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NEW EXACT SOLUTIONS OF COUPLED $(2 + 1)$-DIMENSIONAL NONLINEAR SYSTEMS OF SCHRÖDINGER EQUATIONS

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Abstract

The Exp-function method is applied to construct a new type of solution of the coupled $(2 + 1)$-dimensional nonlinear system of Schrödinger equations. It is shown that the method provides a powerful mathematical tool for solving nonlinear evolution equations in mathematical physics.

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1. Introduction

The investigation of exact solutions of nonlinear equations plays an important role in the study of nonlinear physical phenomena. The importance of obtaining the exact solutions, if available, of those nonlinear equations facilitates the verification of numerical solutions and aids in the stability analysis of solutions. Recently, many approaches have been suggested to solve the nonlinear equations, such as the variational iteration method [8, 11, 15], the homotopy perturbation method [7, 17, 23], the tanh method [28], the extended tanh method [3], the sinh method [25], the spectral collocation method [4, 5, 9, 10, 12], the homogeneous balance method [31, 34], the F-expansion method [32], the extended Fan sub-equation method [30] and the parameter-expansion method [6, 16, 27]. Recently, He and Wu [18] proposed a straightforward method, the Exp-function method, to obtain the exact solutions of nonlinear evolution equations (NLEEs). The Exp-function method has proved to be very effective and convenient for handling many kinds of NLEEs [1, 14, 18–22, 26, 29, 33]. It provides

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the generalized solitary solutions and periodic solutions, as well. Taking advantage of the generalized solitary solutions, we can recover some known solutions obtained by existing methods such as the decomposition method, the tanh-function method, the algebraic method, the extended Jacobi elliptic function expansion method, the F-expansion method, the auxiliary equation method and others [1, 14, 18–22, 29, 33].

In this paper we extend the Exp-function method to a class of nonlinear evolution equations with imaginary number and modulus. We consider the coupled $(2+1)$-dimensional nonlinear system of Schrödinger equations as

$$i\psi_t - \psi_{xx} + \psi_{yy} + |\psi|^2\psi - 2\psi\phi = 0, \tag{1.1}$$

$$\phi_{xx} - \phi_{yy} - (|\psi|^2)_{xx} = 0, \tag{1.2}$$

where $\psi(x, y, t)$ and $\phi(x, y, t)$ are complex-valued functions. Nonlinear partial differential equation systems of the type given by (1.1) and (1.2) play an important role in atomic physics, and the functions ψ and ϕ have different physical meanings in different branches of physics [2, 13, 24]. Well-known applications are, for instance, in fluid dynamics [2] and plasma physics [24]. In the context of water waves, ψ is the amplitude of a surface wave packet while ϕ is the velocity potential of the mean flow interacting with the surface waves [13]. However, in the hydrodynamic context, ψ is the envelope of the wave packet and ϕ is the induced mean flow [2]. In addition, equations (1.1) and (1.2) are relevant in a number of different physical contexts, describing slow modulation effects of the complex amplitude ϕ, due to a small nonlinearity, on a monochromatic wave in a dispersive medium.

2. Method of solution

To obtain the exact solutions of (1.1) and (1.2), we use the transformations

$$\begin{aligned} &\psi(x, y, t) = u(\xi)\exp(i\eta), \qquad \phi(x, y, t) = v(\xi), \\ &\xi = k(x + ly + 2(\alpha - \beta l)t), \qquad \eta = \alpha x + \beta y + \gamma t, \end{aligned} \tag{2.1}$$

where k, l, α, β and γ are constants to be determined. Note that ξ and η are travelling wave variables, not necessarily in the same direction. That is, ξ and η are independent linear functions of x, y and t. Then u and v are assumed to be rational functions of $\exp(\xi)$. When u is positive real, u is the modulus of the complex function ψ, and η is the argument. The modulus and argument are travelling waves but the two waves may be in different directions.

