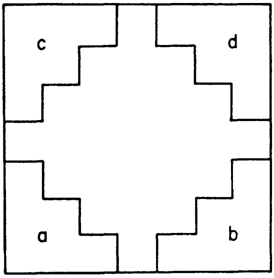
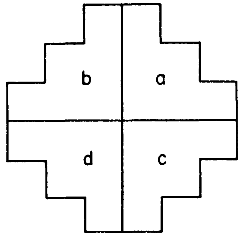
If the corners are removed and rearranged according to the pattern





we obtain the new pattern

		8	2		
	22	16	10	4	
36	30	24	18	12	6
44	38	32	26	20	14
	46	40	34	28	
		48	42		

with "diagonals" 2, 4, 6; 8, 10, 12, 14; 16, 18, 20; 22, 24, 26, 28; 30, 32, 34; 36, 38, 40, 42; 44, 46, 48.

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CORRESPONDENCE

To the Editor, *The Mathematical Gazette*

COMPUTING AT A-LEVEL

DEAR SIR,

At the recent Annual General Meeting of the M.A. there was a panel discussion the primary function of which was to provide suggestions for possible inclusion in the "Applied" section of a core syllabus in mathematics for A-level. Questions and comment from the floor were asked

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for, but came so readily that there was not time for many of these to be answered or further discussed by the panel. Among these, naturally and properly, computers and computing were mentioned with some frequency, but I was—even in an audience consisting mainly of teachers of mathematics—aghast and depressed that so great a lack of knowledge of what computers can and cannot do, and how they do it, seemed to be displayed. On reflection I feel that, perhaps, I should not have been so surprised, since most of the popular and semi-popular writings on these topics are a mixture of wishful thinking, science fiction, and sales talk. May I, therefore, beg some of your valuable space for a few remarks?

In the course of the discussion I had argued that the advisability of teaching computer programming depended upon the availability of a computer upon which programs written by the pupils could be run. Without this, the psychological thrill of getting the computer's print-out is lacking. The checking of programs would also have to be done by the teacher, and—apart from the fact that it is unlikely that he would discover *all* the bugs—this is a misuse of his time, when the compiler's diagnostic facilities will do this checking more surely and much more speedily.

To this I would like to add, first, that "computer" as a title for these machines is fast becoming a misnomer. Apart from Professor Hamming's dictum that "the object of computing is insight, *not* numbers", I would estimate that some seventy or eighty percent of the work done to-day on "computers" is some kind of information processing in which the numerical content is small and the mathematical content is trivial. In the Universities the mathematicians make relatively little use of computers. Scientists, engineers, ... make much more, and there are few disciplines in which some use of computers is not already being made or suggested. Knowledge of what computers can do, and, to some extent, how they do—are made to do—it, should be part of "general studies" for *all* pupils—and can be built up from the first form onwards. It may be even more important to make clear what computers *cannot* do!

"Flow diagrams" were strongly advocated. I am blind, and cannot draw, or use, flow diagrams. But I have done much computing, on desk machines in the past, and more recently on electronic computers and am aware of the need for thoughtful planning of a computation—or, indeed, of any other process or operation. Nevertheless, although I am all for flow diagrams, I would strongly urge that these are a means to an end and *not* an end in themselves. But I would teach these as a useful, graphic, and concise method of reducing any procedure to an orderly logical sequence of simple operations. One can draw a flow diagram for making a telephone call, or (as I hope a few cookery mistresses have already discovered) for making an apple pie! This sort of thing can—should?—be done from the first form upwards, and again, in as many subjects as possible, and certainly not only by mathematical specialists.

To turn the flow diagram into a computer program, however, to-day demands the learning and use of some programming language—a "high-level" one. This is merely a feat of memory, facilitated by practice. One does not learn a foreign language—say Spanish—unless one is going to do some reading or writing in Spanish. This won't help much if you are going to Russia. Now there is *no* universal programming language yet accepted. One depends, in any case, upon the "compiler" (which the *makers* provide) to turn instructions written in the programming language into machine instructions which the machine will obey. Also many programming languages are machine-oriented. Here is another reason which would make it inadvisable to try to teach computer *programming* without access to a computer.

