16

Nonlocality

16.1 Introduction

Our concern in this chapter is *locality* in quantum mechanics (QM). Locality is a heuristic physics principle based on the following propositions.

No Action-at-a-Distance

All the evidence points to the principle that physical actions, taken within restricted (localized) regions of space and time by observers or other agencies such as systems under observation (SUOs), do not cause instantly observable effects on other SUOs at large distances. This does not apply to mathematical/metaphysical concepts such as quantum wave functions or correlations, as these are conceptual objects (Scarani et al., 2000). Statements about instantaneous wave function collapse are vacuous (have no empirical significance) and are therefore *not* an issue of significance in physics. Such statements *are* an issue to theorists who objectivize wave functions, as in Hidden Variables (HV) theory.

Action-at-a-distance is generally regarded as anathema by most physicists. For example, Newton's law of universal gravitation is well known for mathematically encoding action-at-a-distance. There is direct evidence, however, in the form of a letter written by Newton to Bentley, that Newton believed that gravity acting "at a distance through a vacuum without the mediation of anything else" was an absurdity (Newton, 2006).

The *no action-at-a-distance* principle is encoded in quantized detector networks (QDN) by the requirement that labstate preparation and consequent signal detection never occur at the same stage.

Causal Transmission

All physically observable consequences of local actions taken by an observer or SUO are transmitted by identifiable physical processes, such as electromagnetic waves or neutrinos. There is no such thing as magic or *action-at-a-distance*.

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In QDN this proposition is taken into account implicitly in the labstate outcome amplitudes at each stage, as these model how information is propagated from stage to stage. When necessary, the information void can be modeled as if there were fields and/or particles propagating through it, giving scope for different mathematical models, such as Euclidean space, curved spacetime, noncommuting spacetimes, and so on. The structure of the information void is essentially a discussion of whatever modules have to be taken into account between labstate preparation and signal detection.

No Superluminosity

According to the standard principles of relativity, the speed of transmission of any observed physical effect is never greater than the local speed of light, as measured in a standard localized laboratory. To date, no particles or signals that travel faster than the speed of light (tachyons) have been observed.

A necessary, but not sufficient, condition for QM to respect the no-superluminosity principle is that the *no-communication theorem* holds. This theorem in standard QM and in QDN is discussed at the end of this chapter.

The no-communication theorem is insufficient in relativistic QM because it does not mention the speed of light. In relativistic quantum field theory (RQFT), the theorem is replaced by the above-mentioned no-superluminosity principle directly, which involves *lightcones*. These have a special place in physics, having both emergent and reductionist aspects.

Lightcones are obviously emergent structures because they define macroscopic subsets of spacetime consisting of events that are all either time-like or space-like relative to a given event (identified with the vertex of a given lightcone). On the other hand, lightcones are intimately involved in the reductionist formulation of RQFT, such as the postulate that operators representing observables at relatively spacelike intervals have to commute. Interestingly, RQFT places no such restriction on unobservables such as Dirac fields, which obey anticommutation relations. In Chapter 24, we discuss the construction of fermionic quantum fields from a QDN perspective, based on Jordan and Wigner's nonlocal quantum register approach, a manifestly emergent formulation (Jordan and Wigner, 1928).

QDN is not a reductionist approach to QM, as it deals principally with apparatus sitting on the interface between relative internal context (the world of SUO states) and relative external context (the environment in which that apparatus and the observer are situated). The precise relationship between QDN and lightcone structure is not clear at this time. This is consistent with the general situation at this time that a proper quantum theory of spacetime, so-called quantum gravity, has not been established. All attempts to do so from a reductionist approach have failed, to date.[1] It is not even clear at this time what has to be "quantized."

[1] Failure from the point of view of proving empirically vacuous.

We shall show in the last section of this chapter that QDN does obey the no-communication theorem, which is certainly necessary for lightcone physics to work. However, that theorem alone is not sufficient to establish lightcone structure.

Shielding

QDN actually goes further than the no-superluminosity principle, in that it allows for *shielding*. This is the empirical possibility of bypassing *lightcone causality*, the standard relativistic assumption that if event B is inside the forward lightcone of event A, then A can be the location of processes that "cause" effects to be observed at B. In practice it is possible, and often necessary, to materially isolate detectors so that whatever happens at one detector does not affect others, even when they are time-like separated and could in principle interfere with each other. Lest this be thought contrary to standard physics, we point out that all experiments are done on this basis. For example, neutrino detectors are located in deep mines, to filter out noise.

