THE STATISTICAL ANALYSIS OF ANISOTROPIES

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One of the many uses to which a radio survey may be put is an analysis of the distribution of the radio sources on the celestial sphere to find out whether they are bunched into clusters or lie in preferred regions of space. There are many methods of testing for clustering in point processes and since they are not all equally good this contribution is presented as a brief guide to what seem to be the best of them. The radio sources certainly do not show very strong clustering and may well be entirely unclustered so if a statistical method is to be useful it must be both powerful and flexible. A statistic is powerful in this context if it can efficiently distinguish a weakly clustered distribution of sources from an unclustered one, and it is flexible if it can be applied in a way which avoids mistaking defects in the survey for true peculiarities in the distribution of sources.

An ideal survey for statistical analysis is one made with infinitessimal telescope beam area, infinite receiver signal to noise ratio and absolutely constant gain sensitivity across the surveyed region. Measured against this standard every real survey is defective and every catalogue of sources is inaccurate to a greater or lesser degree. First, the finite beam area causes sources to be blended together or 'confused', the principal effect of which is to mistake a close pair of sources for one source: the catalogue thus exhibits an artificial absence of close pairs of sources. Second, the chief effect of noise is the reduction of sensitivity to sources at low galactic latitudes caused by the galactic background radiation. Third, the variation of gain can in principle give a variety of effects but in practice one effect predominates. Most surveys are drift-scan surveys in which the Earth's rotation sweeps a fixed beam across different declination strips each day: variations in receiver calibration from day to day then cause the sensitivity to depend on declination but not on right ascension and it is as a result prudent to be prepared for artificial variations of source density in the direction of increasing declination. Some statistics are better than others in discriminating instrumental effects from celestial effects,

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as will be indicated below.

It is convenient to divide clustering statistics into two classes and to discuss the classes separately:

NUMBER DENSITY STATISTICS

In this class the measured flux density of a source is only used to decide whether the source is bright enough to be included in the analysis; all sources which are included are treated equally, with no further reference to their flux densities. In this class two statistical methods stand out as being the best available.

In binning analysis the surveyed area is divided, somewhat arbitrarily, into a number of disjoint 'bins', and the number of sources in each bin is counted. These numbers are tested by a straightforward application of chi-square (e.g. de Vaucouleurs 1971) or perhaps by a more oblique method such as statistical reduction (Zieba 1975) to find out whether they are consistent with the distribution of sources being a realisation of a Poisson process. Binning analysis is the best method for the very largest scales, such as are met in testing for differences between the north and south galactic hemispheres, because of its simplicity and because it copes with arbitrarily shaped bins. The problem of galactic bachground noise can be met by excluding areas near the galactic plane, and the drift scan effect by choosing bins which have the same shape and size and differ only in right ascension.

Power spectrum analysis consists of defining a spiky function over the surveyed area by erecting a delta function at the position of each source, representing this function by a Fourier series and employing the squares of the values of the coefficients in this series as statistics (Bartlett 1964, Webster 1976a). A spherical harmonic series may in principle be employed instead of the Fourier series (Peebles 1973) but in practice the extra computing involved makes this method less attractive. Power spectrum analysis is powerful and flexible because each wave in the Fourier series contributes information which is practically independent of the information from every other wave, so a large number of waves may be investigated in order to maximise the statistical power, and if the coordinate system is carefully chosen any instrumental effect such as the drift scan effect only contaminates a very few waves which can be discarded from further consideration without significantly weakening the test. The test thus beats binning analysis on all but the largest scales (even if many different binning configurations are tried in order to increase the power analogously to trying many waves) because it is not clear how independent the results of the different binning configurations are, and because most of the configurations are not immune from any given instrumental effect.

The family of neighbour-statistics, and in particular the method of nearest neighbour analysis, is well known but well worth avoiding. Compared with power spectrum analysis this method is weak, inflexible and full of pitfalls in its application (e.g. Webster 1976b).

