THE THEORY OF RADIATION DRIVEN STELLAR WINDS AND THE WOLF-RAYET PHENOMENON

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I. INTRODUCTION

Peter Conti has a tradition of always talking about O-type stars at Wolf-Rayet symposia, and Wolf-Rayet stars at O star symposia. Since there is no well-developed theory for the origin of the winds of WR stars, it is my pleasure to join Peter's tradition, and to talk mainly about the theory of radiation driven winds in OB stars. The advantage of OB stars is that there exists a fairly complete wind theory, which agrees well with the available observations. The question is, can the mass loss observed from Wolf-Rayet stars be explained by a version of this wind theory which is scaled to the conditions found in the envelopes of Wolf-Rayet stars? The topics I consider are:

- The calculated radiation pressure in OB stars, and its dependence on temperature, density, and chemical composition.
- A comparison between predicted and observed mass loss rates and terminal velocities for OB stars.
- The applicability of the standard radiation driven wind models to Wolf-Rayet stars.
- Speculations on how Wolf-Rayet stars achieve their enormous mass loss rates within the context of the radiation pressure mechanism.

II. THE LINE ACCELERATION IN OB STARS

Detailed formulas for the outward radiation pressure on spectral lines have been given by Castor, Abbott, and Klein (1975, hereafter CAK). A more comprehensive discussion of the results discussed in these first two sections is given by Abbott (1981). Conceptually, the net line acceleration can be thought of as depending on two factors

$$\text{Line Acceleration} = \text{function(efficiency of absorption, wind dynamics)}. \quad (1)$$

The "efficiency of absorption" factor describes such things as the total number of absorbing lines, their distribution in frequency, and

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their optical depth. This first factor determines the rate the momentum of the radiation field is transferred to the lines in the wind, i.e., what fraction of the emergent flux is intercepted by lines. The net acceleration is this rate of momentum transfer divided by the mass of the unit column that is doing the absorbing. The size of this unit mass depends on the density and velocity gradient of the wind, i.e. the second factor of equation (1), through the specific combination of variables given by

$$t \equiv \sigma_e \rho \, v_{th}/(dv/dr) = \frac{\dot{M}}{4\pi r^2 v(dv/dr)} \quad . \qquad (2)$$

The calculation of the efficiency of absorption is a well-defined problem. Translating this into a value for the line acceleration is not well-defined, because of the t factor of equation (2), which is only known as a function of radius after solving the hydrodynamic equations. In other words, two models having identical distributions of lines and the exact same emergent fluxes can have completely different values for the line acceleration if their adopted velocity laws v(r) are different. So, what I'm going to describe is how the efficiency of absorption depends on T_{eff}, N_e, and chemical composition. This is accomplished by calculating the line acceleration for a grid of assumed values of t. Then, in the next section, the wind model of CAK is used to translate these calculated efficiency factors into the observable quantities $\dot{M}$ and v_∞.

There are two steps to calculating the line acceleration. First, you must compute the line opacity. This means acquiring atomic data for a large number of lines from a large number of elements, and then coming up with a description of the ionization/excitation equilibrium of the winds of OB stars. The first obstacle has been overcome, as I have compiled/computed a complete list of gf values for lines in the first six stages of ionization for the elements H-Zn. The ionization balance remains an uncertainty, and I've used both the radiative equilibrium models and empirical ionization fractions derived from UV observations. Fortunately, the differences between the various ionization models are less than a factor of 2.

Given the line opacity, one must then solve the radiative transfer problem in the wind. The standard treatment (e.g. Mihalas 1978, section 15-4) ignores two potential uncertainties, both of which are also neglected in this work. First, the velocity law may be nonmonotonic, as in the periodic shock model proposed by Lucy (1981) to explain the X-ray emission from the winds of OB stars. Second, the frequencies of individual lines within multiplets often overlap at medium and high wind velocities because of the Doppler shift. For a typical model with T_{eff} = 40,000 K, I find that 50-75% of the total line acceleration comes from overlapping lines.