The shielding concept is really what underpins the stages concept. Our discussion of the double-slit experiments in Chapter 10 illustrates the point well: when looking at a screen for signals, an observer can do so over extended periods of real laboratory time, provided the screen is not interfered with by processes external to the experiment. The final stage of such a run, then, need not be identified with a definite instant of labtime. This is a form of loss of absolute simultaneity, different from the one discussed in special relativity (SR).

The above criteria are based on current empirical evidence and are subject to constant empirical reexamination: experimentalists continue to search for tachyons, for example. The hard fact is, however, that there have been no observed violations of the locality principle to date. Therefore, since QDN is greatly concerned with signal preparation and outcome detection, the locality principle is one that should be respected by QDN. There are several aspects to be discussed here regarding this.

Observers and Their Apparatus Are Essential

Any discussion of locality or its breakdown (referred as *nonlocality*) is meaningless if there is no mention of the observers involved, their apparatus, and the protocols of observation employed. All of these are necessary in order to define what is meant by the phrase "instantly observable" in the above.

The Information Void Cannot Be an Absolute Void

The information void refers to an absence of detectors, relative to a given observer, but makes no claims about the reality, structure, or otherwise of "empty space." Indeed, what an information void is to one observer may be a seething mass of detectors to another. For example, an observer looking to detect neutrinos faces immense technical difficulties, while an observer looking to detect sunlight need only open their eyes.

Apparatus Nonlocality

A critical attribute of observation that cannot be ignored here concerns the spatial distribution of the observer's apparatus. No apparatus is perfectly localized in space or in time. All equipment consists of vast numbers of atoms, which have spatial extent, and all observations take time. We have in previous chapters formalized this latter fact, that observations take time, into the stage concept.

Correlations

Apparatus nonlocality plays a critical role in quantum correlations. The superluminal transmission of certain types of information, interpreted as correlations, has been investigated and speeds in excess of 10,000 times the speed of light reported (Scarani et al., 2000). One of our aims in this book is to demystify such phenomena. QDN interprets quantum nonlocality as originating from the fact that apparatus is invariably nonlocal, as are the processes of extracting information from it, rather than reflecting strange, nonclassical properties of SUOs. Since apparatus has to be constructed *before* a quantum state or wave function can be given any meaning, or a correlation measured, it is then obvious that nonlocality is built into quantum physics from the word go, in the form of correlations arising from empirical context.

The Interpretation of Relativity

A particular problem with nonlocality arises from the principles of physics embedded in special and general relativity (GR). Relativity holds a strategic position in physics. It has passed all its tests and its principles cannot be dismissed, according to all current empirical evidence. It is as good in its domain of applicability as QM is in its domain.

Before we go further, we should clarify what we mean by *relativity*. There really are two different discussions going on here: the *gravitational* and the *relational*. The former concerns spacetime curvature, while the latter concerns relationships between observers.

Gravitational Side of Relativity

Einstein's field equations in GR couple spacetime curvature to the stress energy tensor. Although phrased in local differential form (thereby giving GR a reductionist flavor), Einstein's equations are really aspects of emergent physics that belong to the relative external context side of any Heisenberg cut: observers will usually think of themselves as moving along time-like worldlines in a GR spacetime, and that is an emergent concept.

Relational Side of Relativity

This side of relativity can be seen as an attempt to formulate a theory of observ*ers* in classical mechanics. However, it is emphatically not a theory of observ*ation*

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per se: there are no prescriptions in SR or GR about apparatus, for instance, beyond how it is related to relative external context. This aspect of relativity gives the rules relating the classical data held by one observer with the classical data held by another, and no more. Put in these terms, we can appreciate why relativity and QM are not contradictory or incompatible. They are frameworks discussing different aspects of observation.

We may summarize these comments by saying that relativity deals with relative external context (REC), whereas QM discusses relative internal context (RIC). Two historical category errors, the quantum gravity program and the Multiverse paradigm, appear to have been made here. Quantum gravity attempts to extend GR into RIC, while Multiverse attempts to extend QM into REC. Both attempts appear to be empirically vacuous at this time.

Our division of empirical context into REC and RIC is not clear cut but is identified in QDN with *Heisenberg's cut*:

> *The dividing line between the system to be observed and the measuring apparatus is immediately defined by the nature of the problem but it obviously signifies no discontinuity of the physical process. For this reason there must, within certain limits, exist complete freedom in choosing the position of the dividing line.* (Heisenberg, 1952)

Relativity impacts on QM because there is a principle in relativity, known as *Einstein locality* or *the principle of local causes* (Peres, 1995), that crosses the line between REC and RIC and has a direct impact on the sort of information that an observer can extract from their apparatus (which we remind the reader has nonlocality built into it even before an experiment starts). This principle asserts that

> events occurring in a given spacetime region are independent of external parameters that may be controlled, at the same moment, by agents located in distant spacetime regions. (Peres, 1995)

The Einstein principle affects the QDN formalism because relativity asserts that detectors that are outside each other's light cones cannot causally influence each other, yet QDN may give amplitude effects between those detectors. Explaining how the classical Einstein locality principle can survive in QM, and indeed in QDN, is a major challenge.

