NONLINEAR, NONRADIAL PULSATION IN RAPIDLY OSCILLATING AP STARS

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Abstract. The rapidly oscillating Ap stars pulsate in high-overtone, low degree p-modes with their pulsation axes aligned with their oblique magnetic axes. They show non-linearity in their pulsation in three ways:

1) The harmonics of the basic pulsation frequency are detectable.
2) The pulsation phase seems to vary stochastically on a time scale of days to years depending on the star.
3) The form of the nonradial surface distortion is not constant with time.

These three effects are illustrated with HR 3831, the best studied of the roAp stars. HR 3831 pulsates in distorted dipole mode which can be modelled as a linear sum of axisymmetric $l = 0$, 1, 2, and 3 spherical harmonics aligned with the magnetic axis. This gives rise to a 7-frequency multiplet split by exactly the rotation frequency. The form of the distortion shows small changes on a time-scale of years. HR 3831 shows a 5-frequency rotationally split first harmonic multiplet, a 3-frequency rotationally split second harmonic multiplet, and a single third harmonic frequency has probably been detected at an amplitude of 0.065 mmag. The first harmonic has changed its form significantly over the last 10 years. A technique for decomposing the fundamental frequency septuplet into its component spherical harmonics is used to fit the pulsation phase as a function of rotation phase. This allows a unique O-C to be defined for any length of light curve. The long term behaviour of the O-C diagram cannot be modelled adequately with a combination of periodic (Doppler shift) and quadratic (evolution) terms; there seems to be a significant stochastic component. The direction of the pulsation phase reversal at rotational phase 0.747 is indeterminate; sometimes it is a positive-going reversal, sometimes negative-going. At present it is not known whether this is a numerical artifact, or a physical effect in the star. If it is a physical effect, it means that small non-periodic differences in pulsation amplitude between the bipolar hemispheres have been detected.

1. Introduction to HR 3831

HR 3831 is an A7p SrCrEu magnetic star with an effective temperature of $T_{\text{eff}} = 8000 \pm 200$ K, a radius of $R = 1.9 \pm 0.1 R_{\odot}$ and an absolute magnitude of $M_{\text{bol}} = 2.0 \pm 0.3$, where the quoted errors are estimates. It has a polarity reversing magnetic field which ranges from about +780 G to −720 G with the rotation period of $P_{\text{rot}} = 2.851982 \pm 0.000005$ day. From $v \sin i = 33 \pm 3$ km s^{-1} and the radius estimate, $i > 38°$. The magnetic field variations require that either i or $\beta > 62°$. Mean light variations occur with the rotation period, but have extrema which lag behind the magnetic extrema. HR 3831 is a visual binary with a separation of 3.29 arcsec; the secondary is a main sequence G2 star. From the absolute magnitude of the secondary and from the parallax, the distance is estimated to be about 60 pc. (Kurtz *et al.* 1992; Kurtz, Kanaan and Martinez 1992).

Astrophysics and Space Science **210**: 207–214, 1993.

HR 3831 is a singly-periodic rapidly oscillating Ap (roAp) star which pulsates with a frequency of ν = 1.4280128 mHz (P = 700.27 s = 11.67 min). (See Kurtz 1990 for a review of the roAp stars.) Observed through a Johnson B filter, the semi-amplitude of the light variation associated with the pulsation ranges from a little over 4 mmag at the times of magnetic extrema to zero at one of the magnetic quadratures (Kurtz, Kanaan and Martienz 1992; Kurtz 1990; Kurtz, Shibahashi and Goode 1990; Kurtz and Shibahashi 1986; Kurtz 1982). The amplitude modulation period is the same as the rotation period, $P_{\rm rot}$ = 2.851982 day, and the times of pulsation amplitude maxima coincide with the magnetic extrema (Kurtz *et al.* 1992).