Examples of the calculated line acceleration are given in figure 1 for typical values of the wind parameter t. The acceleration is

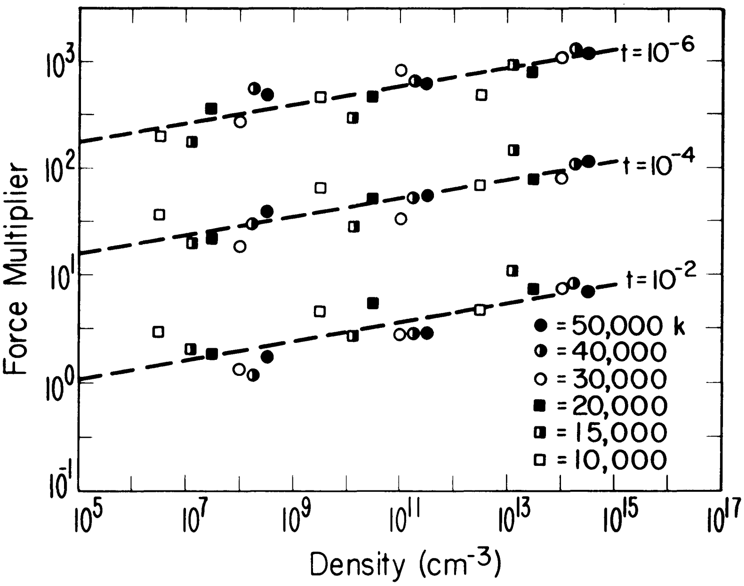

Figure 1. The calculated line acceleration versus density for temperatures in the range 50,000 ⩾ T_{eff} ⩾ 10,000 K. The vertical axis gives the line acceleration in units of the continuum radiation force, $\sigma_e F/c$. The horizontal axis is the electron density in the wind. The acceleration is shown for three typical values of the wind parameter t, defined by equation (2). Dashed lines are least-squares fit, all of which have a slope of 0.1.

remarkably constant with temperature, and depends on wind density to the 0.1 power for all temperatures and t values. The scatter about the mean values is ~0.2 dex. The line acceleration drops off sharply for temperatures cooler than 10,000 K.

Most relevant to the discussion of winds from WR stars is the dependence of the line acceleration on chemical composition, which is shown in figure 2. Two kinds of chemical composition are considered. In the first, the number abundances of all metals are increased or decreased by a constant factor, which is intended to represent stars born in galactic or extragalactic environments of different metallicity. As shown in figure 2, the line acceleration increases with increasing metallicity, which is natural since metals provide the lines that absorb the radiative momentum. Combining this dependence on metallicity with the numerical results shown in figure 1 gives a calculated line acceleration a_L which is fit to within a factor of ~2 by the analytic expression

$$a_L = \frac{\sigma_e F}{c} k\ t^{-\alpha}\ (N_e)_{11}^{0.1}\ (Z/Z_\odot)^{1-\alpha} \quad , \qquad (3)$$

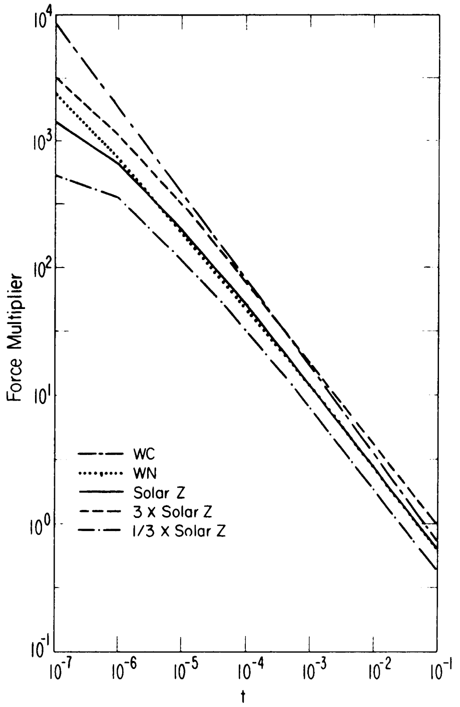

Figure 2. The dependence of the line acceleration on chemical composition. (———) Solar metallicity taken from Cameron (1973). (— — —) All elements heavier than helium had a number abundance 3 times solar. (— • —) All elements heavier than helium had a number abundance 1/3 of solar. (••••) Mass fractions of X = 0.0, Y = 0.96, and CNO number abundances of $[C/H_e] = 2.7 \times 10^{-4}$, $[N/H_e] = 1.0 \times 10^{-2}$, and $[O/H_e] = 2.3 \times 10^{-4}$. The mass fractions of all other elements were unchanged. (— - —) Mass fractions of X = 0.0, Y = 0.0, and C, N, O and N_e mass fractions of C = 0.27, N = 0.0, O = 0.71, and $N_e = 0.024$. All other mass fractions were unchanged. All models are for $T_{eff} = 40{,}000$ K.

where $k = 0.28$, $\alpha = 0.56$, and $(N_e)_{11}$ is the electron density in the wind in units of 10^{11} cm^{-3}.