16.2 Active and Passive Transformations

Before we go further, we need to clarify a fundamental point about transformations, as it concerns the relationship between mathematics and physics.

In general, transformations come in two types, called *active* and *passive*. We illustrate the difference between these by considering some set $\Theta \equiv \{\theta^a\}$ of objects, labeled by an index a.

Active Transformations

An active transformation is something done to the original set Θ, replacing it with a new set, $\Theta' \equiv \{\theta'^a\}$. We represent an active transformation by the rule

$$\Theta \to \Theta' \neq \Theta, \tag{16.1}$$

where the arrow $\to$ means "*is replaced by.*" An active transformation implies the existence of some observer (that is, mathematician, experimentalist, or theorist) who is making a specific change in, or of, a set of objects. Moreover, there is an implicit assumption that some observer (who may be a "superobserver" playing the role of a god) has a memory of the original set and can compare it with the new set.

Passive Transformations

On the other hand, a passive transformation is merely a relabeling of the elements in a set, with no actual change in the contextually significant properties of those elements. For such a transformation, we write

$$\Theta \to \Theta^{''} = \Theta, \tag{16.2}$$

where $\Theta'' \equiv \{\theta^{a'}\}$. A passive transformation therefore concerns how a set is *described*, which, again, implies the existence of some observer (that is, mathematician, experimentalist, or theorist) who is changing their way of describing a given set.

Example 16.1 Consider a d-dimensional real vector space V with orthonormal basis $\{\boldsymbol{e}^i : i = 1, 2, \ldots, d\}$. Then an arbitrary vector $\boldsymbol{v}$ in V may be written in the form $\boldsymbol{v} = v^i \boldsymbol{e}^i$, where the components $\{v^i\}$ are real and the summation convention is used.

An active transformation of $\boldsymbol{v}$ is defined by a change in the components of $\boldsymbol{v}$ but not in the basis: v^i is replaced by an arbitrary v'^i, with the basis vectors remaining the same. Hence for an active transformation, we write $\boldsymbol{v} \to \boldsymbol{v}' \equiv v'^i \boldsymbol{e}^i \neq \boldsymbol{v}$.

On the other hand, a passive transformation leaves the vector $\boldsymbol{v}$ unchanged but the basis vectors are changed for a new set. For example, a change of basis from $\{\boldsymbol{e}^i\}$ to $\{\boldsymbol{e}'^i\}$, defined by the invertible linear relations $\boldsymbol{e}^i \equiv C^{ij} \boldsymbol{e}'^j$, gives $\boldsymbol{v} \to \boldsymbol{v}' \equiv v'^j \boldsymbol{e}'^j = \boldsymbol{v}$. In this case, the new coefficients $\{v'^j\}$ are given by $v'^j \equiv v^i C^{ij}$.

Active transformations are generally the only ones of interest to physicists, because they concern the real, physical world, whereas passive transformations are done in the mind. There are several kinds of active transformation in physics.

Construction of Apparatus

The construction of apparatus is the severest form of active transformation, as this creates physical context. We discuss this form of active transformation in Chapter 25.

Changes in State Preparation

A common form of investigation involves an observer sitting in a fixed laboratory and making active changes in prepared states, keeping the detectors unchanged. For example, switching on an electric field will often be associated with the acceleration of a charged particle in a scattering experiment. In terms of generalized propositions, we may describe such a change by the rule

$$(S, D|\Omega, F) \to (S', D|\Omega, F), \tag{16.3}$$

where S is a proposition about a prepared state, D is the detection apparatus, Ω is the observer, and F is the relative external context (the frame of reference) that defines the observer.

Changes in Outcome Detection

Another common form of investigation involves active changes in detection apparatus, keeping prepared states and relative external context unchanged. For example, the main magnetic field axis of a Stern–Gerlach experiment may be rotated to a new orientation.

We may describe such changes by the rule

$$(S, D|\Omega, F) \to (S, D'|\Omega, F). \tag{16.4}$$

It is commonly assumed that in some cases such as spatial rotations, transformations (16.3) and (16.4) are related by a change in sign in the parameters involved. That is a matter for empirical validation and **not** an absolute truth. The overthrow of parity in 1956 in the Wu–Ambler experiment demonstrated emphatically that symmetry and logic must not be treated as equivalent to empirical truths (Wu et al., 1957).

