A COMPARATIVE STUDY OF GLOBULAR CLUSTERS

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Abstract. This paper is intended for an introduction to the Symposium. In 47 Tuc many milli-second pulsars are found while none has been reported in ω Cen. One might think that in 47 Tuc they have been formed in the collapsed core through tidal capture of a main sequence star by a neutron star. If we use the standard model of gravothermal collapse of globular clusters to integrate the squared stellar density over the core and over its time history, we find, however, the accumulated probability of tidal capture is lower in 47 Tuc than ω Cen. Such contradiction suggests that it will be important to take account of mass segregation as well as stellar evolution in modelling dynamical evolution of star clusters.

1. Introduction

As an introduction to this Symposium I will compare two typical globular clusters, 47 Tuc and ω Cen with each other. The former, 47 Tuc, has a collapsed core with a density cusp. It has 11 milli-second pulsars (Manchester *et al.* 1991) and 21 blue stragglers (Paresce *et al.* 1991). In contrast, the latter, ω Cen, which is the largest globular cluster in the Galaxy has not been reported to contain any pulsars or blue stragglers, though there is a binary star NJL 5 as a cluster member (Margon and Cannon 1990). The existence of pulsars is also reported for the globular clusters having collapsed cores such as M15 and M5, for instance, as summarized by Lyne (1994).

A naive explanation of such observational facts was as follows. The pulsars may have originated from binary stars. The binary star may have been formed by tidal capture of a star encountering close to another star.

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Therefore, more number of binary stars would have been formed in a denser core of the globular cluster.

I will show that it can not be so simple, however. If we look into the theory of a standard gravothermal collapse and integrate the binary formation rate over space and time, we will find that the number of accumulated binaries should be larger in ω Cen than in 47 Tuc because the former has a larger number of stars in its core. Something must be wrong in the model and the integration above.

2. Model of Gravothermal Collapse

Here we use the self-similar solution obtained by Lynden-Bell and Eggleton (1980). The radial density distribution of mass outside the core is expressed as

$$\rho(r,t) = \rho_{\rm c}(\tau)\rho_*(\xi). \tag{1}$$

Here, ξ is the spatial coordinate normalized with the core radius $r_{\rm c}$ as

$$\xi = \frac{r}{r_{\rm c}(\tau)}, \tag{2}$$

and τ is the time remaining to the complete collapse which is expressed using the present time t and the time at the complete collapse $t_{\rm coll}$ as

$$\tau = t_{\rm coll} - t. \tag{3}$$

Similar expressions hold for other quantities such as the mass $M(r,t)$ contained within the sphere of radius r, and for the *rms* velocity of the stars $v(r,t)$ which is proportional to square root of the temperature in a gaseous model.

The similarity solution (Lynden-Bell and Eggleton 1980) is described for their spatial distributions in the halo as

$$\rho_*(\xi) \sim \xi^{-\alpha} = \xi^{-2.21}, \tag{4}$$

$$M_*(\xi) \sim \xi^{3-\alpha} = \xi^{-0.79}, \tag{5}$$

$$v_*(\xi) \sim \left(\frac{M_*}{\xi}\right)^{\frac{1}{2}} \sim \xi^{-\frac{\alpha-2}{2}} = \xi^{-0.11}, \tag{6}$$

where $\alpha = 2.21$ is used to obtain the extreme right-hand sides of the above relations. The temporal changes of the core radius and the other quantities are expressed as

$$r_{\rm c}(\tau) \sim \tau^{\frac{2}{6-\alpha}} = \tau^{0.53}, \tag{7}$$

$$\rho_{\rm c}(\tau) \sim \tau^{-\frac{2\alpha}{6-\alpha}} = \tau^{-1.17}, \tag{8}$$

$$M_c(\tau) \sim \tau^{\frac{6-2\alpha}{6-\alpha}} = \tau^{0.42}, \tag{9}$$

$$v_c(\tau) \sim \tau^{-\frac{\alpha-2}{6-\alpha}} = \tau^{-0.055}. \tag{10}$$

3. Formation Rate of Binaries

The local formation rate of binaries is given by Spitzer (1987, eq. 6-43) as

$$\frac{dn_b}{dt} = 10k \left(\frac{n}{10^4\,\mathrm{pc}^{-3}}\right)^2 \left(\frac{M}{M_\odot}\right)^{1+\frac{\mu}{2}} \times \left(\frac{R}{R_\odot}\right)^{1-\frac{\mu}{2}} \left(\frac{10\,\mathrm{km\,s}^{-1}}{v}\right)^{1+\mu} \mathrm{pc}^{-3}\mathrm{Gyr}^{-1}. \tag{11}$$

Here, n is the number density of the stars which is proportional to ρ, and M and R are the mass and the radius of the individual star. The parameters k and μ depend only on the stellar structure. We shall use the values for a polytrope of index 1.5, *i.e.*, k=2.1 and μ=0.12. From equations (4) and (11) we see that

$$\left(\frac{dn_b}{dt}\right) 4\pi r^3 \sim r^{-2\alpha+3+\frac{(1+\mu)(\alpha-2)}{2}} = r^{-1.30}, \tag{12}$$

implying that the formation of binaries takes place mainly in the core rather than in the outer region of the globular cluster.

