BOOK REVIEWS

WILLIAMSON, J. H. (editor), *Algebra in Analysis* (Academic Press, 1975), xi+312 pp., £10·80, $28.50.

This volume is the proceedings of an Instructional Conference organised by the London Mathematical Society, Birmingham, 1973. The Society is to be congratulated on gathering together a group of experts giving a wide coverage of the subject and with varied and interesting talents for exposition. The resulting volume contains much of importance, and some part will be necessary reading for analysts working on harmonic analysis, operator theory, or several complex variables. With so wide a coverage, a common theme is hardly to be expected. It is tempting to say that an alternative title for the volume could be " Cohomology in analysis ", but this would be unfair to the contributions of R. V. Kadison and L. Waelbroeck. Accordingly I give a brief separate account of each of the lecture courses. In addition to these main lecture courses the volume also contains abstracts of seminar talks (together with full details of three of them).

1. H. HELSON, *Analyticity on compact Abelian groups*

Let K be a compact Abelian group with its dual group Γ ordered in the sense that Γ is a disjoint union $\Gamma_+ \cup \{0\} \cup \{-\Gamma_+\}$ with Γ_+ a semigroup. Let $\{\chi_\lambda : \lambda \in \Gamma\}$ denote the characters on K. A function f on K integrable with respect to Haar measure σ is said to be analytic if

$$\int_K f(x)\overline{\chi_\lambda(x)}d\sigma(x) = 0 \quad (\lambda \in -\Gamma_+).$$

The theory has grown out of the classical theory of the H^p spaces with K the circle group and Γ the integers. Much of the classical theory remains valid in the general case, for example Beurling's characterisation of the invariant subspaces of the unilateral shift and Szegö's formula for the geometric mean of a non-negative integrable function. However, from §2 onwards these lecture notes part company with the classical theory. It is assumed that the order on Γ is Archimedean and that Γ_+ does not have a least element. Thus we are concerned with those subgroups Γ of $\boldsymbol{R}$ that are not isomorphic to $\boldsymbol{Z}$.

The objects of study are the invariant subspaces of $L^2(K)$, that is the closed subspaces of $L^2(K)$ that are mapped into themselves under multiplication by characters χ_λ with $\lambda \in \Gamma_+$. The analysis is carried out in terms of certain " cocycles ". As one would expect, a concept of coboundary enters the theory and hence a cohomology.

Although mainly expository, this article contains several new results, simplified proofs and unsolved problems.

2. B. E. JOHNSON, *Banach algebras: Introductory course*

This is an efficient account of the basic theory of Banach algebras including the Gelfand theory for commutative algebras, the one-variable functional calculus, irreducible representations, uniqueness of the Banach algebra norm topology, and approximate units.

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3. B. E. JOHNSON, *Introduction to cohomology in Banach algebras*

Let A be a Banach algebra, X a Banach A-bimodule, and let $\mathscr{L}^n(A, X)$ denote the space of continuous n-linear mappings: $A \to X$ with $\mathscr{L}^0(A, X)$ taken to be X. Mappings $\delta^n : \mathscr{L}^{n-1}(A, X) \to \mathscr{L}^n(A, X)$ are defined by

$$(\delta^n T)(a_1, \ldots, a_n) = a_1 T(a_2, \ldots, a_{n-1}) - T(a_1 a_2, a_3, \ldots, a_n) + T(a_1, a_2 a_3, a_4, \ldots, a_n)$$
$$\ldots + (-1)^{n-1} T(a_1, \ldots, a_{n-2}, a_{n-1} a_n) + (-1)^n T(a_1, \ldots, a_{n-1}) a_n,$$

where $T \in \mathscr{L}^{n-1}(A, X)$ and $a_1, \ldots, a_n \in A$. The quotient space Ker δ^{n+1}/Im δ^n is the Hochschild Cohomology Group $H^n(A, X)$. In particular $H^1(A, X)$ is the quotient of the space of continuous derivations by the space of inner derivations. Thus

$$H^1(A, X) = \{0\}$$

if and only if all continuous derivations are inner. It is shown that a derivation is inner if and only if a certain mapping has a fixed point, and this is exploited, by way of fixed point theorems, to give sufficient conditions for the vanishing of $H^1(A, X)$. A group G is said to be amenable if there exists a translation invariant mean on the space $l^\infty(G)$. It is shown that if A is the closed linear span of an amenable multiplicative group G, then $H^1(A, X) = 0$ for all dual A-modules X.

These lecture notes also include a discussion of the relation between extensions of A by X and the cohomology group $H^2(A, X)$, and an account of Helemskii's cohomology theory. Much of the development of cohomology in Banach algebras is due to B. E. Johnson, and it is very valuable to have this short and readable account of the subject.

