

ON A TABLE INDICATING THE ANNUAL PREMIUM FROM THE SINGLE, AND VICE VERSÂ.

To the Editor of the Assurance Magazine.

SIR,—We possess, in the late Mr. Orchard's tables, the means of passing from the value of an annuity on any status to those of both the single and the annual premiums for assurance on the same status; and it seems that, to complete our canon, there is still room for a set of tables allowing us to pass from the single to the annual assurance premiums, and *vice versa*. I am, therefore, induced to offer you a specimen of the tables indicated, adapted to the rate of 3 per cent.

Let a denote the value of an annuity on any status, A the value of an assurance on the same status, such that the sum assured is certainly to be paid on the failure or the efflux of that status, then we have

$$A = 1 - (1 - v)(1 + a) \quad . \quad . \quad . \quad (1)$$

and if π denote the annual premium for the same assurance, payable also till the failure or efflux of the status, we likewise have

$$\pi = \frac{A}{1 + a} \quad . \quad . \quad . \quad (2)$$

From (1),
$$1 + a = \frac{1 - A}{1 - v},$$

and
$$\frac{1}{1 + a} = \frac{1 - v}{1 - A};$$

Hence, by substitution in (2),

$$\pi = (1 - v) \frac{A}{1 - A} \quad . \quad . \quad . \quad (3)$$

In accordance with this formula the table was constructed. It must be noted that the expression (3) being obtained by elimination of $(1 + a)$ between the two equations (1) and (2), the table is applicable only in cases in which both these equations hold. Thus, it is applicable to whole life assurances, on one or any number of lives, to assurances on last survivors, and to endowment assurances when the premium is payable until the assurances become due. But it is not applicable to temporary or deferred assurances, or survivorship assurances, as in each of these cases one or other of the specified conditions does not hold.

Each tenth value in the table was formed by a continuous arithmetical process, admitting of check at any point, and interpolation was used to fill up the intermediate terms, two orders of differences being employed in the earlier portion of the table, and three orders in the latter, the change being rendered necessary by the rapid variation in the rate of increase of the tabular values.

The argument, is the single premium for assurance of £1 or £100 according as the decimal point is placed, immediately before or immediately after the two leading figures in the left hand column; and the tabular result, as pointed, is the corresponding annual premium for £100.

The table gives by inspection the result answering to the first three figures only of the single premium, and the remaining two figures have to

be proportioned for. To facilitate this operation the differences (corresponding to the middle of the line on which they are respectively placed) are inserted in the right hand column. They are *additive*, and the rule for their use is:—Multiply the difference by the number composed of the 4th and 5th figures of the single premium, and add one hundredth part of the product to the tabular result corresponding to the first three figures. The sum is the complete result required.

I now give a few examples. I might for these take figures purely at random, but prefer taking actual values, according to various tables, so as to afford the means of easy verification.

Ex. I.—Single life, aged (15). Carlisle 3 per cent.

$$\begin{array}{r}
 A = \cdot 31315 \quad (\text{Gray, Smith, and Orchard}). \\
 \cdot 313 \text{ gives} \quad \quad \quad 1 \cdot 327 \\
 \text{p.p. for 15} = \frac{15 \times 6}{100} = \frac{1}{100} \\
 \pi \quad \quad \quad = \quad \quad \quad \underline{\underline{1 \cdot 328}}
 \end{array}$$

II.—Single life, aged (62). Northampton 3 per cent.

$$\begin{array}{r}
 A = \cdot 70275 \quad (\text{Jones, p. 289}). \\
 \cdot 702 \text{ gives} \quad \quad \quad 6 \cdot 861 \\
 \text{p.p. for 75} = \frac{75 \times 33}{100} = \frac{25}{100} \\
 \pi \quad \quad \quad = \quad \quad \quad \underline{\underline{6 \cdot 886}} \quad (\text{Jones, 289}).
 \end{array}$$

III.—Last survivor of (30) and (35). Carlisle 3 per cent.

