REFRACTION IN TYCHO BRAHE'S SMALL UNIVERSE

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ABSTRACT. Tycho Brahe's work set a new standard of objectivity in empirical science. Aiming at an observational accuracy of one minute of arc he had to introduce corrections of atmospheric refraction. This proved to be anything but trivial, mainly because Tycho adopted the traditional value of 3' for the parallax of the sun. This parameter measured the size of the entire planetary system which Tycho refrained from vizualizing larger than conventionally accepted.

1. ACCURATE AND REPRODUCIBLE OBSERVATIONS

Tycho Brahe (1546-1601) earned fame as the Renaissance reformer of observational astronomy par excellence. During 20 years of happy work, 1576-97, on Hveen he single handedly built a totally new foundation for the age-old practice of positional astronomy. This included pushing the accuracy of angular measurements to the edge of the optical resolving power of the un-aided human eye, i.e. about one minute of arc, which was better than his predecessors by a factor of say ten <5>.

Moreover, he spent a fortune and took pains to ensure reproducibility of any measurement that could be repeated. This included building some 30 astronomical instruments <6>. Having used a particular instrument so that he knew its flaws and merits, Tycho took it as a model for constructing a new instrument of the same type, but perhaps scaled up or down and made out of other materials. He only accepted observational results when determined independently by several instruments, and he only incorporated a new instrument into his observatory when it produced results unambigously compatible with the output of the body of already controlled and reliable equipment of observation.

So Tycho's work set a new standard of objectivity in empirical science.

2. TYCHO BRAHE'S SMALL UNIVERSE

Accordingly, Tycho Brahe had to make whatever corrections necessary

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to reach the one minute of arc he aimed at. Thus he accepted the traditional measure of 3' for the parallax of the sun in the horizon.

Already for the year 1578 his observational diary <1, X, 58> contains a small table of solar parallax corrections which he applied throughout his career. In 1588 he began printing his principal work on the new star of 1572, Astronomiae instauratae progymnasmata. Here you find Tycho's new theory for the solar motion including an elaborate table of parallax for the sun at its minimum, mean, and maximum distance <1, II, 82-85>.

The solar parallax is an additive correction to observed heights of the sun, and its correct value for the sun in the horizon is 9", or only one twentieth of the 3' adopted by Tycho. So he contaminated all his solar altitude measures by errors of (3' - 9")$\cdot$cos h, shown here as the π-curve in Figure 1. As we are going to learn, Tycho's own data could have led him to ignore solar parallax. But instead he

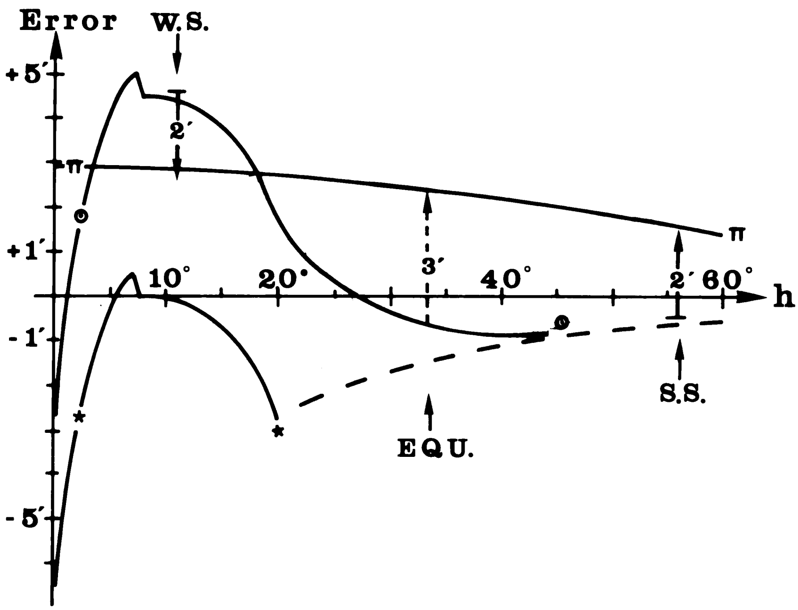

Figure 1. Errors in Tycho Brahe's table of solar parallax, π as a function of the angular height, h, above the horizon, and errors in his tables for the refraction of the light from the sun, ⊙, and from the fixed stars, ★ respectively. The broken line shows the effect of neglecting refraction altogether which Tycho did for solar light above the height of 45°, and for stellar light above the height of 20°. The vertical arrows point to the heights of culmination for the sun at winter solstice, W.S., the equinoxes, EQU., and summer solstice, S.S., at latitude = 56°.

clung on tradition, and as a result he had to invent strange ideas concerning atmospheric refraction to attain consistency in his solar theory.