From (1.1) and (1.2), we may obtain the system of ordinary differential equations

$$k^2(l^2 - 1)u'' + (\alpha^2 - \beta^2 - \gamma)u + u^3 - 2uv = 0, \tag{2.2}$$

$$(1 + l^2)v'' - (u^2)'' = 0. \tag{2.3}$$

Integrating (2.3) with respect to ξ and setting the constants of integration equal to zero yields

$$v = \frac{u^2}{1 + l^2}. \tag{2.4}$$

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Substituting (2.4) into (2.2), we obtain

$$k^2(l^2-1)u'' + (\alpha^2-\beta^2-\gamma)u + \frac{l^2-1}{l^2+1}u^3 = 0. \tag{2.5}$$

Using the Exp-function method [18], the solution of (2.5) may be expressed in the form

$$u(\xi) = \frac{\sum_{n=-c}^{d} a_n \exp(n\xi)}{\sum_{m=-f}^{g} b_m \exp(m\xi)}, \tag{2.6}$$

where c, d, f and g are unknown positive integers to be determined and a_n and b_m are constants. In expanded form, (2.6) becomes

$$u(\xi) = \frac{a_{-c}\exp(-c\xi) + \cdots + a_d \exp(d\xi)}{b_{-f}\exp(-f\xi) + \cdots + b_g \exp(g\xi)}. \tag{2.7}$$

For simplicity of notation, by renaming the variables, equation (2.7) may be rewritten as [18]

$$u(\xi) = \frac{a_c\exp(c\xi) + \cdots + a_{-d} \exp(-d\xi)}{b_f\exp(f\xi) + \cdots + b_{-g} \exp(-g\xi)}. \tag{2.8}$$

In order to determine the values of c and f, we balance the linear term of the highest order in (2.5), u'', with the highest-order nonlinear term, u^3. Simple calculations give

$$u'' = \frac{c_1\exp[(c+3f)\xi] + \cdots}{c_2\exp[4f\xi] + \cdots} \tag{2.9}$$

and

$$u^3 = \frac{c_3\exp[(3c+f)\xi] + \cdots}{c_4\exp[4f\xi] + \cdots}, \tag{2.10}$$

where the c_i are constants. By balancing the highest order of the exp functions in (2.9) and (2.10), we can write $3c+f = c+3f$, which leads to $c = f$.

Similarly, to determine d and g we can write

$$u'' = \frac{\cdots + d_1\exp[-(d+3g)\xi]}{\cdots + d_2\exp[-5g\xi]} \tag{2.11}$$

and

$$u^3 = \frac{\cdots + d_3\exp[-(3d+g)\xi]}{\cdots + d_4\exp[-4g\xi]}, \tag{2.12}$$

where the d_i are constants. By balancing the lowest order of the exp functions in (2.11) and (2.12), we obtain $-(d+3g) = -(3d+g)$, which leads to $g = d$.

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3. Solitary solutions

In this section we discuss the nature of solutions categorically for various values of f and g. We will confirm claims [18, 19] that the final solution does not strongly depend upon the choice of values of c and d.

3.1. Case (i): $f = g = 1$, $c = d = 1$ In this case setting $b_1 = 1$, the trial function (2.8) becomes

$$u(\xi) = \frac{a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi)}{\exp(\xi) + b_0 + b_{-1} \exp(-\xi)}. \tag{3.1}$$

Substituting (3.1) into (2.5) gives

$$\frac{1}{Q} \sum_{j=-3}^{3} K_j \exp(j\xi) = 0, \tag{3.2}$$

where $Q = (l^2 + 1)(\exp(\xi) + b_0 + b_{-1} \exp(-\xi))^3$ and K_j, $j = -3, \ldots, 3$, are constants. Equating the coefficients of the exponential terms in (3.2) to zero, we obtain

$$K_j = 0, \quad j = -3, -2, \ldots, 2, 3. \tag{3.3}$$

Solving the system of algebraic equations (3.3) using MAPLE, we obtain

$$\begin{aligned} a_0 &= -\frac{k}{\sqrt{2}}\sqrt{(l^2+1)(4b_{-1} - b_0^2)}, & a_{-1} &= \frac{ik}{\sqrt{2}}\sqrt{l^2+1}\,b_{-1}, \\ a_1 &= -\frac{ik}{\sqrt{2}}\sqrt{l^2+1}, & \alpha &= -\sqrt{\frac{1}{2}k^2(l^2-1) + \beta^2 + \gamma}, \end{aligned} \tag{3.4}$$

where β, γ, l, k, b_0 and b_{-1} are free parameters.

