ON THE ORBIT MECHANICS OF COMETARY DUST PARTICLES

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It is generally believed that the structure of dust tails of comets can be described in terms of the mechanical theory. The basic parameters of this theory are the repulsive force and initial emission velocity of particles. However, the effect of initial velocity does not seem to have been considered seriously in the discussion of the overall structure of dust tails. Finson and Probstein (1968) have recently proposed an elaborated method to analyse the tail brightness profiles by introducing the distribution function of particles. The initial velocity effect was, however, taken into account only in an approximate way. They assumed that a group of particles are emitted isotropically with a single speed, and afterwards forms a spherical shell expanding uniformly in time. The isotropic emission is suggested by the fluid-dynamical considerations on the gas-dust interaction in the inner head region, but the subsequent spherical expansion implies merely a simplification for the calculation to obtain the line-of-sight integrals analytically. Due to this simplification, the validity of the Finson-Probstein method is seriously limited, particularly when a tail after perihelion passage is concerned.

In order to proceed without such an approximation, we have reconsidered the orbit mechanics of cometary particles. Our problem is not an N-body problem, but a synthesis of a large number of two-body problems. The mathematics involved is rather elementary, but the three-dimensional treatment is required because the numerous orbital planes to be concerned are inclined to each other. If there is an effective formulation availabl to follow, and to synthesize, the orbit mechanics of many particles, then the surface density integrals can be estimated, directly and more rigorous ly, by counting of sample particles.

The underlying physics of the dust emission process suggested by Finson and Probstein (1968) will be retained. According to this picture, the dust emission process can be generally idealized as an isotropic one from a point source. A sample particle is labelled by the following parameters; the effective force parameter, μ_j, the time of emission, t_i, and the relative velocity of emission, $\vec{v}_k$; hence, the initial conditions for its Keplerian motion can be taken as $\vec{r}_0=\vec{r}_c(t_i)$, and $\vec{V}_0=\vec{V}_c(t_i)+\vec{v}_k$; where

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the suffix c indicates the quantity referred to the comet nucleus. The orbital motion of the nucleus will be assumed to be parabolic. This is the case so far as dust-rich comets are concerned. We also note that the ratio (v_k/V_c) is very small ($\lesssim 0.01$) in most cases.

The equation of the Keplerian motion is

$$d^2\vec{r}/dt^2 + (\mu k^2/r^2)(\vec{r}/r) = 0 \quad , \quad \text{where} \quad \mu=1-\gamma \quad . \tag{1}$$

As is well-known, this equation has two vector integrals; the one is the angular momentum vector, $\vec{h}=\vec{r}\times\vec{V}$ ($\vec{V}=(d\vec{r}/dt)$), and the other is given by $\vec{b}=(1/k^2)(\vec{V}\times\vec{h})-\mu(\vec{r}/r)$, which may be termed the eccentric vector (the terminology after Fushimi, 1954).

Employing these two constant vectors of motion, we obtain the equation of orbit in a vectorial form,

$$\vec{r} = q \left\{(1-z^2)\vec{P} + 2z\vec{Q}\right\} / (1-Sz^2) \quad , \tag{2}$$

where $\vec{P}=(\vec{b}/b)$ and $\vec{Q}=(\vec{h}/h)\times\vec{P}$ are unit vectors, $q=(h^2/k^2)/(b+\mu)$ denotes the perihelion distance, and the parameter $S=(b-\mu)/(b+\mu)$ is a modification or a generalization of the eccentricity. The variable $z=\tan(\theta/2)$, tangent of half the true anomaly, is a function of time, and the (z-t) relation is given by the integral of motion,

$$t - T = t_S \, J_S \, (z) \quad , \tag{3}$$

where $t_S=(2q^2/h)$ is a characteristic time of the motion, $T=t_0-t_SJ_S(z_0)$ the epoch of perihelion passage, and $J_S(z)=\int_0^z (1+z^2)/(1-Sz^2)^2dz$. The initial value of z required in the evaluation of T, is determined by the relation $z_0=\left\{(r_0-q)/(Sr_0+q)\right\}^{1/2}\text{sign}(\vec{r}_0\cdot\vec{V}_0)$.

Equations (2) and (3) are convenient and sufficient for our purpose to follow orbital motions of many particles. The generalization with respect to μ is also completed in these equations.