LOG N / LOG S STATISTICS

In this class the slope of the log N / log S relation in one bin of the surveyed area is compared with that in other bins to find out whether the balance of bright sources and faint sources varies with direction. The log N / log S relation for the whole survey can usually be represented quite accurately by a straight line power law fit, so the statistics chosen are usually estimators of the slope of the power law which best fits the sources in each bin. The most powerful statistic therefore is the maximum likelihood estimator of the slope (Crawford et al. 1970) because of a general theorem (Kendall & Stuart 1967) that the ML estimator of a population parameter has a smaller sampling variance than any other estimator of that parameter. Almost as good are the least-squares and 'luminosity-volume' estimators (Pearson 1974) but the most obvious method of comparing the ratio of bright sources to faint in one bin with the ratio in other bins is weak because much of the information in the measured flux densities is wasted.

All of these tests are variants of the method of binning analysis and therefore suffer from the relative inflexibility and weakness of binning analysis mentioned above when comparing it with power spectrum analysis. A variant of power spectrum analysis called cross spectrum analysis (Peebles 1974) retains the power and flexibility of power spectrum analysis but seems never to have been applied to our problem.

Moving on now to mention the results of clustering analyses carried out to date, it seems to me that there is precious little good evidence in favour of significant clustering of the radio sources. Many investigators have indeed reported that they were unable to distinguish the actual distribution of sources in various catalogues from random distributions; my own power spectrum analyses of the 4C, GB, MC1, PKS 2700 MHz, B2 and 5C5 catalogues have led me too to this conclusion. Of the reports of significant clustering:

i) a few have been shown to be due to unanticipated instrumental problems or errors of analysis;

ii) some have not been supported by the results of comparable surveys of the same areas;

iii) many are analyses of surveys which have not been exhaustively shown to be of sufficiently high quality to put the possibility of instrumental error beyond doubt; and

iv) none has produced a result with a statistical significance of more than a few standard deviations anyway so the clustering has never been demonstrated beyond reasonable doubt.

It thus seems to me that there is no good evidence that the radio sources are distributed on the celestial sphere in any fashion other than uniformly, independently at random. This lack of structure is of considerable fundamental significance quite apart from its bearing on whether the measured log N / log S curves are representative of the radio source population in the Universe as a whole. In the first place it is direct evidence for the assumptions of isotropy and homogeneity of the Universe on large scales which underly the Friedmann cosmological models and the Robertson-Walker line element. For example, the power spectrum analysis of the Bologna B2 survey shows that the number of radio sources (and presumably also the density of matter) within a cube of side 1 Gpc or larger varies by less than about 3% as the cube is moved from place to place. This information on the large scale homogeneity is better than that which can be had from the observed isotropy of the microwave background radiation for several reasons. First it is more secure because the background radiation may have been scattered by free electrons after the epoch of recombination and this scattering, depending on circumstances, may make the surface brightness of the sky more or less patchy than it was at recombination. Second, the density contrast of the large scale irregularities is expected to grow with time so an upper limit at a late epoch ($z \sim 1 - 3$ for the radio sources) is more valuable than one at an early epoch ($z \sim 1{,}000$ for the microwave background). The homogeneity revealed by the radio sources confirms a point first made in connexion with the isotropy of the microwave background radiation: the Universe is more homogeneous than it has any known reason to be, in that the density of radio sources in widely separated regions is constant despite the fact that the radio sources formed before a light signal had time to travel from one region to the other. Furthermore the lack of clustering is inconsistent with the local hypothesis for quasars if the quasars are expected to show the clustering and superclustering shown by galaxies in the same region of space. Finally any model of radio sources in which the sources originate in pairs or higher multiples (such as Arp's 1967 model) cannot account for a significant fraction of the radio sources in the Universe because the multiplicity would show up as clear clustering.

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DISCUSSION

Peterson: How does the method of projection of a sphere on to a plane affect the amplitude of the Fourier coefficients? Does it dilute clustering that may be present?

Webster: Scarcely at all. Certainly, the clustering is not diluted. The chief effect is a small distortion of shapes, so that a circular cluster becomes elliptical, but this is an unimportant matter.

Jauncey: If you know what sort of clustering to look for, it seems that you can make a much stronger statement than just the general tests for anisotropy.

Miley: About how many bright radio sources could be haloes of widely spaced doubles which are not recognised as belonging together.

Webster: There cannot be more than about 5% of faint sources which are unrecognised wide doubles, or the power spectrum analysis would show it. This may or may not help decide about the bright sources.