Substituting (3.4) into (3.1) leads to

$$u(\xi) = -\frac{ik\sqrt{l^2+1}}{\sqrt{2}}\left[\frac{\exp(\xi) + \sqrt{b_0^2 - 4b_{-1}} - b_{-1}\exp(-\xi)}{\exp(\xi) + b_0 + b_{-1}\exp(-\xi)}\right], \tag{3.5}$$

where b_0 and b_{-1} are free parameters and

$$\xi = k(x + ly - 2(-\alpha + \beta l)t). \tag{3.6}$$

Therefore we obtain the combined generalized solutions of (1.1) and (1.2),

$$\psi = u \exp[i(\alpha x + \beta y + \gamma t)], \tag{3.7}$$

$$\phi = \frac{u^2}{1 + l^2}, \tag{3.8}$$

where u is determined by (3.5).

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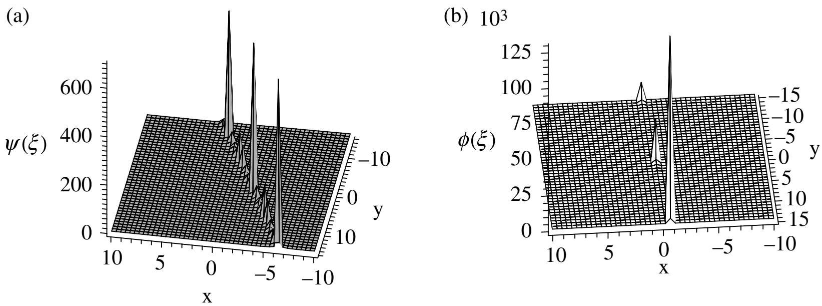

FIGURE 1. Evolution of dromion solutions $\psi(\xi)$ and $\phi(\xi)$ at $t = 10$ for $k = 2$, $\beta = 0.05$, $\gamma = 0.2$ and (a) $l = 0.3$ and $\alpha = 0.1$ and (b) $l = 0.03$ and $\alpha = 0.005$.

Setting $b_{-1} = -1$ and $b_0 = 1$ in (3.5) gives

$$u = -\frac{ik\sqrt{1+l^2}}{\sqrt{2}}\left(\frac{\sqrt{5}+2\cosh(\xi)}{1+2\sinh(\xi)}\right). \tag{3.9}$$

Substituting (3.9) into (3.7), we obtain the exact solutions of (1.1) and (1.2) as

$$\psi(\xi) = -\frac{ik\sqrt{1+l^2}}{\sqrt{2}}\left(\frac{\sqrt{5}+2\cosh(\xi)}{1+2\sinh(\xi)}\right)\exp[i(\alpha x+\beta y+\gamma t)], \tag{3.10}$$

$$\phi(\xi) = -\frac{1}{2}k^2\left(\frac{\sqrt{5}+2\cosh(\xi)}{1+2\sinh(\xi)}\right)^2. \tag{3.11}$$

Figure 1 presents the values of ψ and ϕ from equations (3.10) and (3.11) at $t = 10$ for $k = 2$, $\beta = 0.05$, $\gamma = 0.2$ and different values of α and l. The evolution of the combined generalized solutions is portrayed in the form of dromions.