It is convenient to introduce the concepts of elementary space distributions or structural elements of the tail. First comes an ijk particle. Next, we have synchronic and syndynamic lines, which are familiar examples if constructed on assuming zero-emission velocity. Taking account of the velocity dispersion of emitted particles, we find that synchronic particles extend a three-dimensional volume, which may be termed a "synchronic tube". Similarly, one can define a "syndynamic tube". A "tube element" is also defined as a crossing of two tubes of different kind.

In order to see how these structural elements change their sizes and shapes with time, it is useful to consider the vertical motion of particles relative to the comet orbit plane. The height above the comet orbit plane is given by

$$\zeta = v_{0\zeta} t_s (z-z_0)(1+zz_0)/(1-Sz_0^2)(1-Sz^2) \quad (z \geq z_0) \quad . \tag{4}$$

Combining this with Eq. (3), one can obtain ζ as a function of time. It is readily seen that the linear approximation $\zeta = v_{0\zeta}(t-t_0)$ applies only for particles of small $\tau(=t-t_0)$ or very small μ. We also note that some particles recross the comet orbit plane when the condition $z=-(1/z_0)$ is satisfied. This elementary fact leads to an interesting result, that is, the appearance of a vertically depressed part in the tail, which may be termed the "neckline structure".

For the understanding of such a structural feature, it will prove convenient to define a "neckline" as the locus of particles originally emitted into vertical directions and just recrossing the comet orbit plane. For a vertically emitted particle, it is rather easy to determine the position (r_e, θ_e) and time (t^*) of its recrossing to occur. The results are expressed symbolically as, $t^*=t^*(t_i,\mu_j)$, $r_e=r_e(t_i,\mu_j)$, and $\theta_e=\theta_e(t_i)$. Based on these relationships, one can define several curve families on the comet orbit plane, as follows; $f_1(\theta_e)=c_1(t_i)$, $f_2(r_e,\theta_e)=c_2(\mu_j)$, $f_3(r_e,\theta_e)=c_3(t^*)$. The last one represents the family of necklines.

If further the distribution function of particles, $F(t,\gamma,\vec{v})$ is introduced, one can discuss the tail brightness profiles. With a modification by cross-sections for light-scattering, F can be resolved into three functions, that is, $C_{sca}(\gamma)F(t,\gamma,\vec{v}) = \langle C_{sca}\rangle \dot{N}_d(t) f(\gamma;t)\psi(\vec{v};\gamma,t)$, where $\dot{N}_d(t)$ is the dust emission rate, $f(\gamma;t)$ the modified size-distribution function, and $\psi(\vec{v};\gamma,t)$ the velocity distribution. $\dot{N}_d(t)$ will be assumed to follow the inverse square law of $r_c(t)$ as suggested by the nucleus model of Delsemme and Miller (1971). The velocity distribution is an isotropic one with a single speed v_0, and the time- and γ-dependency of v_0 is assumed to follow Probstein's formulae. The size-distribution $f(\gamma)$ is left to be determined through fitting procedure of the calculation to observed profiles. One can now construct "syndynamic brightness profiles" for different sample values of γ, by means of the counting method. A suitably weighted superposition of them is expected to reproduce the observed feature. If an appropriate weighting function is found, it gives the information on $f(\gamma)$.

We have applied this method for Comet Arend-Roland, and found that both of the main and anti-tails can be reproduced in a unified manner, namely, no temporal anomalies in the source functions need not be introduced. In other words, the anomalous tail of this comet can be well understood in terms of the neckline structure. The nature of the derived and assumed source functions will not be discussed here. The point of the present argument is that the neckline structure is a common feature in the dust tails of comets after perihelion passage, and so it seems responsible for some peculiar appearance of dust tails. The anomalous sunward tail of Comet Arend-Roland, consisted of a sharp spike and a faint extended halo around it, was a striking example.

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Finson, M.L. and Probstein, R.F. : 1968, Astrophys. J., 154, 327 and 353.
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DISCUSSION

Comments by Kiang: 1. The mathematics used is most elegant. The parameter S orders the various forms of the conic sections much more neatly than the usual elements a and e. 2. The neckline structure predicted by the author is a string-like concentration of material starting at the nucleus. As such it should sharpen considerably any interpretation of cometary tails. 3. A complete English translation of the original Chinese paper is now available ("Chinese Astronomy" Vol 1, No 2, December 1977).