The second chemical composition considered is material of solar metallicity, which has been processed by nuclear burning in the stellar core. The model denoted WN in figure 2 has the composition of the end products of hydrogen burning. There is no significant increase in the line acceleration for this case. This is expected, since the main change has been to convert hydrogen to helium, neither of which provides any substantial radiative acceleration. The model denoted WC has the composition of the end products of helium burning. The acceleration is enhanced by roughly a factor of 2. Since the envelopes of WC stars usually contain significant amounts of helium, the enhancement of their acceleration should be intermediate between the solar and WC case pictured in figure 2.

III. COMPARISON TO OBSERVATION

Applying the CAK wind model to the line acceleration of equation (3) gives the predicted terminal velocities and mass loss rates of

$$v_\infty \simeq 1.0\text{–}1.5\ v_{esc}$$

$$\dot{M} \simeq 9 \times 10^{-16} \frac{(L/L_\odot)^2 (Z/Z_\odot)}{(M_{eff}/M_\odot)} M_\odot\ yr^{-1} \quad , \tag{4}$$

where $v_{esc} = [2GM_{eff}/R_*]^{1/2}$, $M_{eff} \equiv M(1-\Gamma)$, and $\Gamma = \sigma_e L/(4\pi GMc)$. A strong correlation is observed between v_∞ and v_{esc}, but the constant of proportionality predicted by equation (4) is a factor of ~2 smaller than observed in the O-type stars. Panagia and Macchetto (1982) explain this discrepancy in terms of multi-scattering of photons in a radiation-driven flow.

The observed mass loss rates are shown in figure 3. Using reasonable estimates for the masses of the stars observed, the predicted and observed mass loss rates are in complete agreement. I conclude that there are sufficient lines to drive the mass loss observed in OB stars, and that the simple wind model of CAK also gives the proper dependence of $\dot{M}$ of L. Observations of the necessary precision are not yet available to test the predicted scaling of $\dot{M}$ on M_{eff} and Z.

IV. APPLICABILITY OF THE RADIATION-DRIVEN WIND MODEL TO WOLF-RAYET STARS

Figure 3 also illustrates the two major challenges that Wolf-Rayet stars pose for radiation-driven wind theory:

1) The Efficiency Problem. Wind theory must explain how Wolf-Rayet stars are roughly 10 times more efficient in converting radiative momentum to wind momentum than OB stars of comparable luminosity and temperature. (The terminal velocities of OB and Wolf-Rayet stars are similar.)

2) The Momentum Problem. Although the observational uncertainty is large, the mass loss rates of Wolf-Rayet stars do not seem to correlate with L_{bol}. One is therefore faced with the problem that the amount of wind momentum ejected from the star does not depend on the amount of radiative momentum available to drive the flow.