There were pleas that our pupils need to be prepared for existence in "a world controlled by computers". *Decision-making* will continue to be controlled by *humans*. They may use computers to give them access to much more relevant information, to organise and abstract from this information. It may be possible to write instructions in the program which will print a "decision", but only when some human has instructed the computer as to how the decision is to be made.

We have, of course, to distinguish between what we *teach*, and upon what we are going to *examine*. There *is* much relevant to computers and computing which can, and well may, be introduced into the teaching of almost every subject. There is no justification for isolating this material and putting it as one option in A-level mathematics: but its practice should improve performance all round.

Finally then, as regards computers and computing, and in view of what has been said above, I suggest that my mathematical friends should prevail upon their non-mathematical colleagues to take as large a share as possible in teaching the relevant skills: and in doing *all* the examinations—if *they* deem these necessary.

Yours sincerely,

W. G. Bickley

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To the Editor, *The Mathematical Gazette*

Dear Sir,

At the recent Annual General Meeting of the Association the session devoted to an account of experiments in schools ended with some questions on transformation geometry. Only a very short time was available for discussion and few people were able to speak. I fear that that such discussion as there was showed some misunderstanding on what transformation geometry is and that, as a result, the more fundamental issues tended to be obscured.

Unfortunately, transformation geometry is assumed by many to be that which is to be found in the SMP and similar O-level texts, and this, I think, was exemplified by the questions asked by Mr. A on geometrical constructions and Miss B on the apparent lack of structure (class-room not algebraic!). There is no reason why constructions should not play a large part in the teaching of transformation geometry (see, for example, Jeger, *Transformation Geometry*, Allen and Unwin) or why the approach should not be more formal and rigid (see, for example, the DDR (East German) school texts, or, at a higher level, Choquet *L'enseignement de la géométrie* (Hermann). Indeed Miss B might well find the formality she seeks in *A School Course in Geometry* by W. J.

Dobbs, Longmans Green, 1913 (although this book has long been out of print, extracts from it can be found in Vol. VII of the *Gazette*). There would be no great difficulty in designing a course in transformation geometry to achieve those objectives which Miss B sets herself when she teaches Euclid's geometry, although it must be borne in mind that theorems relating to triangles which are such a feature of Euclid's geometry will not be proved so elegantly by means of transformations (see, for example, the proof of the nine-point circle theorem given in Jeger)—traditional results are, in general, more readily obtained by traditional methods. It was, I imagine, for this reason that the work of Dobbs had so little impact on the teaching of school geometry, for it is only the inclusion in the syllabus of functions, matrices, groups, etc that so greatly strengthens the case for teaching transformation geometry.

The question asked by Miss B was "Why do you teach transformation geometry?" and one answer to this is hinted at in the preceding paragraph. What, I believe, she wanted to ask was "Why do you teach transformation geometry, or any other topic, in this way?" I think it would help discussion of this latter, more fundamental, question, if it were understood to be different from the former.

Yours sincerely,
A. G. Howson

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To the Editor, *The Mathematical Gazette*

Dear Sir,

J. C. Burns' imaginative "How long is a piece of string?" (*Math. Gaz.*, 1968, **52**, 14), despite its concern with mechanical aids and with what can be done using a pair of scissors and a ball of string, does not lead me, as a scientist, to believe that its content is anything but mathematical fiction.