Kurtz, Kanaan and Martinez (1992) found that HR 3831 pulsates in a single mode which is a distorted dipole with its pulsation axis aligned with the magnetic axis. Kurtz (1992) showed that the distortion can be modelled by a linear sum of axisymmetric spherical harmonic of degree l = 0, 1, 2, and 3. The first second and third harmonics are also present. Table I gives the frequencies derived by Kurtz, Kanaan and Martinez (1992).

TABLE I
A least-squares fit of ν = 1.4280128 mHz and its rotational sidelobes and harmonics to the 1991 data set.

Frequency name	Frequency mHz	Amp mmag	Phase rad
$\nu - 3\nu_{\rm rot}$	1.4158380	0.243±0.022	−3.0430±0.0904
$\nu - 2\nu_{\rm rot}$	1.4198962	0.267±0.022	1.1426±0.0811
$\nu - \nu_{\rm rot}$	1.4239545	1.974±0.022	2.6014±0.0110
ν	1.4280128	0.481±0.023	0.5805±0.0483
$\nu + \nu_{\rm rot}$	1.4320710	1.640±0.022	2.6013±0.0133
$\nu + 2\nu_{\rm rot}$	1.4361293	0.085±0.022	0.0084±0.2549
$\nu + 3\nu_{\rm rot}$	1.4401875	0.126±0.022	−2.9291±0.1742
$2\nu - 2\nu_{\rm rot}$	2.8479090	0.113±0.022	−2.4037±0.1918
$2\nu - \nu_{\rm rot}$	2.8519673	0.128±0.022	2.7427±0.1698
2ν	2.8560255	0.404±0.021	−2.3549±0.0511
$2\nu + \nu_{\rm rot}$	2.8600838	0.054±0.022	1.3846±0.4032
$2\nu + 2\nu_{\rm rot}$	2.8641420	0.121±0.022	−2.3580±0.1788
$3\nu - \nu_{\rm rot}$	4.2799800	0.121±0.020	−0.0447±0.1698
3ν	4.2840383	0.064±0.021	−2.2201±0.3200
$3\nu + \nu_{\rm rot}$	4.2880965	0.144±0.020	−0.1701±0.1426
4ν	5.7120510	0.065±0.023	1.7344±0.3163

t_0 = HJD2448312.24019
σ = 1.7386 mmag per observation

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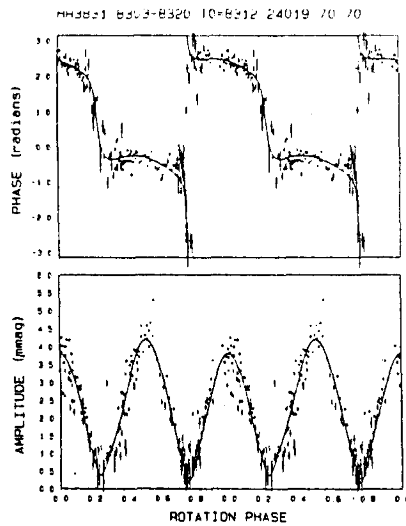

Fig. 1. This diagram plots the pulsation phase and amplitude as a function of the rotation phase for the 1991 data. The rotation phase is calculated from the time of magnetic maximum using the ephemeris given by Kurtz *et al.* (1992) with the rotation period $P_{rot} = 2.851982$ day. Two rotation cycles are plotted. Each point in the diagram has been calculated by fitting the frequency $\nu = 1.4280128$ mHz to 4 cycles (46.685 min) of the high-speed photometric data by linear least-squares. The theoretical lines show the best fit to the low frequency septuplet given in Table I assuming a pulsation mode which can be described by the sum of $l = 0$, 1, 2 and 3 axisymmetric spherical harmonics.

2. Theoretical Fits to the Phase and Amplitude Modulation Curves

Figures 1 to 5 show the theoretical fits of a sum of axisymmetric spherical harmonics of degree $l = 0$, 1, 2 and 3 to the 1981, 1985, 1986, 1990 and 1991 HR 3831 data sets. There appear to be real changes in the surface distortion from year-to-year. This is better illustrated in figures 6 and 7 which are schematic amplitude spectra. See Kurtz, Kanaan and Martinez (1992) and Kurtz (1992) for a complete discussion of the theory and observations which were used to construct these diagrams.

