

$$N_{t+1} = N_t + \nu_t. \tag{33}$$

Since the final good sector and the intermediate varieties production technology are as in the DE, it follows that in equilibrium it is still the case that $P_{t,n} = 1/\lambda$, $X = (\lambda^2 A)^{1/(1-\lambda)}L$, $Y_t = (\lambda^{2\lambda}A)^{1/(1-\lambda)}LN_t$, $\pi = X(1-\lambda)/\lambda$, $V_t^c = \sum_{i=t+1}^{\infty} d_{it}\pi$. As all ideas are equally productive, the free entry condition is now given by $\eta = V_t^c/\theta_t$. Moreover, consumers face the same problem as in the DE. Then, it is easy to see that $g_Y = g_C = g_N = g_\mu = g_\nu$. However, now from the law of motion for ideas, it follows that $g_\nu = M - 1$. Next, using the laws of motion for ideas and varieties, it follows that along the BGP we still have, $\nu/N = M - 1$. Furthermore, from the resource constraint, it follows that $C/N = (1+\lambda)\pi/\lambda - \eta\theta^c(M-1)$. Lastly, using the free entry condition and the Euler equation, it follows that the market tightness is given by $\theta^c = \beta\pi/[\eta(M-\beta)]$.

Next, we can compare the percent of wasteful innovations in the two economies. In the CE there are μ_t innovations and ν_t of those are beneficial. Hence, $\omega^c = 1 - 1/\theta^c$. Then, it follows that

$$\omega - \omega^c = \frac{\eta(M-\beta)}{\beta\pi} - \frac{\eta(1+g-\beta)}{\beta\pi} = \frac{e^{-\theta}(M-1)\eta}{\beta\pi} > 0. \tag{34}$$

□

Proof of Proposition 4

Proof. From $\theta^c = \beta\pi/[\eta(M-\beta)]$ it follows that

$$\frac{\theta^c}{1-e^{-\theta}} = \frac{\beta\pi}{\eta(M-\beta)(1-e^{-\theta})} > \frac{\beta\pi}{\eta(1+(1-e^{-\theta})(M-1)-\beta)} = \frac{\theta}{1-e^{-\theta}}, \tag{35}$$

where the inequality follows because $\beta < 1 \Rightarrow 1+(1-e^{-\theta})(M-1)-\beta > (M-\beta)(1-e^{-\theta})$. Hence, $\theta^c > \theta$. □

Proof of Proposition 5

Proof. The results for the fraction of foregone innovation are immediate from Proposition 2. Next, let us look at the difference in the fraction of wasteful simultaneous innovation. Totally differentiating equation (10) with respect to π, β, η, and M yields

$$\frac{(\omega-\omega^c)}{\pi} = -\frac{\omega-\omega^c}{\pi} - (\omega-\omega^c)\frac{\theta}{\pi} < 0, \tag{36}$$

$$\frac{(\omega-\omega^c)}{\beta} = -\frac{\omega-\omega^c}{\beta} - (\omega-\omega^c)\frac{\theta}{\beta} < 0, \tag{37}$$

$$\frac{(\omega-\omega^c)}{\eta} = \frac{\omega-\omega^c}{\eta} - (\omega-\omega^c)\frac{\theta}{\eta} > 0, \tag{38}$$

$$\frac{(\omega-\omega^c)}{M} = \frac{\eta e^{-\theta}}{\beta\pi} - (\omega-\omega^c)\frac{\theta}{M} > 0. \tag{39}$$

Furthermore, $\frac{(\omega-\omega^c)}{A} = \frac{(\omega-\omega^c)}{\pi} \times \frac{\pi}{A} > 0$.