After some calculations using the relations above, the binary formation rate in the core is estimated as

$$\left(\frac{dN_b}{dt}\right)_{\mathrm{core}} \simeq 30 \left(\frac{N_c}{10^5}\right) \left(\frac{n_c}{10^3\,\mathrm{pc}^{-3}}\right) \left(\frac{10\,\mathrm{km\,s}^{-1}}{v_c}\right)^{1.12} \mathrm{Gyr}^{-1} \sim \tau^{a-1} = \tau^{-0.69}, \tag{13}$$

where the power index to τ is expressed as

$$a - 1 = \frac{6 - 4\alpha + (1+\mu)(\alpha - 2)}{6 - \alpha}. \tag{14}$$

The negative value of $a - 1 = -0.69$ implies that more binaries are formed in the later stages when evaluated *per unit time*. However, the positive value of $a = 0.31$ implies that the later stages are shorter and that the binaries are *accumulated* rather during the earlier stages while τ is still large.

Integrating equation (13) backward from the present τ_0 to the birth of the globular cluster τ_1 , *i.e.*, over the age of the globular cluster,

$$t_{\mathrm{age}} = \tau_1 - \tau_0, \tag{15}$$

we obtain the number of binaries formed so far,

$$\begin{aligned} N_{\mathrm{b}} &= \int_{\tau_0}^{\tau_1} \left(\frac{dN_{\mathrm{b}}}{dt}\right) d\tau \\ &= \left(\frac{dN_{\mathrm{b}}}{dt}\right)_0 \tau_0 \int_0^{\ln(\tau_1/\tau_0)} \left(\frac{\tau}{\tau_0}\right)^a d\ln(\tau/\tau_0) = \left(\frac{dN_{\mathrm{b}}}{dt}\right)_0 \tau_{\mathrm{eff}}, \end{aligned} \tag{16}$$

where the effective time τ_{eff} is expressed as

$$\tau_{\mathrm{eff}} = \frac{\tau_0}{a}\left[\left(1+\frac{t_{\mathrm{age}}}{\tau_0}\right)^a - 1\right]$$

$$= \begin{cases} \left(\frac{t_{\mathrm{age}}}{a}\right)\left(\frac{\tau_0}{t_{\mathrm{age}}}\right)^{1-a}, & \text{for } t_{\mathrm{age}} >> \tau_0, \\ t_{\mathrm{age}}, & \text{for } t_{\mathrm{age}} << \tau_0. \end{cases} \tag{17}$$

4. Comparison between 47 Tuc and ω Cen

For illustrative purpose we apply the results in the preceding section to Model No. 14 of 47 Tuc by Meylan (1988) and to Model No. 13 of ω Cen by Meylan (1987). Though there are other models (*e.g.*, Chernoff and Djorgovsky 1989), and though the estimates given in the preceding section are rather rough, the following discussions will not be altered so far as the qualitative conclusions are concerned. The relevant quantities of Meylan's models are summarized in Table 1. Here, the notations have the following meanings; M_{tot} the total mass of the globular cluster, ρ_{h} the density at the half-mass radius, v_{s} the velocity scale corresponding to the velocity dispersion, $t_{\mathrm{r,h}}$ the two-body relaxation time at the half-mass, and $t_{\mathrm{r,c}}$ the relaxation time in the core of the globular cluster.

Quantities derived using the relations in the preceding sections are summarized in Table 2. Here, the time left to the complete collapse τ_0 is calculated using equation (4-17) of Spitzer (1987), *i.e.*, using

$$\tau_0 = 190\, t_{\mathrm{r,c}}. \tag{18}$$

To convert the mass density to the number density of the stars we used the mass at the main sequence turn-off which is 0.79 $M_\odot$ both for 47 Tuc and ω Cen. We see that τ_{eff} is much shorter than t_{age} for 47 Tuc while it is almost equal to t_{age} for ω Cen. This implies that the binaries are being formed much faster than the past in 47 Tuc while almost at the same formation rate in ω Cen. We see in Table 2 that the number of binaries formed so far, *i.e.*, N_{b}, is calculated to be too large.

Table 1. Model of globular clusters by Meylan (1988, 1987)

cluster	47 Tuc	ω Cen
Model No.	14	13
$t_{\rm age}$ (Gyr)	16	15
$M_{\rm tot}$ ($10^6 M_\odot$)	0.67	3.33
r_c (pc)	0.56	3.9
ρ_c ($10^3\ M_\odot\,{\rm pc}^{-3}$)	77	3.2
ρ_h ($10^3\ M_\odot\,{\rm pc}^{-3}$)	0.58	0.30
v_s (km s−1)	12.1	17.1
$t_{\rm r,h}$ (Gyr)	2.4	16
$t_{\rm r,c}$ (Gyr)	0.0016	0.78

