THE DOUBLE-MODE CEPHEIDS

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Abstract. Recent observations of double-mode Cepheids and proposed candidates are reviewed. It appears that the change of modal content reported in some stars may not be real. Observations show that the double-mode Cepheids are indistinguishable from normal Cepheids of similar period. This poses a severe problem for pulsation theory which predicts very low masses from the ratio of first overtone to fundamental period. Attempts to resolve this problem and the problem of modal selection are discussed.

INTRODUCTION

The phenomenon of double-mode pulsation is common among the lower luminosity stars in the Cepheid instability strip. Most δ Scuti stars are in fact multimode pulsators; those with larger amplitudes usually pulsate in two modes and are sometimes known as AI Vel stars or dwarf Cepheids. Among the Cepheids, the double-mode pulsators comprise nearly half the number of stars in the 2 - 4 day period range.

The double-mode Cepheids pose two serious problems for stellar pulsation theory. Firstly, the masses inferred from the ratio of their periods (presumed to be the fundamental and first overtone radial modes) are about half the value deduced from the theory of stellar evolution. This is by far the most serious of the various mass discrepancy problems encountered among the Cepheids. Secondly, the very existence of these stars is a problem because stable double-mode behaviour has not yet been produced in any Cepheid model.

Because of these problems, the double-mode Cepheids have received much observational and theoretical attention in recent years.

RECENT OBSERVATIONAL STUDIES

In Table 1 we summarize some data on the eleven known double-mode Cepheids. The data are mainly from Stobie & Balona (1979) and from references cited below in the discussion of individual stars. Despite intensive efforts to discover further candidates (Pike & Andrews 1979; Henden 1979, 1980; Barrell 1982b), none have been found. Three stars recently proposed as possible members of this class or closely related to them are also discussed below.

TU Cas. Faulkner (1977) analysed photoelectric observations of TU Cas and suggested the presence of a third periodicity which was interpreted as the second overtone at $P_2 = 1.25246$ days. Subsequently, Hodson, Stellingwerf & Cox (1979) repeated the analysis and concluded that P_2 was just an artifact of the data. More recently, Faulkner (1979) has disputed this conclusion.

The presence of a third mode would have very important consequences as it further constrains the theory. We have re-analysed all photoelectric

Table 1. Periods, period ratios and semiamplitudes of the light and radial velocity variations of double-mode Cepheids. Standard errors of these quantities are given on the second line whenever available.

Name	P_0	P_1	P_1/P_0	$A(V_0)$	$A(V_1)$	$A(RV_0)$	$A(RV_1)$
TU Cas	2.13931	1.518285	0.709708	0.292	0.103	13.5	6.6
	2	12	9	5	5	.9	.9
U TrA	2.568425	1.824876	0.7105037	0.297	0.125	14.7	7.5
	3	3	14	13	12	.4	.4
VX Pup	3.0109	2.1390	0.7104	0.166	0.144	6.3	9.0
	10	5	3	4	4	.7	.7
AP Vel	3.12776	2.19984	0.70333	0.275	0.138	13.2	9.8
	10	5	3	7	7	.8	.8
BK Cen	3.17387	2.22297	0.70040	0.245	0.106	13.2	10.0
	10	5	3	13	13	1.2	1.4
UZ Cen	3.33435	2.35529	0.70637	0.308	0.070	14.7	6.2
	11	6	3	10	9	.9	.8
Y Car	3.63981	2.55954	0.70321	0.266	0.120	11.1	6.8
	13	7	3	7	7	.9	1.0
AX Vel	3.673170	2.592924	0.705909	0.116	0.148	3.6	7.5
	14	5	3	5	5	.5	.5
GZ Car	4.15885	2.93372	0.70542	0.150	0.086	5.3	7.1
	17	9	4	5	4	1.0	1.0
BQ Ser	4.27073	3.01205	0.7053				
	?	?	?				
V367 Sct	6.29307	4.38466	0.696744	0.143	0.137		
	4	2	5	?	?		

observations including an early set by Bahner & Mavridis (1971) which was overlooked by the previous workers. Our conclusion is that the third periodicity definitely does not exist.

Hodson et al. also found that the light amplitude of the first overtone has decayed with respect to that of the fundamental by about forty per cent over the last sixty years. Niva (1979) found a similar, though smaller, trend in the radial velocities. The observational evidence in both cases is weak. The conclusion of the former workers rests entirely on visual observations conducted during the early decades of this century. Photoelectric observations obtained during the last twenty years do not show any trend. The amplitude changes in the radial velocities found by Niva are not statistically significant, differing by no more than one standard deviation.

