CORRESPONDENCE.

INTERPOLATION FORMULAE.

To the Editors of the Journal of the Institute of Actuaries.

SIRS,—The position of Lagrange among the pioneers of interpolation, appears to have been relegated to the background by most of the present-day authorities.

The simplicity and utility of his formulæ have been forsaken for the more complicated though admittedly elegant expressions, commonly known as "Central Differences."

Notwithstanding this I venture to maintain the position taken by Sir J. Burn and myself in our "Elements of Finite Differences", published in 1902 and now out of print, from which I would quote the following:

"If we consider Lagrange's theorem, where the interpolated "term is shown to consist in the algebraic sum of certain proportions "of each of the terms employed, the proportion being greatest in "the case of those terms nearest to the interpolated term" (p. 33).

. . . .

"If the central difference formula is taken as far as first, third, "or any odd order of central differences, we may obtain truer results "than would be possible by the use of the corresponding order of "ordinary differences, but it must be remembered that third central "differences require five given terms which would enable us to "obtain fourth ordinary differences, and so on. If, however, the "central difference formula is taken as far as an even order of "central differences, the result obtained is precisely the same as "when the ordinary differences are used.

"Central differences, therefore, do not appear to possess any "considerable value, since with the same data, we can quite as "easily obtain the same results with ordinary differences.

"If any of the adjustments in central differences are employed, "there is always the risk of obtaining a worse instead of a better "result, as is necessarily the case in any arbitrary adjustment." . . . . (p. 36.)

I am led to refer to this, by a consideration of the Actuarial Note on "Approximation by interpolation to the values of actuarial functions depending on two or more lives", *J.I.A.*, vol. lv., p. 255.

The second central difference formula given on p. 257, although very elegant, appears to be more correctly described as a hybrid first and third difference formula. In fact, the data employed obviously permit of no other interpretation.

This is disclosed by transforming the second central difference formula into one involving the original functions, by the substitution of $(u_{-1}-2u_0+u_1)$ for $\Delta^2_{-1}u_0$, when we obtain

$$u_{\theta\chi}=\frac{(\chi-0)(\chi-1)(\chi-2)}{(-1-0)(-1-1)(-1-2)}\left\{\frac{\theta-1}{0-1}u_{0-1}+\frac{\theta-0}{1-0}u_{1-1}\right\}$$

$$+\frac{(\chi-[-1])(\chi-1)(\chi-2)}{(0-[-1])(0-1)(0-2)}\left\{\frac{\theta-1}{0-1}u_{00}+\frac{\theta-0}{1-0}u_{10}\right\}$$

$$+\frac{(\chi-[-1])(\chi-0)(\chi-2)}{(1-[-1])(1-0)(1-2)}\left\{\frac{\theta-1}{0-1}u_{01}+\frac{\theta-0}{1-0}u_{11}\right\}$$

$$+\frac{(\chi-[-1])(\chi-0)(\chi-1)}{(2-[-1])(2-0)(2-1)}\left\{\frac{\theta-1}{0-1}u_{02}+\frac{\theta-0}{1-0}u_{12}\right\}$$

$$+\frac{(\theta-0)(\theta-1)(\theta-2)}{(-1-0)(-1-1)(-1-2)}\left\{\frac{\chi-1}{0-1}u_{-10}+\frac{\chi-0}{1-0}u_{-11}\right\}$$

$$+\frac{(\theta-[-1])(\theta-1)(\theta-2)}{(0-[-1])(0-1)(0-2)}\left\{\frac{\chi-1}{0-1}u_{00}+\frac{\chi-0}{1-0}u_{01}\right\}$$

$$+\frac{(\theta-[-1])(\theta-0)(\theta-2)}{(1-[-1])(1-0)(1-2)}\left\{\frac{\chi-1}{0-1}u_{10}+\frac{\chi-0}{1-0}u_{11}\right\}$$

$$+\frac{(\theta-[-1])(\theta-0)(\theta-1)}{(2-[-1])(2-0)(2-1)}\left\{\frac{\chi-1}{0-1}u_{20}+\frac{\chi-0}{1-0}u_{21}\right\}$$

$$-\frac{(\theta-1)(\chi-1)}{(0-1)(0-1)}u_{00}-\frac{(\theta-1)(\chi-0)}{(0-1)(1-0)}u_{01}$$

$$-\frac{(\theta-0)(\chi-1)}{(1-0)(0-1)}u_{10}-\frac{(\theta-0)(\chi-0)}{(1-0)(1-0)}u_{11}$$

The foregoing shows that the approximation given in the Note is the algebraic sum of three approximations:

(1) Assuming u_{xy} to be a function of the 1st degree in x and the 3rd degree in y.

(2) Assuming u_{xy} to be a function of the 3rd degree in x and the 1st degree in y.

(3) Assuming u_{xy} to be a function of the 1st degree in both x and y.

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And I venture to think that it is a moot point as to whether the combination of the three expressions is superior to any particular one. In any case, the use of the phrase "second central differences" seems open to objection.

Yours faithfully,

EDWD. H. BROWN.

Grosvenor Cottage,
Denton, near Canterbury.

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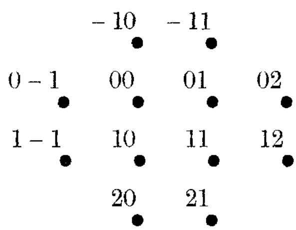
[We do not think there can be much doubt that the surface represented by the central difference formula in question, *i.e.*, the surface determined by the ordinates at the points -10, -11, $0-1$, 00, 01, 02, $1-1$, 10, 11, 12, 20, 21 in the following plan:



will in general give a better approximation to the value of the ordinate at a point in the central square than that given by any of the three component surfaces, *i.e.*, by the surface determined by the ordinates at -10, -11, 00, 01, 10, 11, 20, 21, the surface determined by the ordinates at $0-1$, 00, 01, 02, $1-1$, 10, 11, 12, or the surface determined by the ordinates at 00, 01, 10, 11.

We are glad to publish a plea for "Back to Lagrange", but we cannot agree that the central difference formula in question is "hybrid." The ordinary second difference formula

$$(1 + x\Delta_x + y\Delta_y + \frac{1}{2}x \cdot \overline{x-1}\Delta_x^2 + \frac{1}{2}y \cdot \overline{y-1}\Delta_y^2 + xy\Delta_x\Delta_y)u_{00}$$

can be written in the form

$$(1 + x\Delta_x + \frac{1}{2}x \cdot \overline{x-1}\Delta_x^2 + y\Delta_y)u_{00} + (1 + y\Delta_y + \frac{1}{2}y \cdot \overline{y-1}\Delta_y^2 + x\Delta_x)u_{00}$$
$$+ (1 + x\Delta_x + y\Delta_y + xy\Delta_x\Delta_y)u_{00} - 2(1 + x\Delta_x + y\Delta_y)u_{00}$$

i.e., as a blend of four distinct approximations, but it is not a hybrid first and second difference formula. Nor can we agree that the formula is incorrectly described as a second central difference formula. It is the central difference formula of Everett type (*i.e.*, in terms of u_{00}, u_{01}, u_{10}, u_{11}, and their even central differences) stopping at second differences.—EDS. *J.I.A.*]

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