$$\begin{array}{r}
 A = \cdot 30733 \quad (\text{Gray, Smith, and Orchard}). \\
 \cdot 307 \text{ gives} \quad \quad \quad 1 \cdot 290 \\
 \text{p.p. for 33} = \frac{33 \times 6}{100} = \frac{2}{100} \\
 \pi \quad \quad \quad = \quad \quad \quad \underline{\underline{1 \cdot 292}}
 \end{array}$$

IV.—Endowment and assurance for 20 years on (40). Carlisle 3 per cent.

$$\begin{array}{r}
 A = \cdot 60461 \\
 \cdot 604 \text{ gives} \quad \quad \quad 4 \cdot 442 \\
 \text{p.p. for } \frac{61 \times 19}{100} = \frac{12}{100} \\
 \pi \quad \quad \quad = \quad \quad \quad \underline{\underline{4 \cdot 454}}
 \end{array}$$

V.—Last survivor of (21) and (31). Northampton 3 per cent.

$$\begin{array}{r}
 A = \cdot 32770 \quad (\text{Jones, pp. 288 and 917}). \\
 \cdot 327 \text{ gives} \quad \quad \quad 1 \cdot 415 \\
 \text{p.p. for 70} = \frac{70 \times 6}{100} = \frac{4}{100} \\
 \pi \quad \quad \quad = \quad \quad \quad \underline{\underline{1 \cdot 419}}
 \end{array}$$

For the inverse use of the table the rule is:—Take out the three-figure single premium corresponding to the tabular value next lower than the given annual premium, attach two ciphers to the corresponding tabular difference, and divide the number thus formed by the excess of the given annual premium over the next lower tabular one. The quotient (*prefixing* a cipher if it consist of only one figure) will be the 4th and 5th figures of the single premium.

Ex. I.—Required the single premium corresponding to annual premium 1·952.

$\pi = 1\cdot952$	
Next lower	40·1
Tab. diff.	<u>8)200</u>
	25 <u>25</u>
	A = <u>40·125</u>

II.—Required the single premium corresponding to annual premium 6·996.

$\pi = 6\cdot996$	
Next lower	70·6
Tab. diff.	<u>34)2</u>
	6 <u>06</u>
	A = <u>70·606</u>

I shall be glad if you think this contribution worthy of a place in the *Assurance Magazine*,

And remain, Sir,

Your most obedient servant,

Aberdeen, 4th June, 1863.

H. AMBROSE SMITH.

Annual Premium for Assurance of £100 at 3 per Cent.

Single Premium.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Difference.
30	1·248	1·254	1·260	1·266	1·272	1·278	1·284	1·290	1·296	1·302	6
31	1·309	1·315	1·321	1·327	1·333	1·339	1·346	1·352	1·358	1·364	6
32	1·371	1·377	1·383	1·390	1·396	1·402	1·409	1·415	1·422	1·428	6
33	1·435	1·441	1·448	1·454	1·461	1·467	1·474	1·480	1·487	1·494	7
34	1·500	1·507	1·514	1·521	1·527	1·534	1·541	1·548	1·555	1·561	7
35	1·568	1·575	1·582	1·589	1·596	1·603	1·610	1·617	1·624	1·631	7
36	1·638	1·645	1·653	1·660	1·667	1·674	1·681	1·689	1·696	1·703	7
37	1·711	1·718	1·725	1·733	1·740	1·748	1·755	1·763	1·770	1·778	7
38	1·785	1·793	1·800	1·808	1·816	1·823	1·831	1·839	1·847	1·854	8
39	1·862	1·870	1·878	1·886	1·894	1·902	1·910	1·918	1·926	1·934	8

Annual Premium for Assurance of £100 at 3 per Cent. (continued).