U Tra. Faulkner & Shobbrook (1979) found an increase in amplitude of the first overtone with respect to the fundamental in the photoelectric data prior to 1977. More recently, the same authors (Faulkner & Shobbrook 1983) repeated the analysis in a different manner and concluded that the change in modal content reported earlier was not significant, though their most recent data do show some increase in first overtone amplitude. They concluded that the apparent changes in modal amplitudes may not be real, but are probably due to seemingly random fluctuations in the light curve. They also find that the first overtone is subject to a greater degree of incoherence in phase and variability in amplitude than is the fundamental.

AX Vel. This is the only double-mode Cepheid in which the first overtone light amplitude is larger than that of the fundamental. As Table 1 shows, there are other double-mode Cepheids in which the first overtone predominates in radial velocity amplitude. Shobbrook & Faulkner (1982) obtained new observations of AX Vel. They conclude that, like U TrA, the light variations of the fundamental mode are stable and coherent, but that the first overtone suffers small phase fluctuations which can be as large as 0.07 periods over a five year interval.

Y Car. This is the only double-mode Cepheid known to be a spectroscopic binary (Stobie & Balona 1979). Balona (1983) obtained an orbital period of 993 days, which is typical for binary Cepheids. The mass function leads to a lower limit of 1.2 solar masses for the companion if the evolutionary mass is used for the primary.

PROPOSED CANDIDATES

CO Aur. This star was first observed by Smak (1964) to check its classification as an RV Tau star. He could not detect any periodicity in his photoelectric observations. The same result was obtained by DuPuy & Brooks (1974) who re-observed the star. Recently, Mantegazza (1983) has concluded that CO Aur is a double-mode Cepheid by re-analysing Smak's data. He finds $P_0 = 1.784$, $P_1 = 1.4255$ days. If this star is indeed a double-mode Cepheid, then it has the shortest period of the group, but more important it has the unusual ratio $P_1/P_0 = 0.80$.

This ratio is incompatible with current ideas on double-mode Cepheids and could lead to a reappraisal of the subject. Antonello & Mantegazza (1983) obtained further photometry which is claimed to confirm CO Aur as a double-mode Cepheid.

We have analysed all available photometry for this star including the data by Antonello & Mantegazza which they kindly put at our disposal prior to publication. We confirm that the P_0 period is indeed present in all the data sets, though the alias at P = 2.273 days is almost as strong. However, when the data is prewhitened by P_0, there are no peaks in the power spectrum at or near P_1 or in fact any peak much above noise level. This not only applies to the set of combined data, but also to the individual data sets. The strongest indication for P_1 is in Smak's data, but this peak is only some 10 or 20 per cent stronger than the noise level and can hardly be regarded as significant. We were nevertheless puzzled by a diagram in Mantegazza (1983) and Antonello & Mantegazza (1983) which purports to show the variation of the fundamental prewhitened by the first overtone and vice versa. We could not reproduce this diagram by the normal prewhitening procedure. Instead, if we assumed the presence of the second period, found the best fitting coefficients by a double-mode least squares solution, and removed the coefficients pertaining to P_0 or to P_1, we could reproduce their diagram by prewhitening with the remaining coefficients. This procedure is incorrect, since the inclusion of an arbitrary periodicity distorts the least squares solution in such a way as to reflect the periodicity of the missing coefficients in the prewhitened data.

In conclusion, CO Aur cannot be regared as a double-mode Cepheid at this stage. The 1.78 day periodicity is however certainly real, but further observations are required to elucidate the nature of this star.

HD161796. This is an F3Ib star located at high galactic latitude. Burki, Mayor & Rufener (1980) found a semi-regular variation in both photometry and radial velocities with a characteristic period of about 54 days. Percy & Welch (1981) found a period of about 60 days for the light variations during 1979, but in 1980 the period seemed to have shortened to about 40 days.

Fernie (1983) obtained further photometric data during 1980 and confirmed a period of 43 days for this season. He concludes that the 60 day to 43 day ratio for the two seasons is what one expects for a star in the process of switching from fundamental to first overtone radial pulsation. Takeuti (1983) has calculated that a Cepheid of 30 solar masses would have the observed periods.