Interframe Experiments

An *interframe* experiment is essentially an experiment involving two observers: one of these is associated with state preparation and the other is involved with outcome detection. Such experiments explore perhaps the most spectacular and deepest issues in physics. Examples are the Doppler shift, the Unruh effect (Unruh, 1976), and indeed, the notion that more than one observer is a meaningful topic in physics. Questions about standardization of physics protocols (units and such like), observation of observers, the constancy of physical "constants," Early Universe physics, and so on, come flying at us immediately.

Such experiments may be represented symbolically by

$$(S, A|\Omega, F) \to (S', D'|\Omega', F'), \tag{16.5}$$

where A is the apparatus that creates the state S, relative to observer Ω and D' is the detecting apparatus relative to observer Ω'.

We place a prime on the transformed state/proposition, S', on the right-hand side in (16.5), for two reasons. First, what the detecting observer Ω' detects will appear to have properties different from those of the state that the preparing observer Ω believes they have prepared. Second, that a quantum state means the same thing to different observers is a notion that needs to be questioned in several respects, concerning the standardization of physics and the exchange of context. In QDN, for instance, we do not accept that quantum states are objective "things."

To illustrate our concerns, consider the detection by observer Ω' of a single photon signal. On what basis could Ω' believe that they had detected a signal sent from a distant galaxy? The only plausible scenario is that the observer had already received sufficient contextual information about that galaxy to formulate and justify such a belief.

We could take the view that such an exchange of sufficient contextual information from the source observer to detecting observer is equivalent to having a single observer, encompassing both source and detectors. While reasonable in most cases, that point of view seems bizarre as far as intergalactic processes are concerned.

Active transformations play a role in mathematics, where they are associated with functions, maps, and operations. A function can be regarded as a form of active transformation: given a set Θ, a function f maps elements of that set into some other set, Θ'. We can even think of this as defining a "mathematical arrow of time."

Passive transformations have played a role in physics in some important situations.

Symmetries in the Void

By the fact that there are no detectors in the information void, it is permissible to model signal propagation through it in any theoretically useful way, such as through Minkowski spacetime. It may then be useful to consider passive transformations in that spacetime and explore the empirical consequences. For example, relativistic quantum field theory, which is used to calculate the dynamics of quantum fields in the information void, is generally assumed to Lorentz covariant, meaning that it does not matter which inertial frame is used to do the calculations.

Space-Time Symmetries

Passive coordinate transformations played a critical role in the development of modern physics. Aristotelian physics was firmly based on the proposition that the Earth is an absolute frame of reference. After the works of Galileo and Newton, observers discussed physics more carefully. The observer's frame of reference now became identified as an important ingredient. For convenience, the important frames were usually taken to be inertial frames. It was pointed out by Bishop Berkeley early on in the development of Newtonian mechanics that Newton's laws of motion are invariant to (unchanged by) any passive Galilean transformation of an observer's inertial frame of reference (Berkeley, 1721). Later, this idea was extended to special relativity, leading to the notion that the laws of physics (excluding gravitation) are invariant to passive Lorentz and Poincaré transformations of inertial frames of reference.

We describe this concept in generalized proposition form by

$$(L, \emptyset | \Omega, F) \equiv (L, \emptyset | \Omega', F'), \tag{16.6}$$

where L are Newton's laws of motion in the nonrelativistic case or the laws of relativistic mechanics in the relativistic case, and frames F and F' are related by either a Galilean or Poincaré transformation, as appropriate.

We note the absence of any relative internal context in (16.6), typical of both nonrelativistic classical mechanics and relativistic mechanics. This contextual incompleteness gives the generalized proposition classification of such theories as two.

16.3 Local Operations

In this and following sections, we pin down our meaning of *local operation*. In line with the discussion in the previous section, such an operation is always an active one, taking place over at least one stage and possibly more. We consider an active physical operation $\mathbb{L}_{n+1,n}$ on a rank-r apparatus $\mathcal{A}_n^{[r]}$ at stage Σ_n, an operation that affects a number p of detectors in $\mathcal{A}_n^{[r]}$ and leaves the remaining $q \equiv r - p$ detectors unaffected in a specific way, to be explained below. The affected detectors and their corresponding signal qubits will be called *relatively local*, while the unaffected detectors and their corresponding signal qubits will be called *relatively remote*. By unaffected, we mean that no possible partial measurements on the remote detectors alone would detect any changes due to $\mathbb{L}_{n+1,n}$.[2]

Our approach is to split the quantum register $\mathcal{Q}_n$ at stage Σ_n into two subregisters $\mathcal{Q}_n^{[L]}$ and $\mathcal{Q}_n^{[R]}$, such that $\mathcal{Q}_n = \mathcal{Q}_n^{[L]}\mathcal{Q}_n^{[R]}$. Splitting a quantum register is a purely mathematical operation, discussed in detail in Chapter 22, although the motivation for doing this comes entirely from the physics of the experiment.

