*Relativity in Fundamental Astronomy*
*Proceedings IAU Symposium No. 261, 2009*
*S. A. Klioner, P. K. Seidelman & M. H. Soffel, eds.*

doi:10.1017/S1743921309990421

# The art of precision pulsar timing

**Matthew Bailes**[1]

[1]Centre for Astrophysics and Supercomputing,
Swinburne University of Technology, H39, PO Box 218,
Hawthorn Vic, Australia 3122
email: mbailes@swin.edu.au

**Abstract.** Pulsar timing has proven to be a wonderful tool with which to study neutron stars, providing insights into their ages, distances, proper motions, magnetic field strengths, internal structure, binary histories and evolution, and for tests of General Relativity. Here I describe how to optimise strategies for millisecond pulsar timing to enable the highest timing precision.

**Keywords.** Pulsars, Millisecond Pulsars, Precision Timing, General Relativity, Gravitational Waves.

## 1. Introduction

Almost all of the major advances in pulsar astrophysics have arisen as the result of the practice known as "pulsar timing". Pulsar timing is the practice of observing a known pulsar and "folding" the data at an assumed period to create an average pulse profile. The inherent assumption is that average pulsar profiles are stable (Lorimer & Kramer 2004), and that any given observed profile $O(x)$ is just a phase-shifted version of the intrinsic profile $P(x)$ multiplied by a constant plus noise $N(x)$, ie

$$O(x) = AP(x - \alpha) + N(x) \tag{1.1}$$

where $A$ is a scale factor and $\alpha$ is the phase shift. One then cross correlates this observed profile with a template, and obtains an arrival time and associated error. By fitting a model to the arrival times of the form

$$\phi = \phi_0 + \nu(t - t_0) + (1/2)\dot{\nu}(t - t_0)^2 + (1/6)\ddot{\nu}(t - t_0)^3 \tag{1.2}$$

where $\phi$ is the phase of the pulsar, $\nu$ is the rotation frequency and the dots represent time derivatives, it is possible to parameterise the majority of pulsars' timing history by remarkably few parameters.

The vast majority of the almost 2000 known radio pulsars have been found in either large-scale surveys of our galaxy with the world's largest radio telescopes (Lyne *et al.* 1998, Manchester *et al.* 2001), or from targetted observations of globular clusters (Manchester *et al.* 1991, Ransom *et al.* 2005). Most of the known radio pulsars are young objects with characteristic (indicative) ages of a few million years and estimated magnetic field strengths of $\sim 10^{12}$ G. These tend to be imperfect rotators, and deviate from their few-parameter spin-down models after a few years, exhibiting what we usually refer to as "timing noise".

A sub-class of pulsars however, the so-called "millisecond" pulsars, have remarkably small rotation periods ($\sim$1–$\sim$20) milliseconds, and fields that are much weaker ($\sim 10^8$ G). For these pulsars special instrumentation is required to avoid the deleterious effects of pulse dispersion to obtain the highest precision, and their remarkably small torques and large angular momenta make them clocks of outstanding precision.

The "art" of pulsar timing concerns adopting techniques that maximise the yield from a given set of observations. Telescope time is a greatly sought-after resource, and almost all pulsars have their timing precision limited by it, due to their small radio fluxes that prohibit high signal-to-noise ratio ($SNR$) observations.

## 2. Pulsar Timing soon after Discovery

When a pulsar is first discovered, it is important to quickly establish "phase connection", a term which means that you can unambiguously establish the exact number of rotations between the observations. But very little is usually known about a pulsar when it is first discovered. The original detection provides an approximate period and dispersion measure, and a few arc minute position at best. It is only after a prolonged timing campaign, often extending for 1–2 years that these parameters become more refined, but with perserverence phase-coherent pulsar timing can yield parameters to astounding precision. Without phase connection, pulsar timing fails to capitalize on its greatest strength, and that is that for many of the most interesting parameters, the precision grows much faster than time $t^{-1/2}$. In most astrophysical experiments, continued observing reduces one's relative errors by the total integration time $t^{-1/2}$. In pulsar timing, provided you have maintained phase connection, the error in the pulsar period decreases as $T^{-3/2}$, where $T$ is the entire time span between your first and last observations. This makes period derivatives $\dot{P}$ easy to establish, and these in turn provide both an *estimate* of the characteristic age (from $\tau = P/(2\dot{P})$) and magnetic field strength ($B^2 \propto P\dot{P}$).