In a semi-regular variable, one could expect to find by chance the correct period ratio in two or even more consecutive cycles. To conclude that this star is a Cepheid undergoing mode transition one would have to show that for a long time the star was pulsating in predominantly one period and after an interval in predominantly the other period. We analysed the available photometric and radial velocity data, but failed to confirm this behaviour. Fernie has tried to explain the 54 day periodicity in the data by Burki et al. as an aliasing effect, but we cannot support this conclusion. Their radial velocity data show a clear unaliased peak at 53 ± 1 days. The combined photometric data shows a peak at 56.7 days; in both cases a period near 60 days can be excluded.

HR7308. This star is unique among the Cepheids. Not only does it have the shortest period (1.49 days) but the amplitude itself varies with a period of about 1200 days (Breger 1981). This amplitude modulation can also be interpreted as a beating of two very nearly equal periods. In this case HR7308 could be classified as a double-mode Cepheid, though with $P_1/P_0 = 0.999$, the pulsation modes cannot be the same as in normal double-mode Cepheids.

We have analysed the radial velocity observations of Burki, Mayor & Benz (1982) which constitute a large, accurate and homogeneous body of data with excellent phase coverage over nearly one complete beat cycle. Table 2 shows the results of a periodogram analysis with successive prewhitening. At each stage the frequency range between 0 and 1 cycle day^{-1} was searched for peaks. Except for those in the table, no others were found.

It is apparent that the data cannot be represented adequately by just two periods. The results of Table 2 show that the main pulsation at 0.6709 cycles day^{-1} is flanked symmetrically on each side by a pair of equally spaced frequencies, the spacing being equal to the beat frequency. When all five oscillations are used, the standard deviation of the least squares fit is 0.7 km s^{-1} per observation.

There are two interpretations of this frequency spectrum. If all five frequencies are real oscillations, then the most obvious interpretation is that they are caused by rotationally split quadrupole or higher order nonradial oscillations. On the other hand one could intepret this spectrum as asymmetric amplitude modulation of a single radial pulsation. It is easy to show that in this case the spectrum should consist of the main pulsation flanked by equally spaced components with symmetric decaying amplitudes. The results of Table 1 seem to confirm these conditions within the errors. In this interpretation there should also be a relationship between the phases, but this could not be checked as the phases are strongly affected by small uncertainties in the frequencies. An analysis of the available photometry confirms the pattern shown by the radial velocities, though the smaller amount of data and the uneven phase coverage pose some problems.

Table 2. Frequencies of radial velocity variation in HR7308.

f (day^{-1})	Semiamplitude (km s^{-1})	Indentification
0.6709	4.98 ± 0.07	f_0
0.6717	2.07 ± 0.07	$f_0 + \Delta f$
0.6699	1.82 ± 0.07	$f_0 - \Delta f$
0.6727	1.00 ± 0.07	$f_0 + 2\Delta f$
0.6689	0.61 ± 0.07	$f_0 - 2\Delta f$

The evidence from the power spectrum of the radial velocities and the fact that Burki et al. (1982) were able to perform a Baade-Wesselink analysis assuming radial pulsation strongly suggests that HR7308 is pulsating in a single amplitude modulated radial mode. It is certainly not possible to interpret the observations in terms of a double-mode pulsation.

PHYSICAL AND PULSATIONAL PROPERTIES

The evidence that double-mode Cepheids are Population I objects of high mass is very compelling. Perhaps the best proof of this is V367 Sct which is a member of the young open cluster NGC6649. Barrell (1980) measured the mean radial velocity of the Cepheid and showed that it was the same as that of the cluster stars. The three brightest main sequence stars were found to be emission-line objects, removing an objection to cluster membership (Flower 1978).