Single Premium.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Difference.
40	1.942	1.950	1.958	1.966	1.974	1.983	1.991	1.999	2.007	2.016	8
41	2.024	2.032	2.041	2.049	2.058	2.066	2.075	2.083	2.092	2.100	9
42	2.109	2.118	2.127	2.135	2.144	2.153	2.162	2.170	2.179	2.188	9
43	2.197	2.206	2.215	2.224	2.233	2.242	2.252	2.261	2.270	2.279	9
44	2.288	2.298	2.307	2.317	2.326	2.335	2.345	2.354	2.364	2.373	9
45	2.383	2.393	2.402	2.412	2.422	2.432	2.441	2.451	2.461	2.471	10
46	2.481	2.491	2.501	2.511	2.521	2.532	2.542	2.552	2.562	2.573	10
47	2.583	2.593	2.604	2.614	2.625	2.635	2.646	2.656	2.667	2.678	11
48	2.689	2.699	2.710	2.721	2.732	2.743	2.754	2.765	2.776	2.787	11
49	2.798	2.810	2.821	2.832	2.844	2.855	2.866	2.878	2.889	2.901	11
50	2.913	2.924	2.936	2.948	2.960	2.971	2.983	2.995	3.007	3.019	12
51	3.032	3.044	3.056	3.068	3.080	3.093	3.105	3.118	3.130	3.143	12
52	3.155	3.168	3.181	3.194	3.206	3.219	3.232	3.245	3.258	3.271	13
53	3.284	3.298	3.311	3.324	3.338	3.351	3.365	3.378	3.392	3.405	13
54	3.419	3.433	3.447	3.461	3.475	3.489	3.503	3.517	3.531	3.546	14
55	3.560	3.574	3.589	3.603	3.618	3.633	3.647	3.662	3.677	3.692	15
56	3.707	3.722	3.737	3.752	3.768	3.783	3.798	3.814	3.830	3.845	15
57	3.861	3.877	3.893	3.909	3.925	3.941	3.957	3.973	3.989	4.006	16
58	4.022	4.039	4.055	4.072	4.089	4.106	4.123	4.140	4.157	4.174	17
59	4.191	4.209	4.226	4.244	4.261	4.279	4.297	4.315	4.333	4.351	18
60	4.369	4.387	4.406	4.424	4.442	4.461	4.480	4.499	4.518	4.537	19
61	4.556	4.575	4.594	4.614	4.633	4.653	4.672	4.692	4.712	4.732	20
62	4.752	4.772	4.793	4.813	4.834	4.854	4.875	4.896	4.917	4.938	21
63	4.959	4.981	5.002	5.024	5.045	5.067	5.089	5.111	5.133	5.156	22
64	5.178	5.201	5.223	5.246	5.269	5.292	5.315	5.338	5.362	5.385	23
65	5.409	5.433	5.457	5.481	5.505	5.530	5.554	5.579	5.604	5.629	25
66	5.654	5.679	5.705	5.730	5.756	5.782	5.808	5.834	5.860	5.887	26
67	5.914	5.940	5.967	5.994	6.022	6.049	6.077	6.105	6.133	6.161	28
68	6.189	6.218	6.247	6.275	6.305	6.334	6.363	6.393	6.423	6.453	29
69	6.483	6.513	6.544	6.575	6.606	6.637	6.668	6.700	6.732	6.764	31
70	6.796	6.829	6.861	6.894	6.927	6.961	6.994	7.028	7.062	7.096	34
71	7.131	7.166	7.201	7.236	7.271	7.307	7.343	7.379	7.416	7.453	36
72	7.490	7.527	7.564	7.602	7.640	7.679	7.717	7.756	7.796	7.835	39
73	7.875	7.915	7.955	7.996	8.037	8.078	8.120	8.162	8.204	8.247	42
74	8.290	8.333	8.377	8.421	8.465	8.509	8.554	8.600	8.645	8.691	45
75	8.738	8.785	8.832	8.879	8.927	8.976	9.024	9.073	9.123	9.173	49
76	9.223	9.274	9.325	9.377	9.429	9.482	9.535	9.588	9.642	9.696	53
77	9.751	9.806	9.862	9.918	9.975	10.032	10.090	10.148	10.207	10.267	58
78	10.327	10.387	10.448	10.510	10.572	10.635	10.698	10.762	10.826	10.891	63
79	10.957	11.023	11.090	11.158	11.226	11.295	11.365	11.435	11.506	11.578	70
80	11.650	11.724	11.798	11.872	11.948	12.024	12.101	12.179	12.257	12.337	77