3. The O-C Diagrams

Figures 8 and 9 show the O-C diagrams for the entire data set and for the yearly data sets respectively. The long-term behaviour of these diagrams cannot be adequately modelled with periodic and quadratic terms, indicating that either the pulsation frequency or phase is not constant. Again, see Kurtz, Kanaan and Martinez (1992) for a more detailed discussion.

Other roAp stars also show changes in their pulsation frequencies, ampli-

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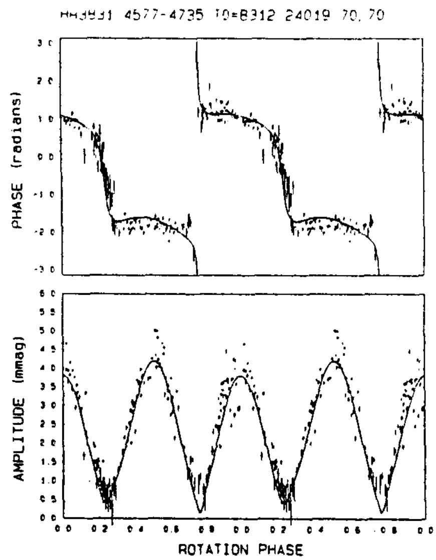

Fig. 2. This diagram plots the pulsation phase and amplitude as a function of the rotation phase for the 1981 data. The fitted curves are from the 1991 data and hence are the same as those in Fig. 1. The phase curve has been adjusted vertically to minimize the residuals to the fit.

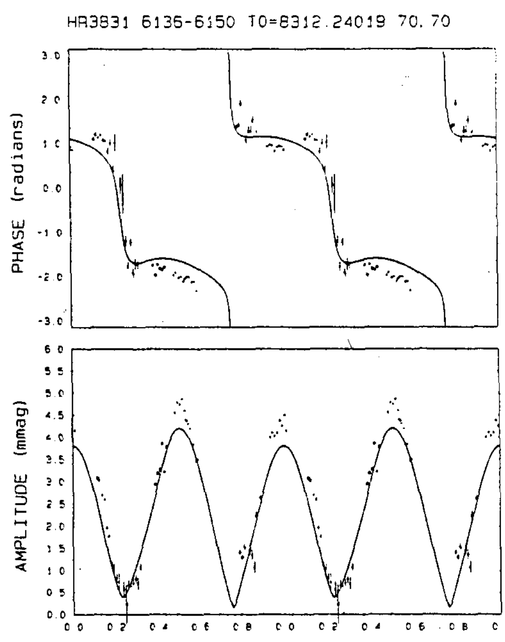

Fig. 3. This diagram plots the pulsation phase and amplitude as a function of the rotation phase for the 1985 data. The fitted curves are from the 1991 data and hence are the same as those in Fig. 1. The phase curve has been adjusted vertically to minimize the residuals to the fit.

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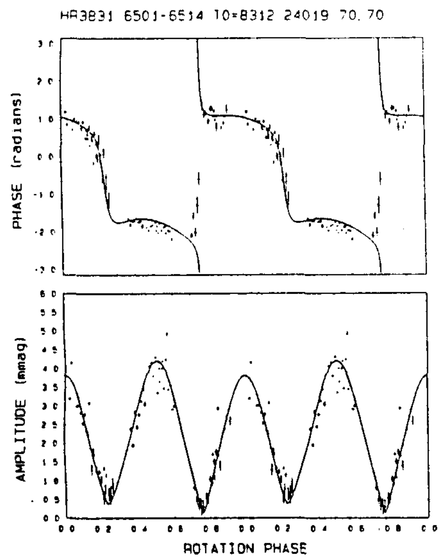

Fig. 4. This diagram plots the pulsation phase and amplitude as a function of the rotation phase for the 1986 data. The fitted curves are from the 1991 data and hence are the same as those in Fig. 1. The phase curve has been adjust vertically to minimize the residuals to the fit.