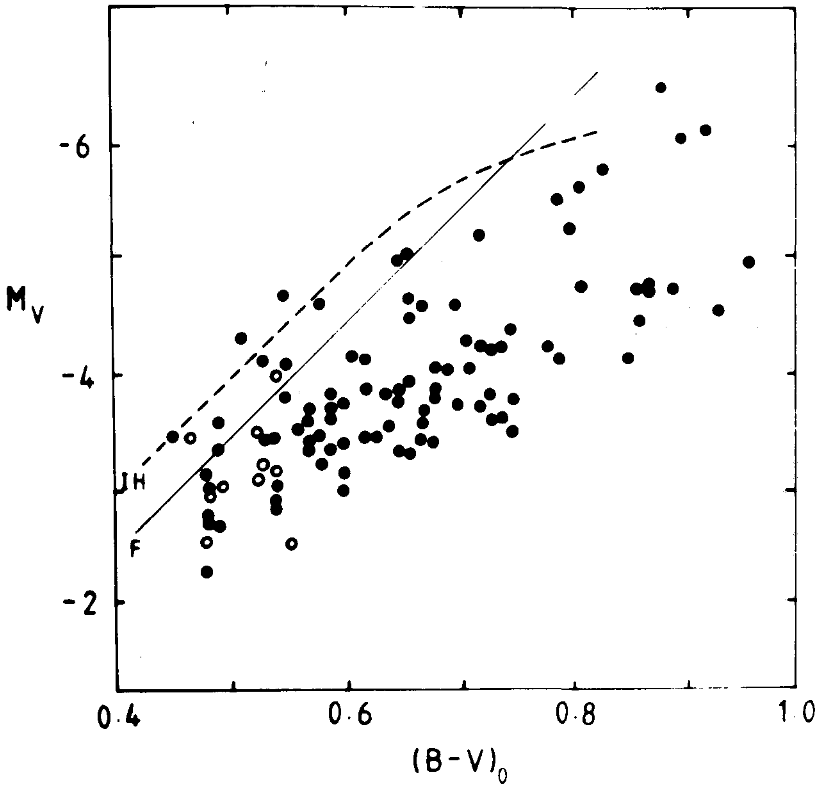
Barrell (1982a) established that the iron abundance in ten double-mode Cepheids is consistent with that of the sun. Balona & Stobie (1979a) obtained Baade-Wesselink radii for eight double-mode Cepheids and showed that they obey the same period - radius relationship as normal Cepheids. Niva & Schmidt (1979) found the same result from the radius of TU Cas.

Barrell (1981) estimated the effective temperatures of ten double-mode Cepheids by measuring the Hα line profiles and comparing them with profiles calculated from model atmospheres. This method is independent of interstellar reddening and insensitive to differences in surface gravity. Within the uncertainties, Barrell found all double-mode Cepheids to have the same mean effective temperature. Balona & Stobie (1979a) used BVRI photometry to estimate the reddening and determine the intrinsic colours of eight double-mode Cepheids. Comparison with single-mode Cepheids (Fig. 1) shows that there is no preferred location for the double-mode Cepheids in the instability strip. The double-mode RR Lyrae variables in M15 also show no preference for a particular location in the instability strip. Unlike the double-mode Cepheids, they lie in a narrow period range in which no single mode RR Lyraes exist (Cox, Hodson & Clancy 1983).

The only observational evidence which distinguishes the double-mode Cepheids from their single mode counterparts is the remarkable discovery by Barrell (1978) of strong Hα emission occuring at seemingly random phases. There is a possibility that this emission may be an instrumental effect since the observations were made during the early days of operation of a new instrument. Barrell subsequently observed these stars for several more nights without finding any emission (Feast, priv. comm. via Barrell). Nevertheless, there is no reason to suspect any fault in the instrument and confirmation of this effect would be most important. Henden, Cornett & Schmidt (1982) could not find any abnormalities in spectra of TU Cas which included the Hα line.

The pulsational properties of double-mode Cepheids are not significantly different from those of normal Cepheids. Both have the same phase lag between the radius and light variation (Balona & Stobie 1979a). The ratio of temperature variation to radius variation, which in normal Cepheids depends on the mean temperature or period (Balona & Stobie 1979b), follows the same relation for the double-mode Cepheids. The ratio of radial velocity to light amplitude, which is different for the fundamental mode and for the first overtone mode, is easily explained as a result of radial pulsation (Stobie & Balona 1979). Balona & Stobie (1979a) find that the fractional radius variation amplitude of the first overtone dominates that of the fundamental for the hotter double-mode Cepheids. The double-mode RR Lyraes seem to follow this pattern: being hotter than any of the double-mode Cepheids the first overtone always has a larger light amplitude than the fundamental mode (Cox et al. 1983).

Fig. 1. The colour - magnitude diagram for normal Cepheids (closed circles and double-mode Cepheids (open circles). The transformed fundamental (F) and first overtone (1H) blue edges calculated by King et al. (1973) are shown.