In the case of the solar parallax too much was at stake because this parameter measures the earth-sun distance, on the average equal to 1150 terrestrial radii according to Tycho, or only one twentieth of the true distance. And Tycho accepted the relative dimensions within the planetary system as established by Copernicus. So his earth-sun distance made for a measuring rod of the entire solar system. Tycho's scientific imagination simply refrained from visualizing a solar system substantially larger than conventionally accepted. This is on par with his resisting the still more dramatic expansion of the sphere of the fixed stars which would result from accepting the Copernican idea of the annual motion of the earth around the sun. It is well-known that the latter led Tycho to produce his own compromise of a world picture <3>.

3. TWO TABLES OF REFRACTION

Tycho Brahe is also known as the first astronomer to apply corrections of atmospheric refraction in his reduction of astronomical observations. Strange to say, in his Progymnasmata he gave three different tables of refraction. They relate to the bending of light from the sun <1, II, 64, 76-82>, the moon <1, II, 136>, and the stars <1, II, 286-287> respectively. Here we shall leave out the lunar table which runs very close to the table for the sun. Tycho's refractions for the stars come to zero already at the altitude of 20° where the correct refraction is about 2.5'. His refractions for the sun, up to the altitude of 20°, are higher than those for the stars by a constant difference of 4.5', and for the rest they drop to zero at the altitude of 45° where the correct refraction is 1'.

Refraction is a subtractive correction to observed heights above the horizon, and the errors committed by applying Tycho's tables for the sun and for the stars are shown in Figure 1. It is seen that his solar refractions were virtually correct very close to the horizon, and again above 20°, whereas the same is true of his stellar refractions corresponding to intermediate altitudes between say 4° and 17°.

Unlike parallax, refraction was a new correction which Tycho introduced only reluctantly and rather late in his career. In his diaries of observation you find substantially sound determinations of solar as well as stellar refractions during the period 1584-1588. For a selection of his data corresponding to a wide range of altitudes <1, X, 223, 293, 352-3, XI, 2, 223> I find an average error of -0.6' ± 0.8'. So it is manifest that Tycho acted in bad faith when adding 4.5' to get his solar refractions at low altitudes. In his Progymnasmata, however, he covered this by relating that he obtained his stellar refractions by subtracting instead 4.5' from the solar ones <1, II, 287>.

4. STARS, REFRACTION, AND GEOGRAPHICAL LATITUDE

Tycho Brahe was the first astronomer since the time of Hipparchos who set out to restore the theories of the wandering stars by establishing improved positions for the fixed ones. His 'Hveenian Durchmusterung' led to the Progymnasmata star catalogue of 777 fixed stars, and near the end of his career, in 1598, he produced an extended version with a thousand entries.

To establish a reliable measure of right ascension Tycho made a large number of fundamental star observations. He used Venus as an intermediate body of observation to relate positions of the sun determined in day time and positions of bright stars observed after sunset. Of course the procedure included proper allowance for the effects of refraction and of the supposed parallaxes for the sun and for Venus.

However, out of more than one hundred such observations Tycho actually used no more than three <1, II, 160-169>. Probably he could not reduce the remaining bulk of material to yield consistent results, but he proved ingenious enough to avoid depending on dubious data. He selected twelve couples of symmetrical observations from the period 1582-1588, with one on the western horizon in the evening, and the other on the eastern horizon in the morning <1, II, 170-196>. Thus whatever be the corrections due to refraction and parallax they would

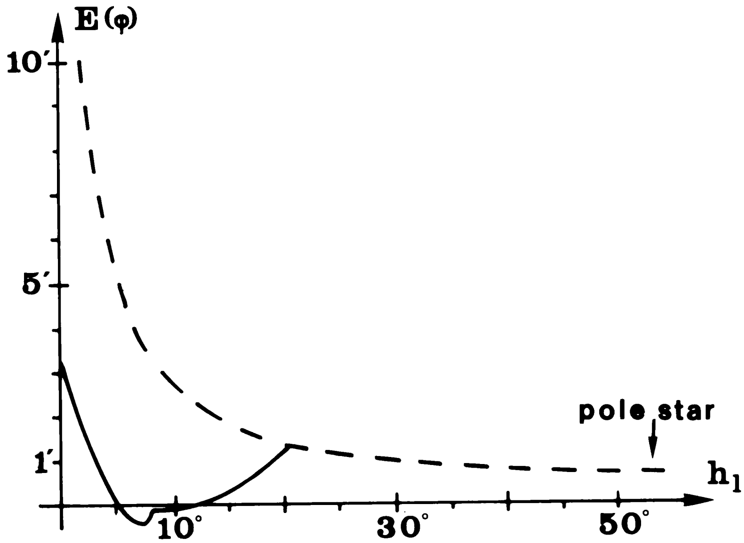

Figure 2. When determining the geographical latitude of the observer, φ from the upper, h_u, and lower, h_l, heights of culmination for circumpolar stars, one introduces an error, E, by neglecting refraction (broken line), or by using Tycho Brahe's table of refraction (full curve).