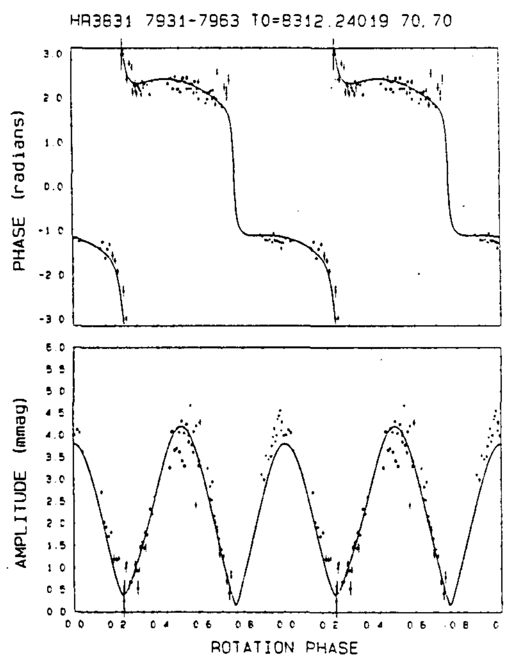

Fig. 5. This diagram plots the pulsation phase and amplitude as a function of the rotation phase for the 1990 data. The fitted curves are from the 1991 data and hence are the same as those in Fig. 1. The phase curve has been adjusted vertically to minimize the residuals to the fit.

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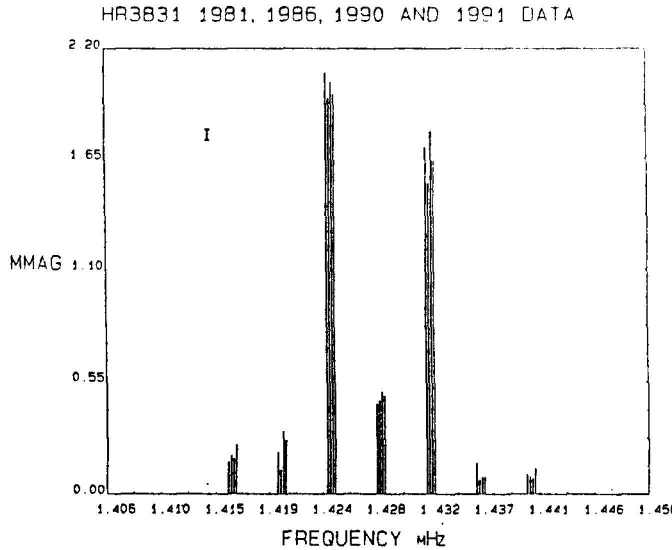

Fig. 6. A schematic amplitude spectrum for the fundamental frequencies showing a linear least-squares fit of the frequency septuplet ν, $\nu \pm \nu_{\rm rot}$, $\nu \pm 2\nu_{\rm rot}$ and $\nu \pm 3\nu_{\rm rot}$ to four data sets: JD2444577-4735 (1981), JD2446501-6514 (1986), JD2447931-7963 (1990) and JD2448303-8320 (1991). The amplitude spectrum for each data set has been shifted by a small frequency to display all four sets on the same diagram. At each frequency the left peak is 1981, the second peak 1986, the third 1990 and the right peak is 1991. *The separation of the four components in frequency is for display only; all four peaks coincide; the left peak is actually plotted at the correct frequency.* It is suggested that some of the apparent variation in the amplitudes of the components is real, but that the basic form of the rotational amplitude modulation has remained the same for 10 years. The error bar is ±0.05 mmag which is twice the internal error.

tudes and/or phases: HD 60435 shows amplitude modulation on a time-scale of days (Matthews, Kurtz and Wehlau 1986, 1987); HD 24712 (HR 1217) on a time-scale of weeks (Kurtz *et al.* 1989); HD 101065 on a time-scale of days or weeks (Martinez and Kurtz 1990); and HD 217522 on time-scales of days and years (Kreidl *et al.* 1991). It is not yet clear what governs the time-scale of the non-periodic component of the pulsation in roAp stars. There is the impression that the fewer the number of pulsation modes, the longer the life-times of the modes, but the statistics are inadequate as yet to be certain of this. More of these stars need to be discovered, and better studies are needed of those already known.

