ABSTRACTS OF THESES

Walter D. Burgess, Ph.D., Group Rings and their Rings of Quotients, McGill University, September 1967. (Supervisor: J. Lambek)

The complete ring of right quotients Q(AG) of the group ring AG is shown to be a ring of functions from G to an injective hull of the right module A_A. When G is finite, $Q(AG) \cong Q(A)G$. Multiplication in Q(AG) and its relation to convolution is examined.

If Q(AG) is left Noetherian, then G has no infinite locally finite subgroups. If, in addition, Q(A) is regular, then Q(A) is completely reducible. If G has a normal subgroup H which is locally normal and G/H is ordered, then Q(AG) is regular if and only if Q(A) is regular and the order of every finite normal subgroup of G is a non-zero divisor in A. It is also shown by these methods that Q(A[x]) is completely reducible if and only if Q(A) is.

When G is finite, A has a classical ring of quotients if AG has, and $Q_{c\ell}(A)G \subseteq Q_{c\ell}(AG)$. A weak analogue of the Faith-Utumi theorem is given for orders in group rings of finite groups.

Commutative semi-perfect group rings are examined. If G is finite and N is an ideal in the Jacobson radical of A, then idempotents can be lifted from AG/NG to AG/N^2G. It follows that, if A is a complete local ring and G is finite, then AG is semi-perfect. A result in the converse direction is also obtained.

Israel Kleiner, Ph.D., Lie Modules and Rings of Quotients, McGill University, September 1967. (Supervisor: J. Lambek).

The category of Lie L-modules over a Lie ring L is isomorphic to the category of W(L)-modules over the universal enveloping ring W(L) of L. Free Lie modules, injective hulls of Lie modules, and rational completions of Lie modules are then determined.

Let R be a commutative ring with 1, Q(R) its maximal ring of quotients. According to Krishna Tewari, every derivation d of R has a unique extension to a derivation $\bar{d}$ of Q(R). Suppose $d_1, \ldots d_n$ are mutually commuting derivations of R. Let $D_n(R)$ consist of all $r_1 d_1 + \ldots + r_n d_n$ with $r_i \in R$, and let $D_n(Q(R))$ consist of all $q_1 \bar{d}_1 + \ldots q_n \bar{d}_n$ with $q_i \in Q(R)$. It is shown that under certain conditions the latter is a (Lie) ring of quotients of the former. The conditions are satisfied, for example, when R is an integral domain. Under the same conditions, the set of all "special" derivations of Q(R) is a (Lie) ring of quotients of the Lie ring of all derivations of R, where a derivation is called special when it has the form $q_1 d_1 + \dots + q_n d_n$, with $q_1 \in Q(R)$ and d_i a derivation of Q(R) which sends R into itself.

David J. Fieldhouse, Ph.D., Purity and Flatness, McGill University, September 1967. (Supervisor: J. Lambek)

P.M. Cohn calls a submodule P of the left A-module M pure if and only if $0 \to E \otimes P \to E \otimes M$ is exact for all rt. modules E. Most of the well known theorems on pure subgroups are valid for pure submodules. Extending a definition of Maranda to arbitrary rings, a module Q is called pure projective if and only if $Hom(Q, M) \to Hom(Q, M/P) \to 0$ is exact whenever P is pure in M. Maranda's results on pure projectivity are extended and a complete structure for pure projective modules is obtained.

Generalizing a known property of regular rings, a (left) A-module is called regular if and only if all its submodules are pure. The ring A is shown to be regular if and only if all left (or all rt.) A-modules are regular. A structure theorem for regular projective modules is obtained. A regular socle is defined, analogous to the semi-simple (= usual) socle, and its basic properties established. Several new characterizations of regular rings are given.

It is known that a left module F is flat if and only if its character module $Hom_Z(F, Q/Z)$ is injective. For (left) noetherian rings, the dual holds: the left module I is injective if and only if its character module is flat. It is also shown that the weak (= flat) global dimension of A is equal to: sup weak dimension E, with the sup taken over all left (or rt.) finitely presented cyclic modules E.

Pure simple and indecomposable rings are related to the PP and PF rings of Hattori. The latter are rings in which every principal (left) ideal is projective (or flat). These rings are characterized both in the commutative and non-commutative cases.

Localization theorems for purity, regularity, PP and PF rings are obtained.

Finally, as an application, flat covers of modules are constructed and their basic properties established. They always exist and coincide with the projective cover for the perfect rings of Bass. However, they are not in general unique.

Jagannath K. Wani, Ph.D., Certain studies on the linear exponential family, McGill University, Montreal, November 1966. (Supervisor: A.M. Mathai)

The linear exponential family is first studied for characterisation through moment relations. The moment relations used for this purpose are further simplified in the case of some of the classical discrete distributions. The problem of estimating a distribution function at a known point has been treated for some of the distributions in the exponential family by using a uniform technique made available by the basic defining property of sufficient statistic. In the last chapter estimation problem for the logarithmic series distribution is examined with special attention to the maximum likelihood estimate.

Victor Byers, Ph.D., Non-Archimedian Norms and Bounds, McGill University, July 1967. (Supervisor: H. Schwerdtfeger)

An application of iterative methods of numerical analysis to p-adic fields is based on a study of matrix norms.

The preliminary general treatment is largely confined to non-Archimedian norms, i.e., norms which satisfy the strong triangle inequality. Norms and pseudonorms on a module over a valuated ring are considered and the existence of a canonical pseudonorm on the quotient module of a non-Archimedian pseudonormed module is proved. With the help of an algebraic definition of boundedness an association is established between a class of non-Archimedian norms on a vector space E over a non-Archimedian valuated field K and certain submodules of E over the valuation ring of K. A discussion of the properties of bounds (special norms on spaces of linear transformations) is followed by a proof that the open unit sphere of a complete non-Archimedian algebra A is an ideal contained in the Jacobson radical of the closed unit sphere of A with resultant effects on convergence and invertibility.

Finite-dimensional vector spaces over an arbitrary valuated field are considered next. Duality is discussed, a new proof of a result of Ljubič's is given and a new necessary and sufficient condition for a matrix norm to be a bound is established. The properties of the "natural" norm for a non-Archimedian field are investigated; the corresponding bound is used to derive an inequality satisfied by non-Archimedian matrix norms and to show the existence (in appropriate cases) of a bound equal to the spectral radius of a matrix. The concept of an $\mathcal{J}$-unitary matrix yields results concerning the non-singularity of a class of matrices over a non-Archimedian field as well as certain properties of their inverses and spectra. There follow estimates for the rate of convergence of certain iterative processes over a non-Archimedian field and examples of the use of methods of successive approximation for the solution of systems of linear equations and the inversion of matrices over the field of p-adic numbers.