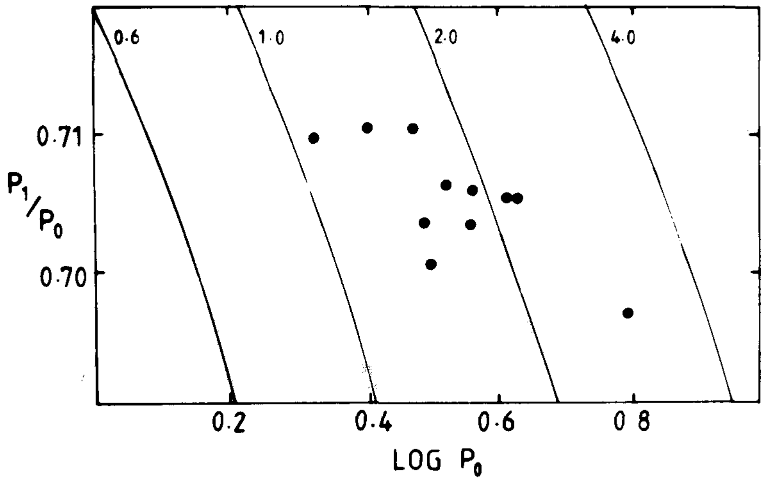


THE MASS DISCREPANCY PROBLEM

The well known mass discrepancy problem for double-mode Cepheids is most easily understood from the diagram of Fig. 2, adapted from Petersen (1979). This shows the variation of period ratio with fundamental period. Also shown are Petersen's (1979) calculations for standard Cepheid models of various masses. It is clear that the predicted masses of double-mode Cepheids lie in the range 1 to 3 solar masses, while standard evolution theory predicts masses in the range 5 to 7 solar masses for models with the same fundamental period. The models with evolutionary masses have a considerably larger period ratio than is actually observed. Inclusion of convection (Cogan 1977; Deupree 1977; Saio *et al.* 1977) or rotation (Cox *et al.* 1977; Deupree 1978) has only a marginal effect on the period ratio.

Simon (1982) has proposed an idea which may solve the mass discrepancy problem without abandoning the assumptions made in standard evolution and pulsation models. He suggests that the present metal opacities may be underestimated. Increasing these opacities by a factor of 2 to 3 in the region 10^5 °K to $2X10^6$ °K would reduce the period ratio of models with evolutionary masses to the observed values. It will also leave intact the good agreement obtained for the double-mode RR Lyrae period ratios (Cox *et al.* 1983) since these stars have a small metal abundance. Furthermore, it may help to solve the long standing problem of β Cep variability.

Fig. 2. The period ratio vs. the logarithm of fundamental period showing the loci of homogeneous composition models (labeled in solar masses) by Petersen (1979).



There are reasons to suspect that the present metal opacities are too low, but whether they could be increased to the required level has not been demonstrated.

If one accepts the present metal opacities, then one can invert the problem and look for unusual physical properties in the outer layers where the effect on the period ratio will be at its greatest.

Stothers (1979) has proposed that a magnetic field of several hundred gauss at the surface could reconcile the masses. The magnetic fields observed in Cepheids are much lower than this, but it is expected that the magnetic field will be tangled by convection so that the observed field will indeed be smaller. Recently, Stothers (1982) found that the presence of magnetic fields will also lead to better agreement in the predicted and observed phases of secondary bumps in many Cepheids. However, the calculated models have larger amplitudes than actually observed. The required field is also much lower than that needed to explain the period ratio in double-mode Cepheids.

The modification which has received the most attention is that proposed by Cox et al. (1977). They found that by increasing the helium abundance in the outer layers of the star the density gradient is reduced and the period ratio can be lowered to the observed values. The abundance required to match the observed ratios is $Y = 0.65$ up to a temperature of 10^5 °K. They suggest that preferential depletion of hydrogen by a stellar wind could produce the required helium enrichment (Cox, Michaud & Hodson 1978). A surface helium enrichment of $Y = 0.75$ is required to match the phases in bump Cepheids. A serious problem in this model is the instability of the enriched helium zone due to the inverted molecular weight gradient which should lead to rapid mixing. Also it is not certain that the rate of hydrogen depletion is high enough to compensate for this effect. Henden et al. (1982) looked for, but could not find, any spectroscopic evidence for a stellar wind in TU Cas. Luck & Lambert (1981) deduced an appreciable helium overabundance in some Cepheids, particularly TU Cas, from the observed depletion of oxygen. This finding does not help the mass discrepancy problem since the helium enrichment is not confined to the outer layers and the density gradient is not appreciably affected by homogeneous changes in composition. Takeuti (1980) has suggested that enhanced helium abundance in the outer layers would result in increased convectional energy. Spontaneous Hα emission as reported by Barrell (1978) could then occur.

