This department welcomes short notes and problems believed to be new. Contributors should include solutions where known, or background material in case the problem is unsolved. Send all communications concerning this department to I. G. Connell, Department of Mathematics, McGill University, Montreal, P. Q.

ON SIMPLE RINGS WITH MAXIMAL ANNIHILATOR RIGHT IDEALS

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THEOREM. If R is a simple ring with 1 which contains a maximal annihilator right ideal then R is the endomorphism ring of a unital torsion-free module over an integral domain.

We first prove the following:

LEMMA. Let R be a ring with 1. If $a \in R$ such that $(a)^r = \{ r \in R \mid ar = 0 \}$ is a maximal annihilator right ideal then $\mathrm{Hom}_R(aR, aR)$ is an integral domain.

Proof. Let $N = \{ r \in R \mid r \cdot (a)^r \subseteq (a)^r \}$. Then by [1; Theorem 1, p. 25], $\mathrm{Hom}_R(aR, aR) = N/(a)^r$. Let $K = N/(a)^r$ and let $k_1, k_2 \in K$ such that $k_i = n_i + (a)^r$ for some $n_i \in N$, $n_i \notin (a)^r$, $i = 1, 2$. If $k_1 k_2 = 0$ then $n_1 n_2 \in (a)^r$ and $(an_1)^r \supsetneqq (a)^r$ since $n_1 (a)^r \subseteq (a)^r$ and $n_2 \in (an_1)^r$ but $n_2 \notin (a)^r$. This is impossible since $(a)^r$ is a maximal annihilator right ideal of R. Thus K must be an integral domain.

Proof of Theorem. Let $a \in R$ such that $(a)^r$ is a maximal

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annihilator right ideal and let $L = \{ r \in R \mid r \cdot (a)^r = (0)\}$. If $\ell \in L$, $n + (a)^r \in K$ where $n \in N$, we define $\ell \cdot (n + (a)^r) = \ell \cdot n$. Since $\ell \cdot n \cdot (a)^r = (0)$, L becomes a right K-module by this definition. If $x \in L$ and $x \neq 0$ then $(x)^r = (a)^r$ since $x \cdot (a)^r = (0)$ and $(a)^r$ is a maximal annihilator right ideal of R. Hence if $x \cdot (n + (a)^r) = x \cdot n = 0$ for some non-zero element $x \in L$ and some element $n + (a)^r \in K$, then $a \cdot n = 0$. Thus L is a torsion-free K-module. Now, define a mapping f from R into $\mathrm{Hom}_K(L, L)$ by $f(r)\ell = r \cdot \ell$ for all $r \in R$ and $\ell \in L$. It is easy to see that f is a ring homomorphism of R into $\mathrm{Hom}_K(L, L)$. The kernel of f is zero since simplicity of R implies that $L^\ell = \{ r \in R \mid r \cdot L = (0)\} = (0)$. Let $t \in \mathrm{Hom}_K(L, L)$ and $\ell \in L$. Since R is simple, $R = LR$ and $1 = \sum_{i=1}^{n} \ell_i r_i$ for some $\ell_i \in L$, $r_i \in R$ and some positive integer n. $t(\ell) = t(1 \cdot \ell) = t(\sum_{i=1}^{n} \ell_i \cdot r_i \cdot \ell) = \sum_{i=1}^{n} t(\ell_i) r_i \ell$ since t is a K-homomorphism and $r_i \cdot \ell \in N$ for all $1 \leq i \leq n$. Let $\bar{\ell}_i = t(\ell_i)$. Then $t(\ell) = (\sum_{i=1}^{n} \bar{\ell}_i r_i)\ell$ and $t = f(\sum_{i=1}^{n} \bar{\ell}_i r_i)$.

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