

1 An Introduction to Contests

Contests are a prevalent aspect of everyday life, where multiple agents compete against each other by investing valuable resources, such as effort, time, or money, to win a prize. Examples of contests can be seen in various scenarios such as securing a taxi by occupying a favorable location, making an informed decision in choosing a university, publishing a scholarly article, or vying for a Michelin star as a chef. And, sporting teams participate in contests to win major events like the Super Bowl in American football, the Champions League in soccer, the World Series in baseball, or the NBA Finals in basketball. Furthermore, nations also engage in contests, either through legal or through illegal means, to attain control over territory or resources.

But contests not only intrude in our daily lives. They are essential to understanding how the world looks. Why is China communist? And Cuba? Why have countries like Germany and Japan, which previously lacked a strong democratic tradition, transitioned into functioning democracies? Why are most countries close to Russia democracies but Russia is not? And why was Russia communist? Why is English the predominant language north of El Paso, while Spanish and Portuguese are commonly spoken to the south of El Paso? Why do so many countries in America, Africa, and Asia resent their recent past?

The answers to these questions stem from historical conflicts – be they violent or more “cold” in nature – invasions, battles, or contests that ultimately terminated previous regimes and instituted new ones. Contests are the blood and the sap going across history.

Standard economic theory has overlooked the role of contests as an allocation mechanism. This is evident, e.g., in the absence of contests in the conventional analysis of natural resources, which focuses on intertemporal optimal allocation, perfect competition, monopoly, and externalities, among other important topics. It is important to note that standard economic theory provides valuable insights into the efficiency and limitations of markets and central planning. However, it is limited in scope, as it does not account for the importance of contests, especially in the context of natural resource allocation, which is often of interest to powerful nations, leading to contests that can be peaceful or violent. Broadly speaking, standard economic theory takes initial endowments as given while contest theory focuses on how these endowments are obtained and or preserved.

In this chapter, we introduce the concept of a contest. We provide examples of contests such as political competition, litigation, wars, lobbying, awarding a

prize or contract, sports, and patent races. Despite their apparent differences, these examples share the same structure. We present our primary tool, the Contest Success Function (CSF). The CSF, given the expenses/efforts/investments of the contestants, yields the probabilities that the contestants will win the contest or, in the case of a shared prize, the shares of the prize. With this concept in hand, we define strategies (efforts/investments) and payoffs, the latter being the expected return of effort minus the cost of effort. Then, we arrive at our main game theoretical tool, namely Nash equilibrium. The chapter ends by discussing some extensions that will and some that will not be pursued in the main text.

1.1 Contests in the Early Economic Literature

Karl Marx (1818–1883) was one of the early thinkers who recognized the significance of contests in economic analysis.¹ He believed that conflict was a crucial factor in understanding both history and the economy, and he developed his system around the notion that the struggle for the appropriation of the surplus was the driving force behind the succession of different modes of production. Marx made substantial efforts to support his claims with both empirical evidence and theoretical models, famously stating in *Capital* (1867) that “[i]n actual history, it is notorious that conquest, enslavement, robbery, murder, briefly force, play the great part” (p. 507). However, by the time he published his seminal work, his methods, particularly his labor value theory, had become outdated.

Mikhail Pavlovich Osipov (1859–?), a Russian general and topographer, and the engineer Frederik Lanchester (1868–1946) published in 1915–1916 the first mathematical model of war between two armies. The model postulates that the losses suffered by an army during a specific period of time are proportional to the strength of its adversary.

In 1967, Gordon Tullock (1922–2014) published a seminal paper which challenged the conventional analysis of the welfare losses caused by monopolies. Tullock argued that this analysis (see, e.g., Tirole, 1988, chapter 1) underestimated the losses by failing to account for the ways in which monopolies were established, such as through patent litigation (like Alexander Graham Bell, who faced 600 lawsuits in securing his telephone patent), through bribery, or by forcing favorable legislation. Tullock posited that the competitive struggle to secure a monopoly position leads to the dilapidation of most monopoly profits, thus rendering monopoly profits a form of welfare loss. Unfortunately, the lack of publication in a prominent journal limited Tullock’s impact and recognition for this contribution (Tullock, 2003).