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counteract and neutralize the effect on the averaged result within each symmetrical couple of observations.

Then Tycho invented a further check of consistency within the basic coordinates for selections of fundamental stars. He added together cyclically the increases in right ascension from one star to the next to get a full circle of 360° <1, II, 198-206>. From an appendix to Tycho's diary of observation for the year 1584 we learn that these checks could succeed only when he used the value 55;54,30° for the geographical latitude of Uraniborg, and when corrections for refraction were applied.

So the final confirmation of Tycho's stellar refractions and of his Uraniborg latitude - the latter correct to within a few seconds of arc - can be taken as a byproduct of his establishing a star catalogue. So this is in principle independent of his solar theory. The geographical latitude is determined from observed upper and lower heights of culmination for circumpolar stars. To obtain the same latitude from observations of different stars you must introduce corrections for refraction. Figure 2 shows how far Tycho succeeded in this matter.

5. SOLAR THEORY, ERRORS IN BALANCE

From his observation of the heights of culmination for the sun at the summer solstice and at the winter solstice, the latter diminished by his stellar refraction, Tycho could have confirmed the latitude of Uraniborg as found above to within a small fraction of a minute of arc. As a further result he could have derived the obliquity of the ecliptic correct to within one third of a minute of arc. All this involves no solar parallax, so from his own data Tycho could have inferred that solar parallax would at most amount to some fraction of a minute of arc. As Y. Maeyama has traced in a penetrating study <2>, this line of thought lies behind a handful of solar theories two generations after Tycho, namely those of Riccioli, Horrocks, Wendelin, Streete, and Mercator.

Tycho himself, however, argued otherwise. From the relation for the declination of the sun at summer solstice, equal to the obliquity of the ecliptic, ε = observed height + parallax - refraction - $(90^{\circ} - \varphi)$, he found in 1584 $\varepsilon = 23;31,30^{\circ}$, too high by 2'. It is indicated by the arrow at S.S. in Figure 1, that this goes closely with his ignoring refraction above 45° and his erroneous assumption of solar parallax. In the sequel, therefore, he had to balance at winter solstice, W.S. in Figure 1, the supposed solar parallax and the error of 2' in the obliquity by assuming an excessive bending of the solar light. So that is why Tycho produced his peculiar table of solar refraction with low altitude entries 4.5' above the corresponding - and sound - ones in his table of stellar refraction.

So far you may read off in Figure 1 that Tycho's corrections introduced an error of 3' into the declination of the sun at the equinoxes. Thus from correct altitude observations he would think that the equinox occurred, when in fact the declination of the sun was minus 3' which

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would happen early by 3 hours in the spring and late by 3 hours in the autumn <1, II, 15>.

All this led Tycho to assume in his solar theory an eccentricity of the solar orbit too large by 6%. So in the end Tycho Brahe's solar theory could predict, to within a fraction of a minute of arc, his sound and reproducible observations of the heights of culmination of the sun. But this was possible only because his peculiar table of refraction for the sun was calibrated to absorb the errors in his solar parallax and in his obliquity of the ecliptic.

Within Tycho's scientific career a dramatic story begins on his 30th years birthday, 14.12.1576, when observation of the sun at winter solstice led him to the value of $55;49^{o}$ for the geographical latitude of his newly founded Uraniborg observatory. For the following day he found $55;55^{o}$, i.e. a value higher by 6', from observation of the pole star <1, X, 42-43>.

In his diaries of observation you may trace how, during the following decade, he struggled with this entirely unexpected problem. His efforts to reconcile the two values include instances of subtracting solar parallax instead of adding it, and of doubling the application of errors in his division of the measuring circles of his instruments <1, X, 55>.

Here we have learned how Tycho finally succeeded in creating a structure of errors in balance.

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Discussion:

CORBIN Did Tycho ever try to reconcile the discrepancies between solar plus stellar observations by assuming some day versus night atmospheric or instrumental difference?

MOSESGAARD No, he mainly tried to account for this through his incorrect value of the parallax, although he did assume that the greater distance of the stars might be a factor.

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