4. R. V. KADISON, *Operator algebras*

This is a short and lucid account of some of the most important aspects of von Neumann algebras. §§1, 2 are concerned with basic properties of C^*-algebras and von Neumann algebras, and §3 with the comparison theory for projections. A final section is mainly a discussion of normal states.

5. J. L. TAYLOR, *Banach algebras and topology*

These lecture notes are a brave attempt to provide an introduction to algebraic topology for the use of analysts and particularly for Banach algebra specialists. §§1, 2, which give a thorough introduction to axiomatic cohomology, open with the remark: ". . . we hoped, on leaving graduate school, never to encounter algebraic topology again. However, the analyst cannot escape topology. Too often he runs into problems where the solution (its existence, uniqueness, or nature) depends on the topology of some underlying space. On the line every continuous function has a continuous anti-derivative; not so on the circle . . .". Many an analyst will echo these sentiments. In §3 the Čech cohomology for a compact space (or a pair of spaces) is constructed, and in §4 the algebraic structure of a commutative Banach algebra is related to the Čech cohomology groups of its carrier space. The rest of the notes are devoted to a discussion of K-theory and its applications.

6. L. WAELBROECK, *The holomorphic functional calculus and non-Banach algebras*

These lecture notes are concerned with the most difficult aspect of Banach algebra theory, the theory of the joint spectrum of an n-tuple of algebra elements and the associated functional calculus for holomorphic functions of n complex variables. The notes are in three parts. Part I concerns the homomorphic functional calculus in Banach algebras, Part II extends the theory to certain classes of algebras for which the results can be derived from the Banach algebra results, and Part III develops the functional calculus again in such a way that it can be applied to algebras in which the spectra of elements need not be compact.

The subject of this course is inevitably highly technical and the author has made great efforts to ease the lot of the reader by a colloquial style and a liberal use of heuristic discussion. However, the reader will still have to work hard.

F. F. BONSALL

DUBIN, D. A., *Solvable Models in Algebraic Statistical Mechanics* (Oxford Science Research Papers, Oxford University Press, 1974), £5·25.

The branches of mathematics most useful to statistical mechanics are functional and complex analysis, ergodic and probability theory, and the theory of C^*- and W^*-algebras. In return, the " rigorous " approach to statistical mechanics has led to new ideas in these subjects. Solvable models, providing a good qualitative picture of actual physical systems, are also useful as mathematical laboratories. In this way, statements plausible " for physical reasons " often become theorems of some interest to pure mathematicians.

This book is a survey of the simplest solvable models, selected and explained by a mathematical physicist.

It is not a book of pure mathematics, and it may be hard reading for a mathematician, because of both the style and the content. It would repay extended study, especially if this leads to new directions and emphasis in the classical subjects.

R. F. STREATER

MASON, J., *Groups, a Concrete Introduction using Cayley Cards* (Transworld Student Library, 1975), 125.pp., £0·85 (soft cover).

This original and engaging little book on elementary group theory draws on the author's experience in presenting the material for a second year course at the Open University, and discusses groups, subgroups, homomorphisms and the first homomorphism theorem, permutations and Cayley's Theorem, using so-called " Cayley cards ".

First, the alternating group A_4 is presented graphically as a pack of 12 cards, each with two sets of 4 dots down two opposite sides, the dots being joined in an obvious way so as to present the permutation involved (3 packs of cards are included with the book). The group operation is juxtaposition of cards, and associativity is inherent. Then, using these cards, ideas are illustrated in the group A_4, and in other groups, before they are introduced abstractly. Thus the author is able to discuss the difficulties encountered when coping with abstractions, and in fact a large part of the book is concerned with the learning process itself. The student is made to work with the Cayley cards so as to render concrete the various notions met, thus avoiding the danger of abstract algebra being too " abstract ". One drawback to this approach is that the student's ability to manipulate abstract symbols is not developed sufficiently, but perhaps this may wait for a more advanced course; indeed, a final section contains suggestions for further reading. Two excellent features (among many) are a final review of the concepts met using the quaternion group of order 8 in place of A_4, and the tightening of Cayley's theorem to show that A_4 can be retrieved as a subgroup of S_4 rather than as a subgroup of S_{12} (in the usual notation).

There are some faults. Major results are buried in worked exercises, cyclic groups are covered too quickly and in an obscure fashion, and at times the argument develops too rapidly. More unfortunately, there are errors in Exercises 2.7, 3.1 and 5.15, the logic in Exercises 2.6 and 3.21 needs to be tightened, and there are some misprints in

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