

MATHEMATICAL ANALYSIS OF POPULATION GROWTH SUBJECT TO ENVIRONMENTAL CHANGE

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Many ecosystems are pressured when the environment is perturbed, such as when resources are scarce, or even when they are over-abundant. Changes in the environment impact on its ability to support a population of a given species. However, most current models do not take the changing environment into consideration. The standard approach in modelling a population in its environment is to assume that the carrying capacity, which is a proxy for the state of the environment, is unchanging.

In effect, this assumption also posits that the population is negligible compared to the environment and cannot alter the carrying capacity in any way. Modelling the interplay of the population with its environments is important when describing the varying factors that exist in the system. This objective can be achieved by treating the carrying capacity as time- and space-dependent variables in the governing equations of the model. Thereby, any changes to the environment can be naturally reflected in the survival, movement and competition of the species within the ecosystem.

In this thesis, detailed investigations of several mathematical models for population growth are presented. Formulating the carrying capacity as being time-dependent was the fundamental approach used to describe a varying environment, which resulted in the investigation of a nonautonomous equation [1, 4]. This approach led to models that directly couple the dynamics of one or two species with their environments.

To obtain this result, the carrying capacity was modelled as a state-variable [5]. In these models, the ultimate state for the ecosystem depends on the resource enrichment parameter that was found to have significant impact on the growth of a population, leading to either coexistence or extinction of a particular species [2, 3]. Other dynamical behaviours, including oscillations in population, have also been found to exist [2].

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Varying the carrying capacity has given a better understanding of population growth when subjected to environmental change. This thesis serves as another platform for ecologists and biologists to further investigate the importance of a varying environment and could be applied in future population-growth studies.

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