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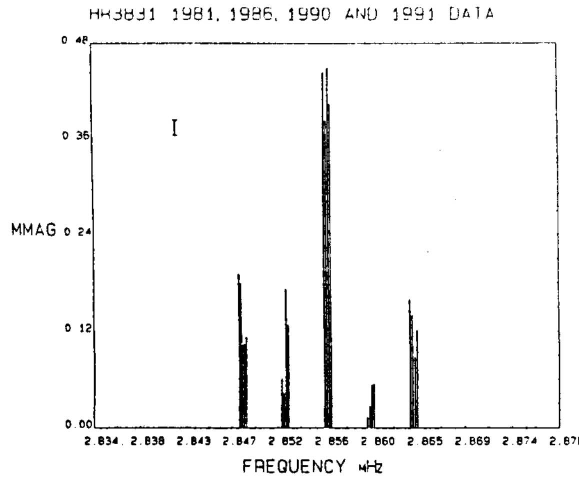

Fig. 7. A schematic amplitude spectrum for the first harmonic frequencies showing a linear least-squares fit of the frequency quintuplet 2ν, $2\nu \pm \nu_{\rm rot}$ and $2\nu \pm 2\nu_{\rm rot}$ to four data set: JD2444577-4735 (1981), JD2446501-6514 (1986), JD2447931-7963 (1990) and JD2448303-8320 (1991). At each frequency the left peak is 1981, the second peak 1986, the third 1990 and the right peak is 1991. The amplitudes are significantly different for the different data sets. The error bar plotted is ± 0.02 mmag.

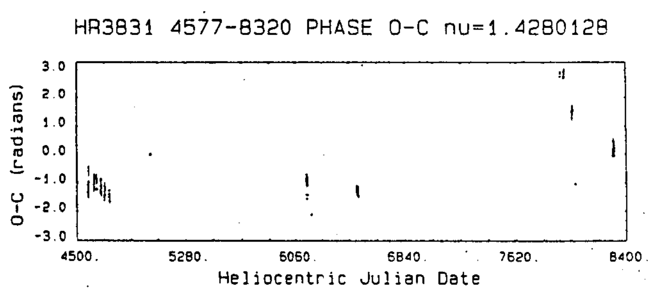

Fig. 8. This is the pulsation phase O-C diagram. The ordinate value of each point is the phase shift necessary to bring the 1991 theoretical phase curve shown in Fig. 1 into agreement with the 4-cycle linear least-squares phases for one night of data. This phase shift can be seen in the fit of the 1991 curve to the yearly data sets in Figs. 2 to 5, but here each point represents just one night.

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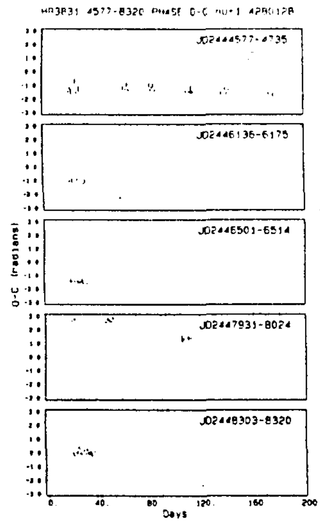

Fig. 9. The phase O-C diagrams for the yearly data sets. Each panel is a blow up of a section of Fig. 8. The top panel is for the 1981 data; the second panel is for the 1985 data; the third panel is for the 1986 data; the fourth panel is for the 1990 data; and the bottom panel is for the 1991 data. All panels are plotted to the same scale in both coordinates. The 1991 data points are flat about O-C = 0 because the fitted frequency $\nu = 1.4280128$ mHz was derived from those data, as was the theoretical curve in Fig. 1 which defines the zero point of the O-C scale.

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