In order to establish and keep phase connection, it is normal to observe the same pulsar $\sim$3 times on a given day, then once a day for 2-3 days, then again, say weekly for a month. For the first year, it is important to maintain frequent $\sim$ monthly observations, in order to decouple the covariant parameters of position ($RA, Dec$), $P$ and $\dot{P}$.

Binary pulsars can prove more difficult to time, as they add another five Keplerian parameters (and sometimes more relativistic parameters) to the mix, and until a complete orbit is observed with phase connection the covariances make pulsar phase prediction often impossible to achieve. Obtaining phase connection early greatly shortens the time until you can report significant findings, hence, the "first law" of precision pulsar timing.

**The first law of pulsar timing.** *Establish phase connection as early as possible as for many of the most interesting parameters, your experiment's precision is often a strong function of the time you have maintained phase connection. A corollary is that profiles smeared by a poor folding model are often of diminished use in the highest precision timing and can lead to unfortunate systematic errors. It is wise to adopt your final timing system instrumentation as soon as practicable.*

## 3. Precision Timing Theory

The real power of pulsar timing relies on extending the time duration of your experiment to many years. Timing studies of relativistic binaries have provided extraordinarily convincing evidence that General Relativity is the correct theory of gravity, both for neutron star-neutron star binaries (Taylor and Weisberg 1982, Stairs *et al.* 2002, Kramer *et al.* 2006), as well as neutron stars with white dwarf companions (Bhat, Bailes & Verbiest 2008, van Straten *et al.* 2001).

Pulsars are intrinsically low-luminosity objects, and pulsar timing requires the world's largest telescopes in many cases just to detect them. From an astrometric point of view, we can think of pulsar timing as an interferometric experiment. The pulsar, due to its massive moment of inertia and small braking torque is a very precise clock. The pulses therefore represent a source of coherent radiation and the Earth-Sun system are immersed in radiation from this coherent source. The wavelength of the radiation can be thought of as having a wavelength $\lambda = cP$, where $c$ is the speed of light. The baseline of our experiment is 2 astronomical units ($AU$). The position of the pulsar can therefore be determined to an accuracy of $\sim\lambda/(2AU)$ times the relative accuracy with which we can determine the arrival time of the pulse at any given epoch. For a gaussian profile this is approximately $w/(2 \times P \times SNR)$ where $w$ is the half-width of the Gaussian and $SNR$ is the signal-to-noise ratio of the pulsar.

The $SNR$ is provided by a modified version of the radiometer equation:

$$SNR = \frac{SG\sqrt{BN_p t}}{T_{\rm rec} + T_{\rm sky}}\sqrt{\frac{P-w}{w}} \tag{3.1}$$

where $S$ is the pulsar flux in Janskys, $G$ the receiver gain in the units of K/Jy, B the receiver bandwith (Hz), $N_p$ is the number of polarisations (usually 2), $t$ the integration time in seconds, and $T_{\rm rec}$ and $T_{\rm sky}$ are the effective temperatures of the receiver and sky respectively.

This means the error ($\sigma$) in the arrival times or TOAs is approximately:

$$\sigma = \frac{w}{2}\sqrt{\frac{w}{P-w}}\frac{T_{\rm rec} + T_{\rm sky}}{SG\sqrt{BN_p t}} \tag{3.2}$$

and for the brightest millisecond pulsars can be below 100 nanoseconds. These types of errors permit not only positions to be determined to a precision of just tens of microarcseconds, but also allow very accurate proper motions and even parallaxes, orbital period derivatives, and relativistic effects such as Shapiro delay to be observed.

For narrow pulsar profiles, $\sigma \propto w^{3/2}$, highlighting the importance of finding pulsars with narrow features or using instrumentation that minimizes any smearing.

The theory of pulsar timing relies on the pulse profile being an invariant. We know it isn't. Observations of slow pulsars with large fluxes demonstrate that pulsar profiles require many rotations to stabilize, but due to their small fluxes, this is difficult to establish for the millisecond pulsars.

Astronomers fit a template, often constructed of either the sum of the best observations or the addition of several Gaussian components that closely approximate the pulse shape to each observation. As we saw in the introduction the "theory" is that each observation is just a scaled version of the template plus *random* noise with a phase shift. Fitting to equation 1.1 for $A$ and $\alpha$ (often in the Fourier domain) yield an arrival time (TOA) and error. These TOAs and errors are often placed into a least squares fitting program like tempo2 (Hobbs, Edwards and Manchester 2006) to yield pulsar parameter estimations from which physical interpretations are made.