The question I wish to raise is: What are the practical, physical counterparts of Mr. Burns' constructions in contemporary mathematics laboratories, or in drawing-offices and engineering and science workshops? There the length of a piece of string is of course given by a rational number of metres, and no-one uses a ball of string to draw straight lines, but instead makes use of ruler and drawing-board, made for him by methods still based, I suppose, on A. B. Kempe, *How to draw a straight line* (1877). Mr. Burns' analogue of the drawing-board is a Euclidean plane (a good name for a mechanical aid!) defined by three pieces of string; so it is possible, in imagination, to carry out constructions in such a plane using just a ball of string and scissors. In the workshop a piece of string will not give a straight line when stretched between any two points on the surface of a sphere, or between most pairs of points on the surface of a cone or cylinder: the drawing-board is required. Similar considerations apply to paper folding.

Mr. Burns' Fig. 2 is also very relevant to practical mathematics, because the distance between the pegs cannot be measured with a piece of string only, a result expressed by William Whewell, *Elementary Treatise on Mechanics* (1819) in the now famous sentence: "It may at first sight seem unlikely that the pull of gravity will depress the centre of a light cord, held horizontally at a high lateral tension;

And yet no force, however great,
Can stretch a cord, however fine,
Into a horizontal line
That shall be absolutely straight."

It was years before someone discovered that the second part of the sentence is a perfect Alfred Tennyson *In Memoriam* rhymed stanza.

Yours sincerely,

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To the Editor, *The Mathematical Gazette*

DEAR SIR,

I was introduced to complex numbers in the sixth form by method (c) of those listed by T. J. Randall in Note 3200, and it was part of my first-year undergraduate trauma to be told that it was nonsense, for the reasons he quotes, and to be retaught by method (b).

However, the idea of finding a "square root of minus one" has a strong appeal, and the essence of method (c) can be put on a sound basis by introducing complex numbers, written $a \,.\, 1 + b \,.\, i$, in terms of a linear algebra with basis $(1, i)$ and commutative multiplication table $1^2 = 1$, $i^2 = -1$, $1 \,.\, i = i \,.\, 1 = i$. The point that students need to be convinced of is that, while -1 has no square root, it is possible to extend the real number field so that one of the additional elements has a square which corresponds closely to -1. In symbols, while $i = \sqrt{(-1)}$ is illegitimate, $i^2 = -1$ is perfectly all right!

The method proposed by Mr Randall has perhaps more fascination about it, and for future algebra specialists it provides an introduction to the ideas of passing from a field to its polynomial ring, and then back to a difference ring; and the teacher has the opportunity of suggesting the generalisations obtainable by considering an arbitrary polynomial in place of $y^2 + 1$.

Some students, however, may find the more direct step from the real field to a finite basis algebra over it more easily acceptable, and this method makes it easier to keep in sight the applications of complex numbers to geometry and dynamics in two dimensions, or alternating current theory.

Future algebraists get an equivalent bonus in an introduction to linear algebras, once widely referred to as hypercomplex number systems.

Yours faithfully,

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To the Editor, *The Mathematical Gazette*

NUMBER PATTERNS IN MULTIPLICATION TABLES

DEAR SIR,

It is gratifying to learn from Mr. Murrell's letter (Feb. 1968) that another lecturer in mathematics at a College of Education approves of the main points of my article on the Newsom Report (Feb. 1967), but I wonder whether these views are generally acceptable to those responsible for training the teachers of the future. May I invite the readers of the *Gazette* to let me know whether or not they consider that it is important for teachers to become aware of the beauty of mathematics, as distinct from its obvious utilitarian values (so that the children can find aesthetic satisfaction in their mathematical activities) and also to know by experience the satisfaction of "discovering" elementary mathematical relations for themselves (so that they are encouraged to provide similar experiences for those they are teaching)?

Very few of us are capable of making original mathematical discoveries, but we can *rediscover* simple but interesting things for ourselves, when opportunities to do so are provided. For example: (i) Why do all prime numbers, except 2 and 5, end in 1, 3, 7 or 9? You can "see" (literally and metaphorically) the reason why, by using a 10 by 10 number square, showing in order the numbers from 1 to 100, and ringing all the prime numbers, except 2 and 5. (ii) Why are all prime numbers greater than 3 either one less or one more than a multiple of 6? Use a number square showing in order the numbers from 1 to 144.