Setting $b_{-1} = 1$ and $b_0 = 2$ in (3.5), (3.7) and (3.8) become

$$\psi(\xi) = -\frac{ik\sqrt{1+l^2}}{\sqrt{2}}\tanh\left(\frac{\xi}{2}\right)\exp[i(\alpha x+\beta y+\gamma t)], \tag{3.12}$$

$$\phi(\xi) = -\frac{1}{2}k^2\tanh^2\left(\frac{\xi}{2}\right). \tag{3.13}$$

Figure 2 is a plot of the evolution of the dromion solutions (3.12) and (3.13) for two sets of parameter values. Figure 2(a) represents the dromion solutions $\psi(\xi)$ having a groove at the centre and Figure 2(b) represents $\phi(\xi)$. In the contour plots the brighter region represents the maximum amplitude and the darker region represents the minimum or zero amplitude.

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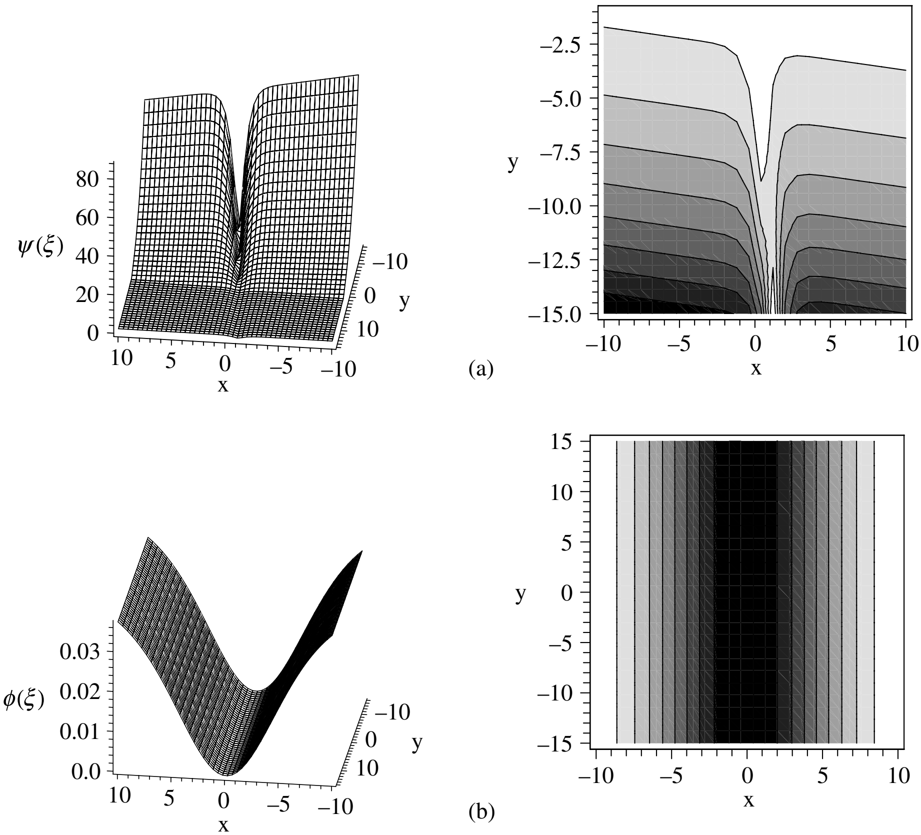

FIGURE 2. Evolution of the dromion solutions $\psi(\xi)$ and $\phi(\xi)$ from (3.12) and (3.13) at $t = 0$ for (a) $k = 2$, $l = 0.08$, $\alpha = 0.5$, $\beta = 1$ and $\gamma = 0.1$ and (b) $k = 0.3$, $l = 0.0003$, $\alpha = 0.005$, $\beta = 0.8$ and $\gamma = 0.2$.

Setting $b_{-1} = -1$ and $b_0 = 2$ in (3.5), (3.7) and (3.8) become

$$\psi(\xi) = -\frac{ik\sqrt{1+l^2}}{\sqrt{2}} \coth\left(\frac{\xi}{2}\right) \exp[i(\alpha x + \beta y + \gamma t)], \tag{3.14}$$

$$\phi(\xi) = -\frac{1}{2}k^2 \coth^2\left(\frac{\xi}{2}\right). \tag{3.15}$$

The dromion solutions representing (3.14) and (3.15) are depicted in Figure 3 at $t = 0$ for two choices of parameters. The figure portrays the homogeneous evolution of dromions which is also evident from the corresponding contour plots.