Here $\mathcal{Q}_n^{[L]}$ is the rank-p tensor product $Q_n^1 Q_n^2 \dots Q_n^p$ of the local signal qubits and is therefore referred to as the *local subregister*, while $\mathcal{Q}_n^{[R]}$ is the rank $q \equiv r - p$

[2] Note that the language here is imprecise. Experiments to detect changes in the remote detectors would actually involve two ensembles of runs, comparing partial measurements on apparatus evolving with the action of $\mathbb{L}_{n+1,n}$ with partial measurements on apparatus evolving without it.

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tensor product $Q_n^{p+1}Q_n^{p+2}\dots Q_n^r$ of the remote signal qubits and is therefore referred to as the *remote subregister*.

Note that the labeling of detectors is arbitrary in principle, so we are always entitled to order local and remote qubits in the way given above.

Example 16.2 An observer has prepared a labstate $\boldsymbol{\Psi}$ in a rank-five quantum register $\mathcal{Q}^{[5]} \equiv Q^1Q^2Q^3Q^4Q^5$, given in the computational basis representation (CBR) by

$$\boldsymbol{\Psi} = ac\underline{\mathbf{24}} + ad\underline{\mathbf{29}} + ae\underline{\mathbf{20}} + bc\underline{\mathbf{10}} + bd\underline{\mathbf{15}} + be\mathbf{6}, \tag{16.7}$$

where each underlined number in bold, such as $\underline{\mathbf{24}}$, represents a single element of the CBR, and a, b, c, d, and e are complex coefficients. Show that this state is separable relative to the split $\mathcal{Q}^{[5]} = \mathcal{Q}^{[L]}\mathcal{Q}^{[R]}$, where $\mathcal{Q}^{[L]} \equiv Q^2Q^5$ and $\mathcal{Q}^{[R]} \equiv Q^1Q^3Q^4$.

Solution

The CBR is not best suited to discuss splits, so we need to find the equivalent of the signal basis representation (SBR) of the state. We translate the above CBR basis vectors into SBR counterparts as follows. By inspection, the binary decomposition of the integers $6, 10, 15, 20, 24$, and 29 is

$$6 = 2+4, 10 = 2+8, 15 = 1+2+4+8, 20 = 4+16, 24 = 8+16, 29 = 1+4+8+16. \tag{16.8}$$

Then for example we have $\mathbf{6} = \widehat{\mathbb{A}}^2\widehat{\mathbb{A}}^3\mathbf{0}$, and so on. Hence we can write

$$\begin{aligned}\boldsymbol{\Psi} = ac\widehat{\mathbb{A}}^4\widehat{\mathbb{A}}^5\mathbf{0} + ad\widehat{\mathbb{A}}^1\widehat{\mathbb{A}}^3\widehat{\mathbb{A}}^4\widehat{\mathbb{A}}^5\mathbf{0} + ae\widehat{\mathbb{A}}^3\widehat{\mathbb{A}}^5\,\mathbf{0} + \\ + bc\widehat{\mathbb{A}}^2\widehat{\mathbb{A}}^4\mathbf{0} + bd\widehat{\mathbb{A}}^1\widehat{\mathbb{A}}^2\widehat{\mathbb{A}}^3\widehat{\mathbb{A}}^4\mathbf{0} + be\widehat{\mathbb{A}}^2\widehat{\mathbb{A}}^3\mathbf{0}.\end{aligned} \tag{16.9}$$

By inspection, this can be operator factorized into the form

$$\boldsymbol{\Psi} = (a\widehat{\mathbb{A}}^5 + b\widehat{\mathbb{A}}^2)(c\widehat{\mathbb{A}}^4 + d\widehat{\mathbb{A}}^1\widehat{\mathbb{A}}^3\widehat{\mathbb{A}}^4 + e\widehat{\mathbb{A}}^3)\mathbf{0}. \tag{16.10}$$

This can now be interpreted as the tensor product of two subregister states; that is, we may write $\boldsymbol{\Psi} = \boldsymbol{\Psi}^{[L]}\boldsymbol{\Psi}^{[R]}$, where $\boldsymbol{\Psi}^{[L]} \equiv (a\widehat{\mathbb{A}}^5+b\widehat{\mathbb{A}}^2)\mathbf{0}^{[L]}$ is in $\mathcal{Q}^{[L]} \equiv Q^2Q^5$, $\boldsymbol{\Psi}^{[R]} \equiv (c\widehat{\mathbb{A}}^4+d\widehat{\mathbb{A}}^1\widehat{\mathbb{A}}^3\widehat{\mathbb{A}}^4+e\widehat{\mathbb{A}}^3)\mathbf{0}^{[R]}$ is in $\mathcal{Q}^{[R]} \equiv Q^1Q^3Q^4$, $\mathbf{0} = \mathbf{0}^{[L]}\mathbf{0}^{[R]}$, and we define signal creation operators for each subregister accordingly.