The use of a Multiplication Square is recommended for children who cannot memorise their tables, even with the aid of the number line, "Cuisenaire" rods, Napier's "Bones" and similar apparatus, not only because it is a Ready Reckoner of utilitarian value, but also because a child, in building up the Square by himself by repeated addition, gains insight into the process of multiplication, and he can have the pleasure of discovering the number patterns in the Square. For example: (i) What pattern is formed by all the odd numbers in the Square, and why does this pattern arise? (ii) What pattern is formed by all the Square numbers? And for the more advanced pupil, (iii) What kind of symmetrical pattern is formed by all the Triangular numbers? Using a 12 by 12 Square, why are there no Triangular numbers in one of the columns, or in the corresponding row? What is the least Triangular number that is a multiple of 8? What is the least Triangular number that is a multiple of 2^p?

As all multiples of 9 have a digit-sum of 9 (or a multiple of 9), it follows that the digit-sum of any other number is the remainder when that number is divided by 9. Use this to find the remainder, when these numbers are divided by 9: 23, 52, 103, 234, 345, 1111, 2000. (In some cases you have to find the digit-sum of the digit-sum.) This is the basis of the ancient practice of "casting out the nines" as a check to arithmetical calculations: let us use it to check our Multiplication Square, the first and eighth rows of which should read

	1	2	3	4	5	6	7	8	9	10	11	12
	8	16	24	32	40	48	56	64	72	80	88	96
Digit-sum	8	7	6	5	4	3	2	1	9	8	7	6

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(i) What do you find, when you add a number in the first row to the corresponding number in the digit-sum row? Can you explain this?

(ii) What patterns do you find by obtaining the digit-sum row of (a) the third row, (b) the sixth row, (c) the second row, (d) the tenth row, (e) the fifth row, (f) the seventh row, (g) the fourth row of the Multiplication Square? (Note: in some cases alternate numbers show a simple pattern.)

(iii) Find the digit-sum pattern for the 12th row: is it the same as for the 3rd row? If so, can you explain why?

It is sometimes useful to know a simple pattern method for constructing multiplication tables for numbers greater than 12, especially when long divisions by such numbers have to be performed frequently. For example, a teacher with a class of 31 children may need the average age of the class, the average of a set of marks, the average attendance, etc. To construct the table of multiples of 31, write down a column of the "3-times table" (giving the "tens" figures) and alongside the"1-times table":

3	1	1	9	2	9
6	2	3	8	5	8
9	3	5	7	8	7
12	4	7	6	11	6
15	5	9	5	14	5
etc		etc			

The tables for 19, 29, etc. have patterns similar to that of the table for 9: we regard 19 as $20 - 1$, 29 as $30 - 1$, etc.

(i) Use the pattern method to write down the multiplication tables for (a) 41, (b) 69, (c) 81, (d) 99, (e) 999. Check your results by finding the digit-sums.

(ii) Devise your own methods for writing down the multiplication tables for (a) 18, (b) 17, (c) 13, (d) 48. Check your results.

Does an average child (or adult) get more satisfaction and insight into mathematics from finding the cost of a carpet 12′6″ by 10′6″ at 38/6 per sq. yd. or from discovering why a sum of money such as £7/7/7 (equal numbers of £, s, and d.) is always exactly divisible by 11 and 23? Opinions about the points raised in this letter will be very welcome.

Yours sincerely,
D. B. Eperson

Christ Church College,
Canterbury, Kent.

OBITUARY

CHARLES ORPEN TUCKEY

I am glad to have this opportunity of paying tribute to the great services of Tuckey to Mathematical Education. He was a lifelong friend, as both of us went to Charterhouse in January 1899, one as a boy of 13, the other as a young master of 23, with an age gap of little