To compare $N_{\rm b}$ with observations of pulsars we have to take account of the followings. The progenitor of the neutron star is a massive star. Taking account of the Salpeter's initial mass function (Salpeter 1955), we estimate the number of binary stars containing neutron star as one of its component stars as

$$N_{\rm b,n} = N_{\rm b} \left(\frac{16}{0.79}\right)^{-1.4}, \tag{19}$$

where the typical mass of the progenitor is taken as 16 $M_\odot$. This yields $N_{\rm b,n} = 290$ (47 Tuc) and 1030 (ω Cen). The neutron star will be spun-up to become a milli-second pulsar. If we assume the lifetime of the millisecond pulsar to be $t_{\rm pul}$, then the number of the milli-second pulsars is estimated as

$$N_{\rm pul} = N_{\rm b,n} \left(\frac{t_{\rm pul}}{\tau_{\rm eff}}\right). \tag{20}$$

Table 2. Parameters for and accumulated numbers of binary formations

cluster	47 Tuc	ω Cen
$N_{\rm tot}$ (10^3)	850	4200
N_c (10^3)	37	520
$N_c n_c$ ($10^9\,{\rm pc}^{-3}$)	3.6	2.1
τ_0 (Gyr)	0.3	150
$t_{\rm age}$ (Gyr)	16	15
$\tau_{\rm eff}$ (Gyr)	2.4	14.5
$N_{\rm b}$	19000	68000

This yields $N_{\mathrm{pul}} = 120$ (47 Tuc) and 71 (ω Cen) for $t_{\mathrm{pul}} \simeq 1$ Gyr. These numbers are still too large.

During the gravothermal collapse the mass segregation must have proceeded in 47 Tuc since its half-mass relaxation time is as short as 2.4 Gyr which should be compared with the age of the cluster, 16 Gyr. Because the mass of the neutron star progenitor is heavier than the average mass of the stars, the time scale of the mass segregation becomes of the order of 0.1 Gyr. This implies that the progenitor star had become a neutron star before it sank down to the core. However, the neutron stars were heavier than the average mass stars so that they have sunk down into the core thereafter.

The occurrence of such mass segregation in 47 Tuc can easily be imagined from the following facts. According to Meylan (1988) a mass function, which is much flatter than Salpeter's (1955) one, better fits both to observation (over smaller mass range) and to theoretical model (over larger mass range) for 47 Tuc. Moreover, the half-mass relaxation time as listed in Table 1 indicates that the time to complete core collapse is as long as $t_{\mathrm{coll}} = 15\ t_{\mathrm{r,h}} = 36$ Gyr (see, *e.g.*, Spitzer 1987, p. 93), *i.e.*, appreciably longer than t_{age}. In spite of such estimate 47 Tuc has an almost collapsed core (Meylan 1988). This implies that the core collapse should have taken place more rapidly than such estimate. As a result of mass segregation, models containing multi-mass components are shown to collapse more rapidly than the single-mass component models, if they are compared with the models that consist of stars of mass corresponding to the average mass of the multi-mass component models (Inagaki and Wiyanto 1984; Funato *et al.* 1993).

We shall assume that all the neutron stars sank down into the core in 47 Tuc while no mass segregation has taken place at all in ω Cen. Then, the number of milli-second pulsars should be increased by a factor of $N_{\mathrm{tot}}/N_{\mathrm{c}}$ for 47 Tuc to yield $N_{\mathrm{pul}} = 2800$. When this is compared with $N_{\mathrm{pul}} = 71$ for ω Cen, the ratio of them may not be inconsistent with observation. However, their absolute values are too large. This may imply that some of the neutron stars should have escaped from the cluster at the time of their formation by sling shot effect or escaped from the core in later stages by exchange collision. In this relation it is interesting to see that the depth of the core gravitational potential, which is proportional to $N_{\mathrm{c}}/r_{\mathrm{c}}$, is about a half for 47 Tuc of the value for ω Cen because of the smaller total and core masses of the former. This implies that the stars escaped more easily from the core of 47 Tuc than ω Cen. This will reduce the ratio of the expected number of pulsars to some extent. Though it reduces also the absolute values of the numbers of pulsars at the same time, there will be enough margin for them.

5. Conclusion

We are surprised how many uncertain points and wide uncertainties are involved in the problem. Even with the rough estimate and discussion that I have given in this talk, we see that the following points await for further studies to understand the evolution of actual globular clusters.

Theoretical model should include the mass-spectrum of the constituent stars to follow the mass segregation and evolution of each star. Such a model requires much larger N-body simulations and, correspondingly, much higher speed of computers, since each mass-bin and higher-mass bins, in particular, should have appreciable number of stars. It is desirable that young globular clusters as exist in the Magellanic Clouds are studied more extensively to clarify the dependence of the local mass functions on the radial distance from the center of the cluster (Kontizas 1994). In order to clarify the escape of the stars from the cluster it will be necessary to think about a scaling law to be applied to N-body simulations. From the side of the origin of pulsars we need to investigate more about their spin-up process and their lifetime. Comparison between pulsars and blue stragglers will be useful not only for the difference in their histories of origin but also for their roles as independent indicators of dynamical evolution of the cluster. Now, I think, it is matured to tackle such problems. We now have a plenty of information from observations on the one hand, and a tera-flops computer dedicated to N-body problem (Taiji 1995) on the other hand.

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