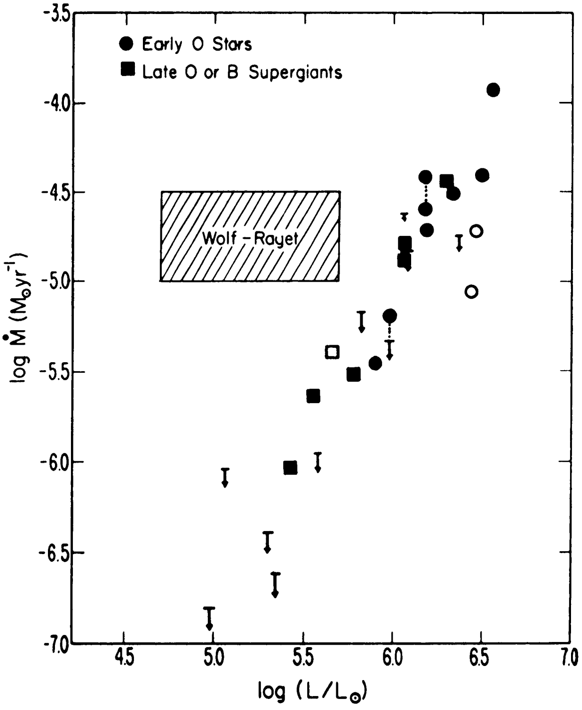

Figure 3. Observed mass loss rates from VLA observations at 5 GHz from Abbott, Bieging, and Churchwell (1981, and this symposium). Filled symbols are detections, open symbols are probable detections, and arrows are upper limits.

The envelopes of Wolf-Rayet stars differ from those of OB stars in several ways that affect the radiation pressure. Table 1 summarizes the most important differences, and their probable effect on the predicted mass loss rates of Wolf-Rayet stars.

Table 1. Differences Between OB and WR Stars That Affect the Radiation Pressure

Factor	OB Stars	WR Stars	Estimated Increase in $\dot{M}$
Chemical Composition	~Solar	Enriched	WN = none WC $\leqslant$ 3
Mass	Pop. I	Undermassive	$\leqslant$2
Ionization/ Excitation	~Radiative Equilibrium	Radiative+Collisional (Very Broad Range)	$\leqslant$2
Radiative Transfer	Core/Halo	Extended Continuum, Creation of Photons	?

The first, and most obvious, difference is chemical composition. As discussed by many contributors to this symposium, all Wolf-Rayet stars are hydrogen deficient to some degree. The atmospheres of the WN sequence are enriched by the products of hydrogen burning. As shown in figure 2, this produces no increase in the line acceleration, and hence in the predicted mass loss rate. The atmospheres of WC stars are enriched by the products of helium burning. This time there is an increase in the line acceleration of up to a factor of 2, which translates into, at most, a factor of 3 increase in the predicted mass loss rate. I conclude that differences in chemical composition are not the major factor in producing the high efficiency outflows from Wolf-Rayet stars.

A small effective mass also leads to an enhanced mass loss rate, as shown by equation (4). A summary of observations of binary systems by Massey (1981) indicates that Wolf-Rayet stars are undermassive for their luminosity by typically a factor of 2 when compared to OB stars. In principle, an unlimited increase in $\dot{M}$ can be achieved by decreasing the $(1-\Gamma)$ factor. However, a drastic reduction in $(1-\Gamma)$ would also produce a drastic reduction in the terminal velocities, because $v_\infty \propto M_{\rm eff}^{1/2}$. This is not observed. I conclude that roughly a factor of 2 increase in $\dot{M}$ of Wolf-Rayet stars is attributable to their reduced mass.

The bulk of the gas in the winds of OB stars is in an ionization state which approximates radiative equilibrium. A much broader range of ionization and excitation is observed in Wolf-Rayet stars. It is likely, therefore, that Wolf-Rayet stars have more lines to intercept the radiation flux than do comparable OB stars. Since the OB stars already block roughly half of the emergent flux, the maximum increase in $\dot{M}$ expected from this effect is a factor of ~2.

The largest uncertainty in radiation pressure models of Wolf-Rayet stars is in the radiative transfer. With few exceptions the winds of OB stars are optically thin, so that the core/halo approximation applies to the radiative transfer. One assumes that each point in the wind sees unattenuated radiation from the stellar core, with no contribution to the continuum radiation field from gas in the extended envelope. In Wolf-Rayet stars, on the other hand, it is clear from the emission line spectrum that the winds are not transparent, and several studies conclude that optical depth unity in electron scattering occurs at high velocities (e.g. Castor and Nussbaumer 1972; Hartmann and Cassinelli 1977). However, the lack of emission at the continuum opacity jumps argues that the thermalization optical depth in the continuum occurs at small velocities, where the extension effects of the atmosphere are negligible.

There are no quantitative calculations of the effects the above differences will have on the radiation field, and hence on the line acceleration. I discuss below some of the qualitative effects the envelope structure of Wolf-Rayet stars might have on the mass loss rate.