Unfortunately, once this fit is performed, the reduced chi-squared is often far from unity, limiting the interpretation of the physical parameter errors, and ultimately the pulsar timing methodology. A conservative approach to this problem is to keep adding a systematic error term in quadrature to each TOA until the reduced chi-squared is unity. A more dangerous (but more often used approach) is to linearly increase the size of each error until the reduced chi-squared is unity. This is an optimistic assumption, but reduces the size of the errors on the physical parameters.

Investing in instrumentation that minimizes systematic errors can be greatly worthwhile. Pulsar fluxes often vary greatly (by factors of $\gg$10) and when such amplifications occur, capitalising on them is essential. The radiometer equation tells us that an

https://doi.org/10.1017/S1743921309990421 Published online by Cambridge University Press

observation at scintillation maximum with 10x the flux of the average theoretically has 100x the influence in the fit. Instrumentation that eliminates the need for artificially adjusting the error estimates can therefore be very powerful.

There are a number of factors which lead to poor residual chi-squares, and most are related to distortions of the profile. These fall into two categories, natural, and man-made.

**Natural sources of error:** For the younger pulsars the assumption that the pulsar is a consistent rotator is just flawed. Unfortunately, on moderate $\sim$ yearly timescales, the imperfect rotation can be masked by the fit for other parameters, such as position or proper motion. Fitting for these parameters often gives nonsensical values and should be avoided. Other sources of error are pulse shape variability, variable scattering and pulsar dispersion due to the changing interstellar medium. In principle variable delays due to changing dispersion measures can be removed by multi-frequency observing, thus observing with two or three different frequencies removes most of this source of error. Unfortunately, multi-path scattering means there is no "one true dispersion measure" for observations conducted at different frequencies, a point first made by Don Backer. Nevertheless when trying to get sub-microsecond timing one should observe the second law of pulsar timing:

**Second law of pulsar timing:** *Eliminate the effects of variable pulse dispersion measures by conducting observations at widely-spaced radio frequencies as closely as possible in time. This gives a history of the pulsar's dispersion measure, which can be removed by the fitting program.*

**Man-made sources of error:** As pulsar radiation traverses the ionized interstellar medium it gets delayed by a frequency-dependent amount. Integration of the pulsar's radiation over finite bandwidths smears the profile. Pulsar astronomers seek to eliminate this by either building multi-channel filterbanks with very narrow filters, or by sampling the voltages and performing the feat known as "coherent dedispersion" (Hankins and Rickett 1975), that removes the dispersive effects "perfectly". Until this century computers were too slow to perform coherent dedispersion on all but narrow bandwidths and filterbanks were too costly to mass produce. This greatly degraded the quality of pulsar timing data.

This has all changed rapidly with the advent of the field programmable gate array and the march of Moore's law producing cheap computer power. It is now possible to build 1024- or 2048-channel digital filterbanks at reasonable cost such as the Australia Telescope's DFB series. It is also possible to attempt real-time coherent dedispersion on large bandwidths using moderately priced clusters in tandem with an A/D convertor, an FPGA and 10 Gb ethernet. These systems are leading to greatly increased timing precision and a reduction in systematic errors (see Verbiest *et al.* 2009 for his analysis of 20 millisecond pulsars with the coherent dedispersion system CPSR2). CPSR2 is a 2x2x64 MHz coherent dedispersion system that achieves theoretical errors that provide reduced chi squared fits near unity for many pulsars. This enables the use of high $SNR$ observations in the fit, and thus more reliable physical parameters. Early results from the ATNF's DFB series of instruments that exploit more bandwidth than CPSR2 are also yielding smaller reduced chi squares than pre-2000 instruments.

**Third law of pulsar timing:** *Eliminate as many distortions to the profile as you can by using digital (not analogue) electronics, and coherent dedispersion where practicable, or at worst multi-channel digital filterbanks at higher frequencies.*

It is very difficult to combine data from different telescopes and instrumental "backends", as each introduce their own unique distortions to the profile, leading to an arbitrary phase jump. This leads to the fourth law of pulsar timing that concerns the fact that we are conducting difference experiments.