THE MODAL SELECTION PROBLEM

The second major problem, one which has received much attention recently, is the cause of double-mode pulsation. One possibility which must be rejected is that these stars are in a state of mode transition. The expected lifetime of double-mode pulsation is much too short to explain the observed numbers of double-mode Cepheids (Stellingwerf 1975).

One of the problems which any theory of modal selection must attempt to answer is the very narrow range of observed period ratios. A range in magnetic field strengths or helium abundances in the outer layers will lead to a much larger variation in this ratio. The most promising answer to this problem and one which probably underlies the modal selection problem is Simon's (1979) resonance hypothesis. He suggests that the double-mode phenomenon involves a resonant interaction between the fundamental and first overtone modes when the sum of their frequencies is close to or equal to the frequency of the third overtone. This hypothesis might also underlie the Hertzprung progression of bumps in the light curves of normal Cepheids. In this case the resonance is between the fundamental and second overtone modes, the latter having nearly twice the frequency of the former. Resonance may also be one of the factors determining the limiting amplitude in Cepheids.

In the resonance theory, the double-mode Cepheids should occupy a region in the instability strip where a close resonance with the third overtone is possible. Simon, Cox & Hodson (1978) examined a series of nonlinear pulsation models of double-mode Cepheids near this resonance, but could not find sustained double-mode behaviour. Petersen (1979, 1980) found that with normal composition the resonance condition did confine the period ratio to a narrow range, but only models with unrealisticly high helium enrichment in the outer layers could have period ratios in the observed range if evolutionary masses are assumed. Takeuti & Aikawa (1980) and Uji-Iye (1980) have found that for a reasonable helium enrichment only models with masses much smaller than evolutionary masses could simultaneously satisfy the period ratio and resonance conditions.

Aikawa (1984) investigated the two and three-mode resonances analytically. The approach to resonance is accompanied by mutual period changes in the models involved, but the change in period ratio is too small to be of significance. Dziembowski (1982) has developed a second-order theory of nonlinear mode coupling in oscillating stars. He obtains an explicit formula for the coupling coefficient and finds an equilibrium solution for two and three interacting modes and develops stability criteria.

Regev & Buchler (1981) and Buchler & Regev (1981) have developed the two-time formalism. This is a useful analytical and computational tool for finding stable double-mode pulsation and for studying the evolution towards it. In addition, this formalism has the advantage of making more apparent the physical significance of various factors which is normally hidden in numerical simulations. Regev & Buchler (1981) suggest that persistent double-mode pulsation may not necessarily require the presence of internal resonances, but these resonances might play an important role in the evolution towards such pulsations. Regev, Buchler & Barranco (1982) include the treatment of resonances in the two-time formalism. Buchler (1983) finds that near resonance a steady state solution where only one amplitude is non-zero does not exist. He suggests that Simon *et al*. (1980) may have made an unfortunate choice for their stellar model.

CONCLUSIONS

Our understanding of double-mode pulsation in Cepheids is still very limited, though some progress has been made. The mass discrepancy problem has still not been solved satisfactorily. The theories which have been proposed suffer from several weaknesses, not the least being that they involve too many free parameters and are practically impossible to disprove. Simon's (1982) idea of enhanced metal opacities seems to be the most promising line of investigation at present.

Until numerical models have been constructed which undergo persistent double-mode behaviour, the cause of double-mode pulsation will remain a problem. Resonances seem to play a very important role in this process, but their effect is still not fully understood. It is particularly important to study the behaviour of the pulsations during the evolution towards and away from the resonance condition.

It is of great importance to confirm the presence of Hα emission in double-mode Cepheids as reported by Barrell (1978), as there is some suspicion that this could have been an instrumental effect. It would also be very useful to extend the method of temperature determination by measuring the Hα profiles to normal Cepheids. This could allow more accurate definition of the location of the double-mode Cepheids within the instability strip.

Further photometric and radial velocity observations of these stars is highly desirable. It is of great importance to verify the finding of Faulkner & Shobbrook (1983) that the first overtone is subject to greater instability and phase jitter than the fundamental. This has cast doubt on the reality of changes in modal content which have been reported in some double-mode Cepheids. It also appears to offer a clue to the resonance hypothesis.

Finally, it would be of great interest to search for double-mode Cepheids in the Magellanic Clouds. This could lead to new insights into these enigmatic objects.

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