The next contribution was by Anne Krueger (1934–) in 1974. She found that the explanation of foreign trade policies, in particular tariffs, is not in the standard reasons

¹ Marx was the first prominent economist for whom the conflict was key to understanding history. But before him many writers, beginning, at least in classical Greece, studied the causes and consequences of war and conflicts; see Vela Tejada, 2004. In the Renaissance, these studies were revitalized following in the footsteps of Machiavelli (1532).

found in International Trade manuals, but in what she called rent-seeking, i.e., the effort of domestic producers to limit foreign competition by pushing for laws and regulations that protect the domestic market.²

Edward Lazear (1948–) and Sherwin Rosen (1938–2001) in 1981 analyzed a ranking-based payment scheme, which awards prizes to both the winners and the losers of labor market contests. In this scheme, earnings are determined by the relative ranking of contestants rather than by the level of output produced during a particular contest. As a result, salaries are not tied to individual performance, as prizes are predetermined. This approach has been applied in the design of executive compensation packages as a means of providing performance-based incentives.

Finally, Gary Becker (1930–2014) studied lobbying in his 1983 paper. Until then, the impact of lobbies on resource allocation was ignored. Income redistribution was seen as a result of marginal productivity and government intervention, which transfers wealth from the rich to the poor. While government intervention does result in a more equal distribution of income, as indicated by the Gini coefficient before and after taxes,³ lobbying also plays a role in redistributing income, often benefiting organized groups over unorganized ones.

With this background knowledge, we are now equipped to further examine contests in-depth.

1.2 Examples

At the outset of this chapter, we presented several examples to illustrate the reasons behind our interest in contests. It is now time to be more systematic and clearly define the scope and focus of our study.

- (1) **Political competition:** This refers to the competition between political parties to attain political office through various means such as political campaigns, vote buying, etc.
- (2) **Litigation/War:** This refers to the situation where two individuals or nations engage in a competition for resources, land, or similar assets. This competition often involves the allocation of significant resources toward the hiring of lawyers, building armies, etc.⁴
- (3) **Lobbying:** This refers to the competition for a specific public policy outcome, such as the allocation of subsidies, regulations, or laws. Contestants in this competition often employ strategies such as bribery, advertising, and similar efforts to influence the outcome.

² In their book *Interest Groups and Trade Policy*, Grossman and Helpman (2002) examine the role of lobbying and rent-seeking in shaping international trade flows and trade agreements.

³ See, e.g., <https://ourworldindata.org/grapher/inequality-of-incomes-before-and-after-taxes-and-transfers-scatter>

⁴ For contests on property rights, see Ekelund, Street, and Tollinson (1997). And for legal contests, see Farmer and Pecorino (1999).

Table 1.1 Table of contest examples: prizes and required efforts or investments

Contest	Prize	Effort/Investment	Example
Politics	Office	Political campaigns/Vote buying	Elections
Litigation	Stake	Lawyers	Inheritance/Resource
War	Resources	Military expenses	Civil/World Wars
Lobby	Law, contract	Ads/Bribes	Regulation/Trade policy
Beauty C	Prize	Build-up	TV/Civil servants
Sports	League, Cup	Hiring expenses/Training	Soccer/Basket
R&D	Patent/Grant	R&D expenses	Drugs/Technology

- (4) **Awarding a prize/contract:** This refers to the competition for a prize or contract, such as the Nobel Prize or the awarding of a procurement contract. Contestants in these competitions often invest effort, incur expenses related to bribery, or incur costs associated with training, such as hiring personal coaches, etc.
- (5) **Sports:** This refers to athletic competitions such as tennis, the Olympic games, soccer, basketball, etc. Participants – whether individuals, teams, or nations – compete to win a contest and expenses include team budgets.⁵
- (6) **Patent race:** This refers to a competition among multiple firms or researchers to secure a patent or grant for an invention. Competition can lead to significant investment in research and development (R&D).⁶

See Table 1.1 for more details.