**Fourth law of pulsar timing:** *Never change any equipment unless it leads to a very substantial improvement in timing, and where possible, overlap your timing to eliminate the need for arbitrary phase jumps. Your data reduction software is just as important to freeze as the hardware in this respect. Even subtle changes in binning algorithms can lead to undiagnosed DC offsets in timing which limit the ultimate timing precision.*

Unfortunately, even the best timing equipment in the world cannot undo smearing introduced by a bad pulsar ephemeris. Until one obtains phase connection, it is easy to artificially smear the pulse profile by observing with a bad ephemeris. Similarly, unnoticed changes to the observatory (a new cable can add many 10s of nanoseconds to an arrival time), clock errors etc, all conspire to reduce your knowledge of your true arrival time errors. The best timing experiments check the quality of their (reduced) data regularly, include at least one well-behaved pulsar for comparison purposes and inspect the data by eye as well as by automated reduction processes. Finally, be patient. Pulsar timing takes time to yield results.

## 4. The Future

We are still learning about how to get the most from our instruments. The effect of radio frequency interference (RFI), both periodic and impulsive are increasingly frustrating sources of error for pulsar astrophysicists. New instrumentation has higher dynamic range A/Ds than earlier models to reduce distortions of the profile. This however makes them subject to large distortions of the profile in literally microseconds during bursts of RFI. In the future, monitoring both the total power and the accuracy with which each sub-integration obeys the fundamental assumptions about pulsar timing (equation 1.1) will help reduce these effects. Others are pioneering the use of reference antennae to subtract sources of known RFI (Kesteven *et al.* 2005, Manchester 2008). At the high fidelity end, van Straten (in preparation) has recently demonstrated that polarization can be used to increase timing precision by using a pulsar to calibrate the receiver, and hence remove the distortion of profiles induced by imperfect receivers. van Straten used the bright millisecond pulsar PSR J0437–4715 to calibrate the Parkes 20cm receivers over many years, determined appropriate coefficients of the Jones' matrices, and then improved the timing of the millisecond pulsar PSR J1022+1001 to below 1 microsecond RMS residuals.

Other second-order contributions to our systematic errors become increasingly important as our instrumentation and sensitivities improve. To maximize our signal-to-noise ratio, we integrate over increasingly large bandwidths. Since the pulse width is a function of frequency, time-dependent scintillation varies the width of the final profile leading to a systematic change in width after "scrunching" in frequency, which can lead to systematic errors in the shift and error estimate. Doppler shifts and the time delay between the top and bottom of the band make "labelling" an integration with a specific time and frequency something to be used with caution. Dispersion measures can appear to change if Doppler corrections are not properly taken into account.

We must not forget about the importance of an accurate solar system barycenter, nor the provision of accurate clocks (see various authors in these proceedings). All of these incremental improvements are leading astronomers to timing below 100 nanoseconds

https://doi.org/10.1017/S1743921309990421 Published online by Cambridge University Press

in precision. With the advent of the Square Kilometre Array (see Kramer *et al.* these proceedings), and lessons learned over the past 40 years, pulsar timing is set to search for even more subtle effects, such as the effect of a gravitational wave background on millisecond pulsar timing (see Hobbs these proceedings) and experiments on relativistic gravity (see Stairs these proceedings).

## References

Bhat, N. D. R., Bailes, M., & Verbiest 2008, *Phys. Rev. D*, 77, 12, 124017

Hankins, T. H. & Rickett, B. J. 1975, *Methods in Computational Physics*,Vol 14, Radio Astronomy, 55.

Hobbs, Edwards & Manchester 2006, *MNRAS*, 369, 655

Kramer *et al.* 2006, *Sci.*, 314, 97

Kesteven *et al.* 2006, *Radio Science 40*, RS5S06, 1

Lorimer, D. R. & Kramer, M. 2004, *The Handbook of Pulsar Astronomy*, Cambridge University Press.

Lyne *et al.* 1998, *MNRAS*, 295, 743

Manchester *et al.* 1991, *Nature*, 352, 219

Manchester *et al.* 2001, *MNRAS*, 328, 17

Manchester 2006, *http://www.nanograv.org/presentations/IPTA/IPTAAug2008Presentations.aspx*

Ransom, S. *et al.* 2005, *Science*, 307, 892

Stairs *et al.* 2002, *ApJ*, 581, 501

Taylor, J. H. & Weisberg, J. M. 1982, *ApJ*, 253, 908

van Straten *et al.* 2001, *Nature*, 412, 158

Verbiest *et al.* 2009, *MNRAS*, in press