Next, let us look in the difference in the market tightness. Given $\theta^c = \beta\pi/(\eta(M-\beta))$, it follows that

$$\frac{\theta^c}{\pi} = \frac{\beta}{\eta(M-\beta)}, \tag{40}$$

$$\frac{\theta^c}{\eta} = -\frac{\beta\pi}{\eta^2(M-\beta)} = -\left(\frac{\pi}{\eta}\right)\frac{\theta^c}{\pi}, \tag{41}$$

$$\frac{\theta^c}{\beta} = \frac{M\pi}{\eta(M-\beta)^2} = \left(\frac{M\pi}{\beta(M-\beta)}\right)\frac{\theta^c}{\pi}. \tag{42}$$

Then, using equations (27) and (39) and straightforward algebra, it follows that

$$\frac{(\theta^c - \theta)}{\pi} < 0. \tag{43}$$

Hence, $\frac{(\theta^c-\theta)}{A} = \frac{(\theta^c-\theta)}{\pi} \times \frac{\pi}{A} < 0$.
Similarly, equations (29) and (40) imply that

$$\frac{(\theta^c - \theta)}{\eta} = -\left(\frac{\pi}{\eta}\right)\frac{(\theta^c - \theta)}{\pi} > 0. \tag{44}$$

Lastly, equation (28) implies that

$$\frac{\theta}{\beta} = \left(\frac{\eta\theta}{\beta(1-e^{-\theta})}\right)\left(1 + \frac{\pi}{\eta}\left(\frac{1-e^{-\theta}}{\theta}\right)\right)\frac{\theta}{\pi},$$

$$\frac{\theta}{\beta} = \left(\frac{\pi}{\beta} + \frac{\pi}{1-\beta+g}\right)\frac{\theta}{\pi},$$

$$\frac{\theta}{\beta} = \left(\frac{(1+g)\pi}{\beta(1-\beta+g)}\right)\frac{\theta}{\pi}. \tag{45}$$

Since $(1+g)\pi/(\beta(1-\beta+g)) > M\pi/(\beta(M-\beta))$, it follows that

$$\frac{(\theta^c - \theta)}{\beta} < \left(\frac{M\pi}{\beta(M-\beta)}\right)\frac{(\theta^c - \theta)}{\pi} < 0. \tag{46}$$

□

Proof of Proposition 7

Proof. The government imposes a tax on R&D activities at a rate τ and subsidizes the purchase of intermediate varieties at a rate s. Furthermore, it keeps a balanced budget through the means of lump-sum transfers to households in the amount T_t. Thus, the government's budget constraint is given by $T_t = \sum_{n=0}^{N_t} sP_{t,n}X_{t,n} - \tau\eta\mu_t$.

The final good firm chooses labor and intermediate inputs to maximize profits, now given by $Y_t - w_tL - \sum_{n=0}^{N_t}(1-s)P_{t,n}X_{t,n}$. The first order conditions yield the same labor demand equation as in the DE.

At stage three of the innovation process, the monopolist faces an analogous problem as in the DE. The only difference now is in the inverse demand function. Hence, in equilibrium, $P = 1/\lambda$, $X = [A\lambda^2/(1-s)]^{1/(1-\lambda)}L$, $\pi = (1-\lambda)X/\lambda$, $Y_t = [A(\lambda^2/(1-s))^{\lambda}]^{1/(1-\lambda)}LN_t$.

As in the economy without government intervention, all ideas are equally profitable, so the matching technology is as in the DE. The free entry condition is now given by

$$\eta(1+\tau) = \frac{1-e^{-\theta_t}}{\theta_t}V_t, \tag{47}$$

where the value of the monopoly position, V_t, is defined as in the DE.

The laws of motion for ideas and varieties, and the Euler equation are as in the DE. Hence, along the BGP, we still have that $\nu_t/N_t = M - 1$. Then it is easy to see that the solution for C/N and the tightness is given by

$$\frac{C}{N} = \frac{1-s-\lambda^2}{(1-\lambda)\lambda}\pi - \eta\theta(M-1), \tag{48}$$

$$1 + (1-e^{-\theta})(M-1) = \beta\left(1 + \frac{\pi}{\eta(1+\tau)}\left(\frac{1-e^{-\theta}}{\theta}\right)\right). \tag{49}$$

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Then, setting $s = s^*$ implies that $\pi = \pi^*$ and setting $\tau = \tau^*$ implies that $\theta = \theta^*$. Thus, $C/N = (C/N)^*$.

To see that the optimal tax rate is positive, observe that $\eta\tau^* = \mathcal{ST}^* - \mathcal{SS}^*$, where the star subscript indicates the magnitude of the externalities is evaluated at the socially optimal level of the tightness. Then, by Proposition 6, $\tau^* > 0$. □

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