To conclude this section, it is worth mentioning that animals also engage in contests for resources such as territory, food, and mating. This interesting topic will not be covered in this book, but those interested can consult Hardy and Briffa (2013) for more information.

1.3 The Main Concept

We are ready to offer a more synthetic definition of our field of study.

DEFINITION 1.1 *A contest refers to a scenario where multiple agents compete to attain a prize or prizes, through the expenditure of costly actions such as effort or investments.*

In other words, a contest is the allocation of scarce resources through a fight!

It is time to delve into the distinction between contests and standard microeconomics. Standard microeconomics examines scenarios where property rights are clearly defined and agents voluntarily exchange rights over goods or produce rights for new goods. This approach has produced significant insights into market and planning processes, including the proof of the existence and efficiency of

⁵ See Szymanski (2003) for a comprehensive overview of the economic design in sports contests.

⁶ For the relationship of innovation and contests, see Baye and Hoppe (2003).

perfect competition, the study of the causes and consequences of international trade, the role of prices as information providers, the design of incentive-compatible mechanisms, and the understanding of imperfectly competitive markets. However, standard microeconomics does not encompass situations in which agents do not trade property rights but instead compete for property rights. These scenarios are referred to as contests. As noted by Jack Hirshleifer (1925–2005), a prominent early contributor to contest theory, contests are *The Dark Side of the Force* (1994).

Given its strategic nature, contest theory is a branch of the expansive field of game theory, along with industrial organization, political economy, and mechanism design, among others. It is a distinct mechanism for resource allocation, separate from market and authority mechanisms.

1.4 Contest as a Normal Form Game

Let n be the number of contestants. Each contestant exerts effort, $g_i \in \mathbb{R}_+$, to win a prize. In some applications, like sports or wars, g_i is the monetary investment made by contestant i . We will refer to g_i as the effort or investment of contestant i indistinctly. The value of the prize for each contestant is V_i . There are two possible interpretations of a contest, either the prize is indivisible and is awarded to contestant i with probability p_i or the prize is divisible and each contestant, say i , receives a share p_i of the prize.⁷ The function that determines p_i as a function of efforts, $\mathbf{g} = (g_1, \dots, g_n)$, is called the Contest Success Function (CSF from now on).⁸ Mathematically,

$$p_i: \mathbb{R}_+^n \rightarrow [0, 1]$$

with $\sum_i p_i(\mathbf{g}) = 1$ in most of the contests studied in this book.⁹ This concept is due to Tullock (1980). It is the tool that allows a unified treatment of the various examples of contests that we have seen so far. The CSF is increasing in g_i and decreasing in g_j for all $j \neq i$. Additionally, sometimes CSFs are assumed quasiconcave or concave in g_i and continuous in \mathbf{g} whenever $\mathbf{g} > 0$.

In most of this book, we will assume that payoffs for each contestant i are additively separable in expected revenue, $p_i(\mathbf{g})V_i$, and cost, $c_i(g_i)$,

$$\pi_i(\mathbf{g}) = p_i(\mathbf{g})V_i - c_i(g_i). \quad (1.1)$$

A payoff function like (1.1) is the building block of a **Contest Game** in which efforts are the strategies. For simplicity, we will refer to a contest game as a contest. We usually assume that costs are linear in effort, $c_i(g_i) = g_i$.

Given that no rational contestant will make a costly effort greater than the value of the prize, we assume that $c_i(g_i) \in [0, V_i]$.

⁷ These two interpretations can be applied generally and interchangeably without impacting the strategic behavior of agents or payoffs. However, there are cases where differences arise, especially in dynamic games with the possibility of bankruptcy. For more details, refer to Exercise 7.20.

⁸ We write vectors in bold letters.

⁹ Notable exceptions include contests with draws, which are examined in Chapter 9.

Our equilibrium concept was defined by John Nash (1928–2015) in a seminal paper (Nash, 1950a). It is a generalization of a previous idea due to Agustin A. Cournot (1801–1877) in a book published in 1838.

