In particular, for $a = d = 1$, from (3) we obtain

$$S_k^{1,1}(n) = 1^k + 2^k + \dots + n^k = \sum_{j=0}^{k} j! \left\{ {k \atop j} \right\} \binom{n+j}{j+1}.$$

References

1. M. Griffiths, More on sums of powers of an arithmetic progression, *Math. Gaz.* **93** (July 2009) pp. 277–279.
2. D. Laissaoui and M. Rahmani, An explicit formula for sums of powers of integers in terms of Stirling numbers, *J. Integer Seq.* **20** (2017), Article 17.4.8, 6 pp.
3. J. L. Cereceda, Generalized Stirling numbers and sums of powers of arithmetic progressions, *Internat. J. Math. Ed. Sci. Tech.* **51** (2020), pp. 954-966.

10.1017/mag.2023.71

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107.26 A difference theorem involving *k*-gonal and centred *k*-gonal numbers

Proposition

Where $p(n, k)$ and $c(n, k)$ denote the k-gonal number of n sides and the centred k-gonal number of n sides, respectively; for $n \in \mathbb{N}$, the following identity holds:

$$p(n, k) - c(n, k - 2) = n - 1.$$

Proof: For $n = 6$, the proof is demonstrated for $k = 10$.

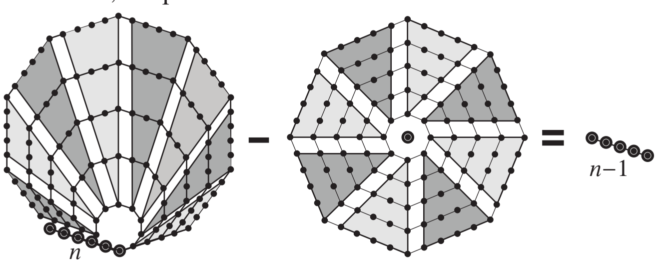

FIGURE 1

Corollary: Using the fact that a star number of n sides is isomorphic to a centred dodecagonal number of n sides [1], we further deduce the following result: where $\tau(n)$ and $\sigma(n)$ denote the tetradecagonal number of n sides and

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the star number of n sides, respectively, for $n \in \mathbb{N}$, the following identity holds:

$$\tau(n) - \sigma(n) = n - 1.$$

Proof: The proof is demonstrated for $n = 6$.

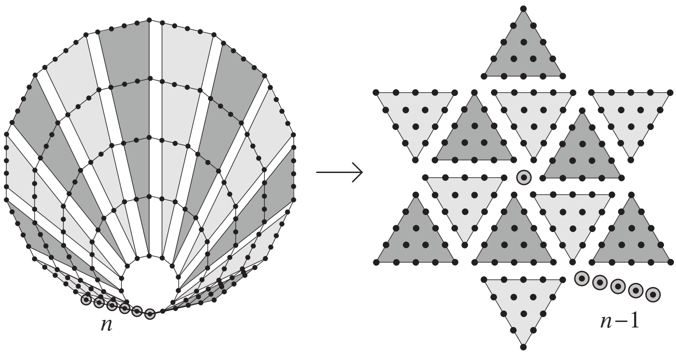

FIGURE 2

In general, where $p(n, k)$ and $c(n, k)$ are expressed in terms of triangular numbers $T_n = \frac{1}{2}n(n + 1)$ via $p(n, k) = n + (k - 2)T_{n-1}$ and $c(n, k) = 1 + kT_{n-1}$ respectively, we simply deduce

$$p(n,k) - c(n,k-2) = n + (k-2)T_{n-1} - \left[1 + (k-2)T_{n-1}\right] = n - 1.$$

Reference

1. G. Caglayan, Visualising the star number - centred dodecagonal number isomorphism, *Math. Gaz.*, **104** (November 2020) pp. 543.

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107.27 The discrete renewal theorem with bounded inter-event times

Probabilistic Sequence

The purpose of this Note is to prove the celebrated Discrete Renewal Theorem in a common special case, using only very elementary methods.

To introduce the problem, consider a class of board games in which a player's counter makes a sequence of moves in a fixed direction along a line of squares S_n, $n \geqslant 0$. The counter starts from S_0, with the sizes of successive moves determined by the roll of a die (or multiple dice), which may be biased.