1) Thermalization Depth. At points deeper in the atmosphere than the thermalization depth in the continuum, there is no line acceleration, because the radiation field becomes isotropic. The maximum wind density that can be driven by radiation pressure is therefore that which gives a thermalization optical depth of unity for the wind. No further increase in $\dot{M}$ is possible once this maximum density is attained. Observationally, Wolf-Rayet stars are at, or near, this maximum wind density. I speculate that this mechanism may be the thermostat that limits all Wolf-Rayet stars to nearly the same maximum rate of mass loss.

2) Electron Scattering Depth. In Wolf-Rayet stars the radius of electron scattering optical depth unity, R_{es}, is larger than the radius of the stellar core, R_c, which I define as thermalization optical depth unity. This means that the core/halo approximation completely breaks down in Wolf-Rayet stars, because the continuum radiation field will be modified between the stellar core and the absorbing lines in the wind. What, if any, effect this will have on the line acceleration is unknown.

To the extent that $R_{es} >> R_{th}$, photons scattered by electrons at a point in the envelope can traverse to the opposite side of the envelope without being occulted by the stellar core. This creates the possibility of multi-scattering of photons by the continuum, with a corresponding increase in the continuum radiation pressure, in a manner analogous to that of multi-scattering in lines described by Castor (1979).

3) Photon Creation. Densities in the winds of Wolf-Rayet stars are large enough to collisionally produce line photons in the wind, as evidenced by the large ratio of emission to absorption observed in the P-Cygni profiles of resonance lines in Wolf-Rayet stars. These additional line photons, coupled with the lack of photospheric absorption profiles in the stellar core, make a very favorable environment for line acceleration. By contrast, in OB stars there is a lack of line photons, because collisions are negligible and the absorbing lines often have strong photospheric absorption features. This difference could produce a large differential in the mass loss rates between OB and Wolf-Rayet stars of otherwise comparable luminosity.

I conclude that within the confines of the CAK wind model, Wolf-Rayet stars are at most a factor of 4 more efficient at driving mass loss than OB stars. To attain the observed mass loss rates of Wolf-Rayets using a radiation pressure mechanism requires a more sophisticated approach than the CAK model. I have identified two aspects of the radiative transfer that are likely to yield higher mass loss rates for Wolf-Rayet stars, when a physically more realistic treatment is employed. Whether or not the enhancements are sufficient to explain the winds from Wolf-Rayet stars is an open question, but one which I hope to answer in time to present the results at the next O star symposium.

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DISCUSSION

Maeder: The question raised by Dr. Abbott was what makes WR stars more efficient for mass loss than O stars. In that context shouldn't we remember vibrational instabilities, which for a given stellar mass become more and more severe as one moves from the Main Sequence stars towards helium stars ?

Abbott: I agree. I think this mechanism should also be considered as a means to remove the hydrogen envelopes in the advanced stages of hydrogen burning and the early stages of central helium burning. At present, there is no way to directly calculate the effect of underlying pulsations on the radiation-driven envelope.

Hummer: On your last viewgraph, you showed a very wide range of spectral characteristics corresponding to a fixed (to within a factor of 3) value of the mass loss rate. However, the particular value you chose of 3×10^{-5} $M_\odot$/y, is the maximum value observed, at least for the majority of the stars. Is it possible that the " throttling effect " to which Dr. Cassinelli referred, is responsible for this diversity of spectral features ? In other words, if you had chosen a smaller value of mass loss rate, would the picture be much different ?

Abbott: Several OB stars in Cyg OB2 have measured mass loss rates exceeding 3×10^{-5} $M_\odot$/y, so the "throttling effect" does not appear to operate at the 3×10^{-5} $M_\odot$/y level for OB stars.

Stenholm: Hogg and Abbott have observed WR stars with the same telescope at the same wavelength. Hogg explained how he selected his stars. Now I have a late question to Abbott: what criteria did you use in your selection of stars ?

Abbott: We essentially selected a distance-limited sample of stars, with a few extra thrown in to give a complete coverage of spectral types. However, many of the distances have gotten revised since we first started.

Van der Hucht: You have presented the best available radio fluxes of WR stars. In the derivation of the mass loss rates now the parameter with the largest uncertainty is the distance which goes to the power of 1.5 in the Wright and Barlow mass loss rate formula. From the small scatter in the average mass loss rate for the stars you observed, one could conclude that the photometric distances are remarkably well determined, i.e. that we could safely use the best available absolute magnitude law, or putting it another way, that there exists a good relation between subclass and intrinsic parameters. Could you comment on that ?