16.4 Primary and Secondary Observers

There is an issue here, the physical implications of which are deep to say the least and that underpins many debates about the nature of reality. In the above section we considered splitting a quantum register into a local subregister and a remote subregister. What we have in mind, of course, is to associate different "observers" with each of these subregisters, as this is of interest in various branches of physics and information theory. Typically we would call the local observer *Alice*, and the remote observer *Bob*.

What needs to be addressed is the following: if Alice and Bob have no connection, meaning that they have no channels of communication between them, then for whom is the combined scenario *Alice and Bob* meaningful? We have argued elsewhere in this book that truth values are contextual. So if we want to discuss Alice and Bob together, we have to specify the context in which we are doing so.

The only answer that makes empirical sense is that there must be (implicitly, if not explicitly) some third observer *Carol* who has the contextual information to know about Alice and Bob and what they are observing and the outcomes that they have found.

In CM, such an overseeing observer is generally not specified, a factor that contributes to the essential contextual incompleteness of that discipline. In QM, we cannot allow such contextual incompleteness. Whatever is asserted must have some empirical basis for its truth values. Quantum entanglement runs directly into this issue.

This chain of reasoning leads us to the *primary observer* concept. A primary observer is the overseer and custodian of all relevant context: the buck stops with a primary observer, there is nothing behind them, in the given context. So when we discuss Alice and Bob as local and remote observers respectively, they are by implication **not** primary observers. We will refer to them as *secondary observers*, or *subobservers*.

A fundamental difference in QM between a primary observer and any secondary observers is that dimensions of Hilbert spaces do not follow an additive rule: if Alice thinks she is dealing with a p-dimensional Hilbert space and Bob thinks he is dealing with a q-dimensional space, then primary observer Carol, who is overseeing Alice and Bob, is dealing with a pq-dimensional space, not a $p + q$-dimensional Hilbert space. What is additive is qubit register *rank*.

Exercise 16.3 If Alice models her experiment with a rank-a qubit quantum register and Bob models his with a rank-b qubit quantum register, prove that if Carol models both experiments by a rank $c = a + b$ qubit quantum register such that the dimension of Carol's quantum register equals the sum of the dimensions of Alice and Bob's registers, then $a = b = 1$.

There is an interesting question here: *is it possible for a primary observer to observe themselves completely, that is, know everything about themselves*? Our resolution of this is that it is not possible to do this in a complete way: real empirical information always comes in discrete form and real observers have finite capacity to store information. According to Sen (Sen, 2010), a finite set cannot be mapped bijectively to any proper subset (but an infinite set can be so mapped). Therefore, a real observer cannot faithfully observe themselves in a complete way.

An observer can observe themselves partially, however: we do this every time we look in a mirror. What we see is only part of ourselves (our face, clothes, and so on), and that information is stored not in those parts but in other parts that serve as memory (our brains). In essence, part of us plays the role of an SUO and other parts play the role of a primary observer.

16.5 Subregister Bases

Contextuality is equivalent in many respects to information. Given the split of the quantum register $\mathcal{Q}_n$ into the tensor product of a local subregister $\mathcal{Q}_n^{[L]}$ and a remote subregister $\mathcal{Q}_n^{[R]}$, discussed in Section 16.3, the question of basis set for each subregister in the split arises. It turns out that QDN gives an immediate answer as follows.

As we saw in previous chapters, context gives a preferred basis set for every signal qubit component of a quantum register. Factoring such a register into two subregisters, each consisting of an integral number of signal qubits, does not change this. A preferred basis is determined by physical context, whereas a split is a purely mathematical operation. Therefore, each signal qubit retains its preferred basis during a split.

Suppose we have a detector array consisting of $r = p + q$ detectors, where p and q are at least one and we intend to split it into the tensor product of a local subregister $\mathcal{Q}_n^{[L]}$ consisting of p detector qubits and a remote subregister $\mathcal{Q}_n^{[R]}$ of q detector qubits. Now in principle there is no natural way to order any array of detectors, because they are located in three-dimensional space, and there is no natural ordering in such a set. We are free to label the original array in any way that we want. In the context of a split such as we envisage, it seems sensible to do it as follows. The p detectors that we will include in the local subregister will be labeled 1 to p and the remote detectors will be labeled $p + 1$ to r when we discuss the original register, but from 1 to q when we are discussing the remote subregister as a separate entity.