When k is an imaginary number, all the solutions obtained above may be converted into trigonometric solutions. If we use $k = i\Omega$, then solutions (3.10) and (3.11)

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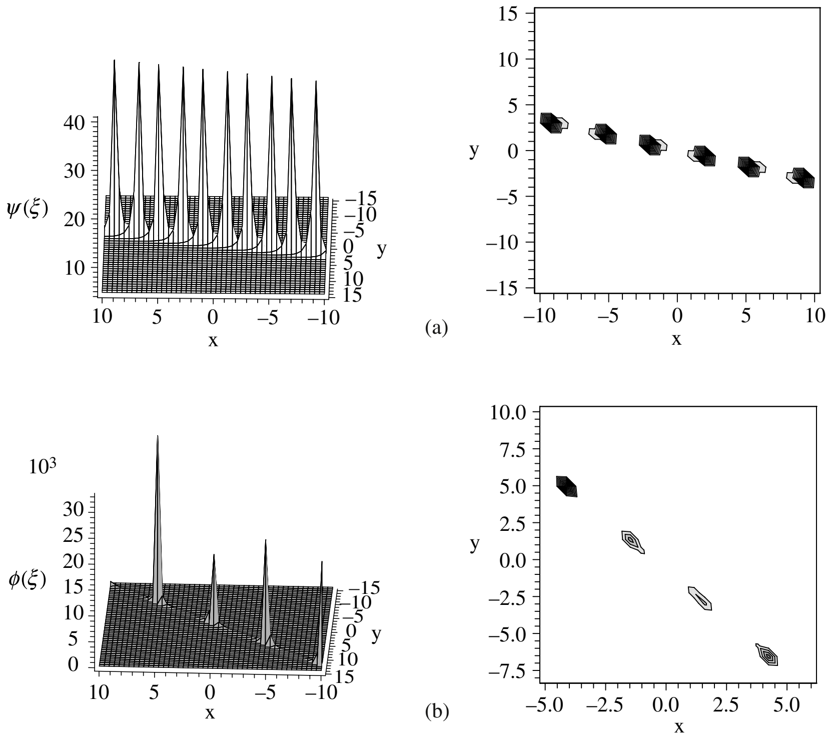

FIGURE 3. Evolution of dromion solutions $\psi(\xi)$ and $\phi(\xi)$ given by (3.14) and (3.15) at $t = 0$ for (a) $k = 2$, $l = 3$, $\alpha = 0.5$, $\beta = 1$ and $\gamma = 0.08$ and (b) $k = 0.05$, $l = 0.73$, $\alpha = 0.61$, $\beta = 0.8$ and $\gamma = 0.2$.

become

$$\psi(\zeta) = -\frac{i\Omega\sqrt{1+l^2}}{\sqrt{2}}\left(\frac{\sqrt{5}+2\cos(\varepsilon)}{-i+2\sin(\varepsilon)}\right) \times \exp\left[i\left(-\sqrt{\frac{\Omega^2}{2}(1-l^2)+\beta^2+\gamma x+\beta y+\gamma t}\right)\right],$$

$$\phi(\zeta) = -\frac{1}{2}\Omega^2\left(\frac{\sqrt{5}+2\cos(\varepsilon)}{-i+2\sin(\varepsilon)}\right)^2,$$

where

$$\zeta = \Omega(x + l(y - 2t\beta) - t\sqrt{4(\beta^2+\gamma) - 2(-1+l^2)\Omega^2}).$$