Abbott: The mass loss rates have a dispersion of roughly a factor of two. Whether this is small or large probably depends on the eye of the beholder.

De Loore: I would like to give an answer to the remark of Anne Underhill about evolution. The fact that stars have a mantle, or a corona, has nothing to do with evolution. Stellar evolution is determined by the nuclear reactions occuring in the interiors. These reactions influence the behaviour of the stars: radius, energy transport (radiation convection), and have as a consequence that the chemical composition of the star changes. The existence of coronae has nothing to do with evolution. I agree that due to evolution, and the behaviour of the different regions of the stellar interior according to this, can have their effect on the nature and extent of these outer regions, if as a consequence of the evolution, turbulence or convection is created, these regions can create a supplementary (or different) energy source (mechanical energy) leading to hot outer layers. But this is not an intermediate effect of the evolution, but a consequence of the changing character of the stellar structure.

Lortet: I would stress that the absence of any trend of mass loss with spectral type is not amazing. We really need to avoid commenting on plots of anything as a function of spectral subtypes, as if their sequence was representative of one evolutionary scenario. Rather, we have to discover how to group and order spectral subtypes into families, using the different possible evolutionary scenarios available (as described by Dr. Maeder) and a close examination of the spatial distribution of WR stars by subtypes in our galaxy and nearby ones.

Underhill: If the continuum optical depth is unity at all wavelengths in a wind, then you are close to the bottom layers of what is usually called the photosphere of the star. It is ridiculous to speak of such optical depths due to electron scattering. Consider a density of 10^{10} electroms $/cm^3$ extending for 100 $R_\odot$. This distance is as long as anything suggested from the study of eclipsing binaries for the length of a WR atmosphere, and the suggested "average" (constant) density is a generous estimate. Then $\tau = N_e \sigma_e l$ is equal to 0.046.

Cassinelli: The electron scattering optical depth is greater than unity in a WR wind. For example if one assumes a standard CAK velocity law one gets $\tau = 1$ where v = 400 km/s for $\dot{M} = 3\times10^{-5}$ $M_\odot$/y.

Underhill: The measured infrared and free-free fluxes tell us simply that a plasma with a gradient in density is present. Another way of obtaining a "suspended" plasma of the required type is possible than outflow with conservation of mass in spherical shells. Suchpossibilities should be considered before concluding that $\dot{M}$ has the values you have quoted. These values are upper limits to what is needed to explain the observations. For more than two years I have been saying that to understand the spectra of luminous early-type stars, it is necessary to think of the atmosphere as being divided into two parts: a photosphere and a mantle. Conditions in the photosphere can be described in terms of T_{eff} and log g; conditions in the mantle are due to the deposition of non-radiative energy and momentum which have come from the envelope of the star below the photosphere. We do not yet understand the details of how this happens, but the process appears to be related to the stage of evolution of the star.

Carrasco: Within the framework of radiatively accelerated winds, how can you produce the important observed changes in both the mass loss rates and the velocity laws in time scales of months to years while the stellar luminosity has remained constant.

Abbott: These calculations are for steady-state, so they cannot address the question of time variability.

Cassinelli: Your explanation of the "WR phenomenon" is an interesting one. I wonder if the strength of the emission lines and IR excess couldn't be explained as just due to differences in the stellar radius ? That is, with $\dot{M}$ fixed, the optical depth in the wind increases as R increases. The continuum calculations that I have carried out show that the star's IR excess is increased if the star's radius is decreased. Also the strength of the recombination emission lines depends on N_e^2, so these should also increase with a decreasing size of the star.

Abbott: That is true, but to get the contrast in profiles shown here one needs a greater density enhancement than can be provided by a shrinking radius.

Vanbeveren: I am a little surprised why in a review paper concerning stellar wind theories nobody ever mentions the fluctuation theory of K.Andriesse.

Nussbaumer: Andriesse's formula would indeed be the solution if it also provided the physical explanation why his mass loss should happen. Someone with a strong background in thermodynamics should find out whether the crucial assumption in Adriesse's work can be filled with physics.