DEFINITION 1.2 A vector of efforts, $\mathbf{g} = (g_1^*, \dots, g_i^*, \dots, g_n^*)$, is a Nash equilibrium if

$$\pi_i(g_i^*, \mathbf{g}_{-i}^*) \geq \pi_i(g_i, \mathbf{g}_{-i}^*) \quad \forall g_i \in \mathbb{R}_+, i \in \{1, \dots, n\},$$

where \mathbf{g}_{-i} is a list of efforts of all contestants minus i .

Standard arguments showing the existence of a Nash equilibrium in pure strategies cannot be applied here because the CSF in some cases is discontinuous in \mathbf{g} , or payoffs are not quasiconcave. In Chapters 2 and 3, we will present several approaches to the existence of equilibrium.

Assuming differentiability, the first-order condition of payoff maximization for contestant i is

$$\frac{\partial p_i(\mathbf{g})}{\partial g_i} V_i - \frac{\partial c_i(g_i)}{\partial g_i} = 0.$$

This equation is valuable as, in many cases, a candidate equilibrium can be calculated directly from the first-order condition by assuming a specific form for the costs and the CSF. The final step is to verify that this candidate indeed maximizes payoffs for each contestant.

1.5 Extensions

In this book, the basic model introduced in this chapter will be expanded to incorporate intergroup contests (Chapter 5), sabotage (Chapter 6), dynamic contests (Chapter 7), asymmetric information (Chapter 8), and contests with draws large contests and entry in contests (Chapter 9).

There are numerous other extensions to the basic model that are not covered in this book but are nonetheless significant. For example:

(1) Several prizes

In many contests, there are prizes for contestants other than the winner. For instance, in the Olympic games, the three best athletes in a competition receive medals and those who compete in the final receive diplomas. In soccer finals, there is a monetary prize for the runner-up, and so on. The basic model presented in this book can be understood as considering two prizes, with the second prize normalized to zero. Chapter 9 explores contests that may end in a draw, such as some wars, trials, or sports matches, with the draw representing a distinct prize. Nevertheless, the literature on contests with multiple prizes is extensive. Readers interested in further knowledge can consult the comprehensive reviews by Sisak (2009) and Fu and Wu (2019).

(2) **Multidimensional effort**

In professional sports, effort encompasses not only the physical exertion of the athletes on the field but also the investments made by clubs to recruit talented players, construct facilities, and so forth. Additionally, firms engage in various forms of effort such as funding R&D and incurring legal expenses for rent-seeking and patent protection. The design of contests with multidimensional effort is explored by Clark and Konrad (2007) and by Arbatskaya and Mialon (2010), among others. Faria et al. (2014) provides an application of this concept in the context of the US telephone market.

(3) **Variable prize**

The value of a prize may fluctuate in response to the level of effort exerted during the contest. This is due to contestants' subjective valuation of the prize, which may be affected by the difficulty of winning, as well as the monetization of substantial effort by the contest organizer through higher admission fees. Furthermore, investments made by contestants can increase the value of the prize, thereby enhancing its quality. This phenomenon has been studied by Chung (1996) and Amegashie (1999a). When the prize is a function that increases with effort, it can be viewed as output, and the corresponding problem is referred to as "Cooperative Production." This term was first coined by Sen (1966) and was inspired by an early paper by Ward (1958). A comprehensive review of this subject can be found in Beviá and Corchón (2018).

(4) **The role of contests in the philanthropic industry**

Philanthropists and governments have used prizes to drive the best possible philanthropic project. These prizes are meant to correct some market or societal failures. Prizes may be educational (Fulbright, Rhodes), artistic (literary, musical, painting), scientific (Nobel, Abel, Fields), or just aimed to foster human values (justice, fraternity, peace, freedom, equality, etc.). The directory of awards, honors, and prizes lists about 30,000 worldwide, but they are only the tip of the iceberg since there are literally millions of small prizes and awards that are important for both donors and recipients like Doctor Honoris Causae for some department or university. The interested reader may consult the data and the analysis offered by McKinsey & Company (2009).