We now have an obvious way of organizing each of our subregister bases. We can define CBRs and SBRs for each. Moreover, we can define signal projection operators $\mathbb{P}_n^i, \widehat{\mathbb{P}}_n^i, \mathbb{A}_n^i, \widehat{\mathbb{A}}_n^i$ for each separately, just as if they were independent registers (as indeed, they could be).

A natural question is: given a basis element $\boldsymbol{i}_n^{[L]}$ in the CBR for the local register, where $0 \leq i < 2^p$ and a basis element $\boldsymbol{j}_n^{[R]}$ in the CBR for the remote register, where $0 \leq j < 2^q$, to what element $\boldsymbol{k}_n$ of the CBR for the original register does the tensor product $\boldsymbol{i}_n^{[L]} \otimes \boldsymbol{j}_n^{[R]}$ correspond? It is easy to show that with the ordering described above, we have the rule $k = i + 2^p j$.

We may readily construct operators that act on elements of a given subregister and not on elements in another subregister. For example, if $\mathbb{U}^{[L]}$ is a subregister operator that acts over $\mathcal{Q}^{[L]}$, then relative to the original register $\mathcal{Q} \equiv \mathcal{Q}^{[L]}\mathcal{Q}^{[R]}$, its action is equivalent to that of the operator $\mathbb{U}^{[L]} \otimes \mathbb{I}^{[R]}$, where $\mathbb{I}^{[R]}$ is the identity operator for $\mathcal{Q}^{[R]}$.

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16.6 Local and Remote Evolution

When a primary observer discusses operations performed by local and remote secondary observers, such actions are never implemented instantaneously but take laboratory time, so should be discussed in terms of stage to stage dynamics.

Suppose we split a register $\mathcal{Q}_n$ into local and remote subregisters as above, with their respective CBR bases. Consider further a process of evolution from stage Σ_m to stage Σ_n, for $n > m$, with the local secondary observer Alice performing an action $\boldsymbol{U}^{[L]}_{n,m}$ on her apparatus and remote secondary observer Bob performing an action $\boldsymbol{U}^{[R]}_{n,m}$ on his apparatus. The question is, how does Carol, the primary observer, describe both of these processes?

For simplicity, we shall assume that the rank of each subregister is stage independent. Following our discussion above on subregister bases, we can write

$$\boldsymbol{U}^{[L]}_{n,m} \equiv \sum_{i,j=0}^{2^{r_L}-1} \boldsymbol{i}^{[L]}_n U^{[L]i,j}_{n,m} \overline{\boldsymbol{j}^{[L]}_m}, \quad \boldsymbol{U}^{[R]}_{n,m} \equiv \sum_{i,j=0}^{2^{r_R}-1} \boldsymbol{i}^{[R]}_n U^{[R]i,j}_{n,m} \overline{\boldsymbol{j}^{[R]}_m}. \qquad (16.11)$$

Then Carol can describe what Alice and Bob are doing by the evolution operator

$$\mathbb{U}_{n,m} \equiv \boldsymbol{U}^{[L]}_{n,m} \otimes \boldsymbol{U}^{[R]}_{n,m} = \sum_{i,j=0}^{2^{r_L}-1} \sum_{k,l=0}^{2^{r_R}-1} \boldsymbol{i}^{[L]}_n \otimes \boldsymbol{k}^{[R]}_n U^{[L]i,j}_{n,m} U^{[R]k,l}_{n,m} \overline{\boldsymbol{j}^{[L]}_m} \otimes \overline{\boldsymbol{l}^{[R]}_m}. \qquad (16.12)$$

It is straightforward to rewrite this expression in terms of the CBR for Carol's quantum register. Likewise, all subregister operators acting on local or remote labstates can be readily rewritten in terms of the global register (that is, from Carol's perspective).

Two important conclusions can be drawn from this analysis. (1) It is consistent to apply QM to parts of the universe, while ignoring the rest, even though all of it is subject to the laws of QM, and (2) it is the possibility of isolating apparatus that gives rise to the SUO concept in the first place

16.7 The No-Communication Theorem

We have stressed the critical significance of the no-communication concept in QM. We shall discuss it in simplistic terms in both standard QM and in QDN.

The QM Account

Consider two observers, Alice and Bob. Alice will be our *local* observer, conducting active transformations on locally prepared signal states, while Bob will be our *remote* observer, conducting observations at remote detectors. The aim is to see if anything Alice does can affect what Bob observes.