3.2. Case (ii): $f = c = 2, d = g = 2$ In this case the trial function (2.8) becomes

$$u(\xi) = \frac{a_2\exp(2\xi) + a_1\exp(\xi) + a_0 + a_{-1}\exp(-\xi) + a_{-2}\exp(-2\xi)}{b_2\exp(2\xi) + b_1\exp(\xi) + b_0 + b_{-1}\exp(-\xi) + b_{-2}\exp(-2\xi)}. \tag{3.16}$$

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There are free parameters in (3.16). For simplicity we set $b_{-1} = b_1 = 0$ and $b_2 = 1$, giving

$$u(\xi) = \frac{a_2 \exp(2\xi) + a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi) + a_{-2} \exp(-2\xi)}{\exp(2\xi) + b_0 + b_{-2} \exp(-2\xi)}. \tag{3.17}$$

Substituting (3.17) into (2.5), we obtain a rational function in powers of $\exp(\xi)$. Equating the coefficients of $\exp(n\xi)$ to zero and solving the system of algebraic equations yields

$$\begin{aligned}
a_{-2} &= -\frac{4ka_0a_1^2 + (i\sqrt{2}a_1^4/\sqrt{1+l^2})}{8k^3(1+l^2)}, & a_{-1} &= \frac{i2k\sqrt{2(1+l^2)}a_0a_1 + a_1^3}{2k^2(1+l^2)},\\
a_2 &= -\frac{i\sqrt{k^2(1+l^2)}}{\sqrt{2}}, & b_{-2} &= \frac{i2k\sqrt{2(1+l^2)}a_0a_1^2 + a_1^4}{4k^4(1+l^2)^2},\\
b_0 &= \frac{ik\sqrt{2(1+l^2)}a_0 + a_1^2}{k^2(1+l^2)}, & \beta &= -\frac{\sqrt{k^2(1-l^2) + 2\alpha^2 - 2\gamma}}{\sqrt{2}},
\end{aligned} \tag{3.18}$$

where $a_0, a_1, \gamma, \alpha, l$ and k are free parameters.

Substitution of (3.18) and $a_1 = i\sqrt{k^2(1+l^2)}/2$ into (3.17) yields a solution of (2.5),

$$u(\xi) = -\frac{ik\sqrt{1+l^2}(-8 + 9\sqrt{2}\cosh(\xi) + 7\sqrt{2}\sinh(\xi))}{2(7\cosh(\xi) + 9\sinh(\xi))}, \tag{3.19}$$

where

$$\xi = k\left(x + ly + 2\left(\alpha + \frac{\sqrt{k^2(1-l^2) + 2\alpha^2 - 2\gamma}}{\sqrt{2}}l\right)t\right).$$

Substitution of (3.18) and $a_1 = i\sqrt{2k^2(1+l^2)}$ into (3.17) yields another solution of (2.5),

$$u(\xi) = -\frac{ik}{\sqrt{2}}\sqrt{1+l^2}\tanh\left(\frac{\xi}{2}\right). \tag{3.20}$$

Setting $a_1 = b_1 = b_2 = 0$ for simplicity, (3.16) reads

$$u(\xi) = \frac{a_2 \exp(2\xi) + a_1 \exp(\xi) + a_0 + a_{-2} \exp(-2\xi)}{b_0 + b_{-1} \exp(-\xi) + b_{-2} \exp(-2\xi)}. \tag{3.21}$$

By the same manipulation as illustrated before, we obtain

$$\frac{\sqrt{k^2(1+l^2)}(a_0 + a_{-2}\exp(-2\xi))}{\sqrt{2}(ia_0 - 2\sqrt{a_{-2}a_0}\exp(-\xi) - ia_{-2}\exp(-2\xi))}, \tag{3.22}$$

where a_0 and a_{-2} are free parameters and

$$\xi = k\left(x + ly - 2\left(\frac{\sqrt{k^2(l^2-1) + 2\beta^2 + 2\gamma}}{\sqrt{2}} + \beta l\right)t\right).$$

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It should be noted that if we set $a_0 = -k^2(1+l^2)$ and $a_{-2} = k^2(1+l^2)$ in (3.22), we can recover the solution (3.20).