Arp: The statement that steep spectrum sources seen at high frequency arise preferentially from relatively bright galaxies makes it seem natural that the Northern Hemisphere anisotropy is due to the greater number of local supercluster galaxies in the Northern Hemisphere.
In that case it is unsophisticated to talk about North-South differences. The brighter galaxies actually are in the projected area of the supercluster. That is a sharper, more sensible test of the anisotropy which would resolve the problem. Along that line, and in contradiction to what Adrian Webster claimed, if you plot the 3CR quasars between V = 17 and 19 Mag. you see they are missing in the 13 to 17^h region and fall in the 8^h - 12^h region with the bulk of the local super cluster galaxies.

Kellermann: I would like to make some historical and perhaps provocative remarks which may stimulate further discussion.

About twenty years ago, not too far away from this room a radio source survey was made at frequency of 81 MHz. Only a few of the approximately 2000 sources were identified (indeed as it turns out only a few more were ever real), but nevertheless profound cosmological conclusions were reached based on the unexpected large deviations of the N(S) relation from the "expected" -1.5 power law, and it was claimed that the observed isotropy excluded interpretations based on a local anomaly. Later surveys give results very much closer to the canonical -1.5 law, especially when differential source counts are used in place of the misleading integral counts used previously.

Today the experimental results are very much improved : Surveys now exist over a wide range of wavelengths which actually measured flux densities down to very low values. The data presented today by Pauliny-Toth and Wall at 5 GHz are very different from the old data. Except for the strongest 100 or so sources, the results agree quite well up to ~ 10^5 sources sr^{-1} with the -1.5 slope corresponding to a random distribution of sources in a hypothetical Eucledean Universe. And for the strongest 100 sources which do derivate from this law, the evidence for isotopy is not clearly established. Although the evidence for anisotropy is only marginal, the important thing is that neither is the evidence that these sources are isotropically distributed established. Since the derivation from the -1.5 law is no greater than the apparent anisotropy, it is not clear that this apparent steep slope is of any cosmological significance. The high degree of isotropy which is observed for the weak sources is not relevant.

But although the experimental situation has changed drastically during the past twenty years, the conclusions drawn from the source counts has not! What has changed is the argument, which now goes that because the radio sources are so distant, the expected effect of the redshift is to depress the counts below the -1.5 law, so that even the observed value of -1.5 requires evolution. But most of the identified radio galaxies are not very distant, and there are still some (perhaps only one or two) who question the cosmological interpretation of the redshift. The question which we really want to answer is the same as was originally posed years ago : What can we learn from the source counts alone, independent of any assumption about the nature of the redshift? After all, if the quasars are cosmological, the great abundance of large redshifts is immediate and obvious evidence without further analysis.

I cannot help but be impressed by the apparent coincidental agreement of the 5 GHz source counts with that expected from a random distribution of sources which are either relatively nearby or which are located in a non-expanding universe. I often wonder how the course of radio astronomy and cosmology might have been changed, had the advance of radio technology been reversed, and the 2C survey made at centimeter rather than meter wavelength.

Rees: It seems to me that the anisotropy problem differs in one important respect. Whereas most of the other tests are controversial because they involve both the physics of the sources and the cosmological model, the question of large scale inhomogeniety is essentially independent of the physics. So it might be better to examine the extent to which it can be improved.

Webster: To improve the isotropy tests one simply wants more and more radio sources. Give me a catalogue with 10^6 sources, then the accuracy can be improved by a factor of 10 over the results from existing catalogues. I feel it won't get much better than that because a radio telescope with sufficient resolving power to find a million sources starts to split up the double radio sources. Then we would not be able to separate out multiplicity within radio sources from multiplicity within groups of sources. I suspect that the limit would not then come down very much farther.

Rees: I would like to question the usual assumption that the tests of large scale isotropy and homogeniety are more powerful when applied to the microwave background rather than to radio sources. When we look at the microwave background, we are looking back to redshifts of the order of 10^3. In the standard models for the evolution of density perturbations, these have had time to grow by a factor of 10^3 since then. So, in order to make a test as good as a 10% test of a scale of 1 Gpc you have to look for fluctuations of order 10^{-4} in the microwave background. Looked at this way, it is not so obvious that the radio source tests lose out.

Another point is that, particularly in low density cosmologies, you can look for fluctuations in the density that would not necessarily give rise to fluctuations in the velocity field. This is something that is not so easily done with the microwave background. So it seems to me that the Webster type tests on radio sources have certain advantages over tests on the microwave background.