First, we put ourselves in the position of a primary observer, Carol, who has an overview of what Alice and Bob do. They are now to be regarded as secondary observers, relative to Carol, although each of them believes themself to be a

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primary observer. Suppose Alice models the states she can prepare by elements of a Hilbert space denoted $\mathcal{H}^A$, and suppose Bob models the states he observes by elements of a Hilbert space $\mathcal{H}^B$. We will suppose that Carol has enough information to model the combined system by the tensor product space $\mathcal{H}^C \equiv \mathcal{H}^A \otimes \mathcal{H}^B$.

In the following we shall deal with pure states only, assuming that mixed states present no exceptional concerns. This is on account of the fact that mixed states involve no more than the extra complication of classical probabilities, and these do not interfere in the way that entangled quantum states do.

Suppose Carol arranges for an entangled state to be prepared, described by a normalized element in $\mathcal{H}^C$ given by

$$|\Psi^C) \equiv \alpha|\psi^A\rangle \otimes |\psi^B\rangle + \beta|\phi^A\rangle \otimes |\phi^B\rangle, \tag{16.13}$$

where $|\psi^A\rangle$ and $|\phi^A\rangle$ are normalized elements in $\mathcal{H}^A$, $|\psi^B\rangle$ and $|\phi^B\rangle$ are normalized elements in $\mathcal{H}^B$, and $|\alpha|^2 + |\beta|^2 = 1$. The actions of Alice and Bob are respectively as follows.

Alice

Alice performs a local active operation U^A on states in $\mathcal{H}^A$ that she has access to, changing them according to the prescription

$$|\psi^A\rangle \to |\psi'^A\rangle \equiv U^A|\psi^A\rangle, \quad |\phi^A\rangle \to |\phi'^A\rangle \equiv U^A|\phi^A\rangle. \tag{16.14}$$

Bob

Bob performs a measurement of an observable O^B on states in $\mathcal{H}^B$ to which he has access.

Carol

From Carol's perspective, a holistic account has to be given, in terms of states in, and operators over, the total Hilbert space $\mathcal{H}^C$. From her perspective, Bob's observable corresponds to the operator $O^C \equiv I^A \otimes O^B$, where I^A is the identity operator over $\mathcal{H}^A$, while Alice's local operator corresponds to the operator $U^C \equiv U^A \otimes I^B$, where I^B is the identity operator over $\mathcal{H}^B$.

Before Alice performs her operation, Carol calculates that Bob's expectation value $\langle O^B\rangle$ will be given by

$$\begin{aligned}\langle O^B\rangle &\equiv (\Psi^C|O^C|\Psi^C) \\ &= |\alpha|^2\langle\psi^B|O^B|\psi^B\rangle + |\beta|^2\langle\phi^B|O|\phi^B\rangle \\ &\quad + \alpha^*\beta\langle\psi^A|\phi^A\rangle\langle\psi^B|O|\phi^B\rangle + \alpha\beta^*\langle\phi^A|\psi^A\rangle\langle\phi^B|O^B|\psi^B\rangle,\end{aligned} \tag{16.15}$$

where we note the presence of interference terms.

After Alice performs her operation, Carol calculates that Bob's expectation value $\langle O^B\rangle'$ will be given by $\langle O^B\rangle' \equiv (\Psi'^C|O^C|\Psi'^C)$, where now

$$|\Psi'^C) \equiv U^C|\Psi^C) = \alpha|\psi'^A\rangle \otimes |\psi^B\rangle + \beta|\phi'^A\rangle \otimes |\phi^B\rangle. \tag{16.16}$$

Then we find

$$\langle O^B \rangle' = |\alpha|^2 \langle \psi^B | O^B | \psi^B \rangle + |\beta|^2 \langle \phi^B | O | \phi^B \rangle + \alpha^* \beta \langle \psi'^A | \phi'^A \rangle \langle \psi^B | O | \phi^B \rangle + \alpha \beta^* \langle \phi'^A | \psi'^A \rangle \langle \phi^B | O^B | \psi^B \rangle. \quad (16.17)$$

Comparing (16.15) and (16.17), it is easy to see that $\langle O^B \rangle' = \langle O^B \rangle$, because $\langle \psi'^A | \phi'^A \rangle = \langle \psi^A | U^{A\dagger} U^A | \phi^A \rangle = \langle \psi^A | \phi^A \rangle$.

The prediction, therefore, is that no active unitary transformations performed by Alice would affect Carol's calculation of Bob's measured expectation value. In other words, Alice could not transmit any signals to Bob using entanglement.

The QDN Account

QDN has no problem in fully accommodating the no-communication theorem, because as discussed in previous sections, it is straightforward to define concepts of local and remote subregisters that can model all the necessary requirements for the no-communication theorem to hold. Indeed, we encounter such processes in our discussion of the quantum eraser experiments in Chapter 14.