3.3. Case (iii): $f = c = 2, d = g = 1$ In this case (2.8) may be expressed as

$$u(\xi) = \frac{a_2 \exp(2\xi) + a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi)}{b_2 \exp(2\xi) + b_1 \exp(\xi) + b_0 + b_{-1} \exp(-\xi)}. \tag{3.23}$$

There are free parameters in (3.23). For simplicity we set $b_2 = 1$ and $b_1 = 0$. After lengthy algebra, we obtain

$$\begin{aligned} a_{-1} &= 0, \quad a_0 = \frac{i a_1^2}{2k\sqrt{2(1+l^2)}}, \quad a_2 = -\frac{i\sqrt{k^2(1+l^2)}}{\sqrt{2}}, \\ b_{-1} &= 0, \quad b_0 = \frac{a_1^2}{2k^2(1+l^2)}, \qquad \beta = -\frac{\sqrt{k^2(1-l^2)+2\alpha^2-2\gamma}}{\sqrt{2}}, \end{aligned} \tag{3.24}$$

where a_1, α, γ, k and l are free parameters. Substituting (3.24) into (3.23) yields

$$u(\xi) = \frac{-(i\sqrt{k^2(1+l^2)}/\sqrt{2}) \exp(2\xi) + a_1 \exp(\xi) + i a_1^2/(2k\sqrt{2(1+l^2)})}{\exp(2\xi) + a_1^2/(2k^2(1+l^2))}, \tag{3.25}$$

where a_1 is a free parameter and

$$\xi = k\left(x + ly + 2\left(\alpha + \frac{\sqrt{k^2(1-l^2)+2\alpha^2-2\gamma}}{\sqrt{2}} l\right) t\right).$$

If we set $a_1 = \sqrt{2k^2(1+l^2)}$, then (3.25) becomes

$$u(\xi) = \frac{-ik}{\sqrt{2}} \sqrt{1+l^2} \operatorname{sech}(\xi)(i + \sinh(\xi)). \tag{3.26}$$

Also if we set $a_1 = i\sqrt{2k^2(1+l^2)}$ in (3.25), then (3.25) recovers the solution (3.20).

4. Extrema and points of inflection of the solutions

In this section, we give some general descriptions of the solutions $u(\xi)$ obtained. An inflection in the path of a point can also be described as a location where the path has zero curvature or where the curve is osculating to first order with a straight line.

4.1. Case (a): $f = g = 1, b_0 = 0, b_{-1} = -1$ We try to elucidate the extremal points by differentiating equation (3.9), which yields

$$u'(\xi) = \frac{ik\sqrt{2+2l^2}}{1+2\sinh(\xi)} \left[\sinh(\xi) - \frac{(\sqrt{5} + 2\cosh(\xi))\cosh(\xi)}{1+2\sinh(\xi)}\right]. \tag{4.1}$$

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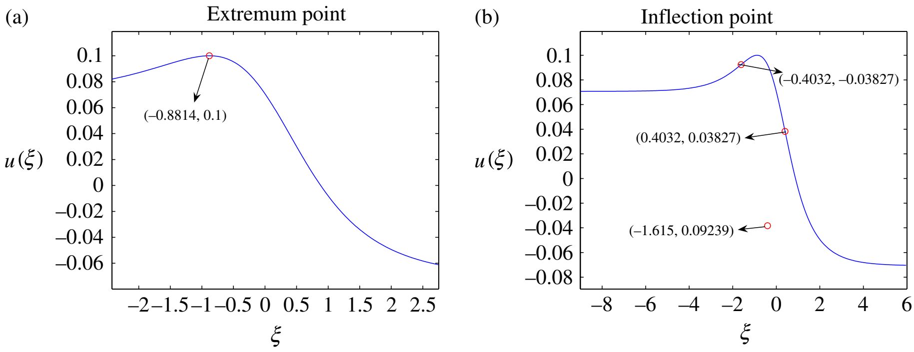

FIGURE 4. (a) Extremum point and (b) inflection points for $k = 0.1$ and $l^2 = -2$.

By choosing $k = 0.1$ and $l = -2$, setting $u'(\xi) = 0$ and using symbolic computation, the extremal points are found to be

$$(\xi, u(\xi)) = \left(\log\left(\frac{-500}{309} \pm \frac{\sqrt{19}}{309}\right), -0.0707\left(\frac{2.2360 + 2\cosh(\xi)}{1 + 2\sinh(\xi)}\right)\right)$$
$$\approx (0.4725 + \pi i, -0.031\,12) \quad \text{and} \quad (0.4899 + \pi i, -0.031\,12).$$

Points of inflection are obtained by setting the second derivative of u to zero in the usual manner to give $(-0.4815, 1.201 \times 10^{-5})$, $(0.4468, -0.030\,64)$ and $(0.5504, -0.033\,57)$. Similarly, for the case of $k = -0.1$ and $l^2 = -2$, the extremum point is $(0.4725, 0.032\,11)$ and the points of inflection are $(-0.4815, -1.201 \times 10^{-5})$, $(0.4468, 0.030\,64)$ and $(0.5504, 0.033\,57)$.

4.2. Case (b): $a_1 = \sqrt{2k^2(1 + l^2)}$ Upon differentiating equation (3.26),

$$u'(\xi) = \frac{k\,\mathrm{sech}(\xi)\sqrt{1 + l^2}}{2i}[-\tanh(\xi)(i + \sqrt{2}\sinh(\xi)) + \sqrt{2}\cosh(\xi)], \tag{4.2}$$

and once again differentiating to obtain u'', extremal and inflection points are found in the usual manner. When $k = 0.1$ and $l^2 = -2$, the extremum point is $(\xi, u(\xi)) = (-0.8814, 0.1)$ which is also represented in Figure 4(a). The points of inflection are $(-1.615, 0.092\,39)$ and $(\pm 0.4032, \pm 0.038\,27)$ for $\xi = -0.4032, -0.4032 + \pi i, -1.6149, 1.6149 + \pi i$. The points of inflection are shown in Figure 4(b).

Similarly, when $k = -0.1$ and $l^2 = -2$, the extremum point is $(-0.8814, -0.1)$ and the inflection points are found to be $(-1.615, 0.092\,39)$ and $(\pm 0.4032, \pm 0.038\,27)$ as portrayed in Figure 5.

5. Conclusions

In this paper, the Exp-function method is employed along with a computerized symbolic computation to obtain the single and combined generalized solutions of a

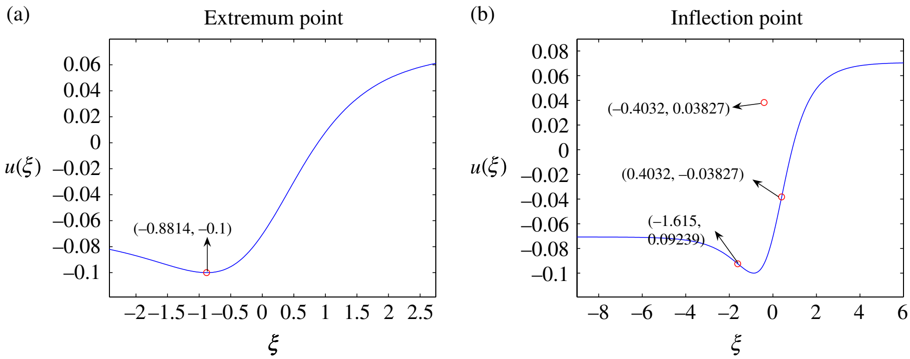

FIGURE 5. (a) Extremum point and (b) inflection points for $k = -0.1$ and $l^2 = -2$.

coupled $(2+1)$-dimensional nonlinear system of Schrödinger equations. We have also constructed the extremum point and points of inflection in order to address the general description of the solutions obtained. The results show that the Exp-function method is a powerful and promising new method to solve nonlinear evolution equations.

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