PANEL DISCUSSION

IVAN R. KING
Astronomy Department, University of California
Berkeley, CA 94720-3411, U.S.A.
GEORGES MEYLAN
European Southern Observatory
Karl-Schwarzschild-Strasse 2
D-85748 Garching bei Muenchen
Germany
FRANK VERBUNT
Astronomical Institute, Utrecht University
Postbox 80.000, 3508 TA Utrecht, the Netherlands
PIET HUT
Institute for Advanced Study, Princeton, NJ 08540, USA
AND
DAIICHIRO SUGIMOTO
Department of Earth Science and Astronomy
College of Arts and Sciences, University of Tokyo
3-8-1 Komaba, Meguro-ku, Tokyo 153, Japan

Ivan R. King

My first reaction, at the end of this conference, is that it has not produced nearly as much "Confrontation of Theory and Observations" as one might have wished. We have all made our separate presentations, but no one has yet made the connections that are so much to be desired. Perhaps it is just that we need time to absorb what we have heard, and react to it.

What I can present here are a few impressions of issues that I think are important:

1. We have a lot of new information on the low-mass end of globular-cluster mass functions. It shows similarities between clusters, and also differences. It is important to understand the differences: which of them reflect

P. Hut and J. Makino (eds.), Dynamical Evolution of Star Clusters, 319–329.
© 1996 *International Astronomical Union. Printed in the Netherlands.*

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

differences of origin, and which of them are results of dynamical evolution. Regarding the latter, we understand qualitatively the effects of internal relaxation and external tidal shocks, but our quantitative knowledge of these processes is still inadequate. It is possible to follow a cluster such as NGC 6397 and calculate its detailed evolution, as stars rearrange themselves and some escape, while Galactic tidal forces change from moment to moment—but this has not yet been done, and I think it is very much needed.

2. A great deal can be done, and remains to be done, in fitting equilibrium models of clusters to the observed data. It is clear that good fits can be made, to clusters that are in a pre- or a post-collapse stage, and that the departures from equilibrium found in the Fokker–Planck calculations do not seriously invalidate the models. (A likely exception to this statement, however, is the late stage of collapse, which might be exemplified by the present state of M15.) An interesting corollary of this fitting exercise is the possibility of detecting, through the density perturbations of their gravitational potential, the presence and number of massive remnants at the center of a cluster. Collapsed cores, however, present a frustrating observational problem: they have too few stars in them to allow us to distinguish a pure cusp from one with a small core radius. It may be that a detailed modeling of the entire mass mixture will introduce large enough numbers of stars to make this problem tractable.

3. It is very satisfying to see core oscillations actually demonstrated in an N-body calculation. But it remains to examine in detail what happens in this phenomenon, as a function of time and for the individual mass groups. An important related question is, how are we to distinguish between a cluster that has never collapsed and one that has collapsed and re-expanded? At a level of even greater detail, will it be possible to determine at what phase of the collapse-and-re-expand cycle a particular cluster is? —and even more difficult, how many times has it collapsed and re-expanded?

4. Finally, the role of binaries is crucial. We are still debating the workings of the two-body capture mechanism; when will this be settled? Perhaps we can relegate this to a side issue, however; the necessities of dynamical evolution dictate that the collapse must, sooner or later, be turned around by the formation of binaries through *some* process, so that perhaps the how and the when merely affect details of the time scale. A separate important issue, however, is the number of primordial binaries, which is not at all well known and has a very serious effect on the evolutionary path that a cluster will take. A final point—which may reflect only my own lack of understanding—is a lack of clarity about the history of an individual binary as it reacts dynamically with its surroundings, and the statistics of such pairs as the cluster evolves.

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

Georges Meylan

NEW THEORETICAL RESULTS

GRAVOTHERMAL OSCILLATIONS IN N-BODY CALCULATIONS

For about a decade, Fokker-Planck and conducting-gas-sphere evolutionary models of globular clusters have been computed well into core collapse and beyond, leading to the discovery of possible post-collapse oscillations. Nevertheless, there have been until now some doubts about the presence of such gravothermal oscillations in a pure large N-body calculation, because of its realistically grainy stellar distribution. This IAU Symposium # 174 has seen the first announcement of the long awaited confirmation of this theoretical prediction, a confirmation which is the consequence of hardware and software improvements of N-body codes. J. Makino (these proceedings) does indeed observe gravothermal oscillations in his N-body simulations (32,000 equal-mass bodies). His fundamental and beautiful results are very similar to those predicted by Fokker-Planck calculations.

Open cluster N-body simulations

Similar hardware and software improvements of N-body codes have allowed S.J. Aarseth (these proceedings) to simulate open cluster dynamical evolution with increasingly realistic physical details such as, e.g., mass loss by stellar evolution, chaotic tidal interaction, Roche lobe mass transfer, common envelope evolution, magnetic braking, and gravitational radiation. Collision outcomes predict blue stragglers and other exotic objects. This represents the beginning of the micro- and macroscopic understanding of the dynamical evolution of genuine star clusters by using N-body codes, which should soon reach N = 50,000.

Stellar collision simulations

There is also an increasing number of very detailed studies of stellar flybys, encounters, collisions, and mergers. See the contributions by D.F. Chernoff, M. Davies, R.A. Mardling, S. McMillan, and F. Rasio (these proceedings).

NEW OBSERVATIONAL RESULTS

The new, very impressive observational results constraining the dynamics of globular clusters, presented at this IAU Symposium # 174, are essentially the consequences of data obtained with two satellite observatories: HST and ROSAT.

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

GCs luminosity and mass functions from HST data

For four of the nearby globular clusters, deep imaging provides luminosity and mass functions for faint low-mass stars, nearly down to the mass which should represent the lower limit of hydrogen burning, i.e. at about 0.08 $M_\odot$. Mass segregation is clearly observed, in a quantitative way, between stars of different masses at different locations in the clusters. For the low-mass stars in the lower part of the main sequence, significant differences exist between the mass functions of NGC 6397 and M15. This could be a direct consequence of the very different galactic orbits followed by these two globular clusters, making NGC 6397 much more susceptible to lose stars via tidal shocking. See the fundamental contributions by I.R. King, A. Cool, G. Piotto, and C. Sosin (these proceedings). See also the beautiful white dwarf sequence observed in M4, the nearest globular cluster (G. Fahlman, these proceedings).

The above results come from the first year of observations with the refurbished HST. This is only the beginning of the use of such data, and there is no doubt that many more studies will follow, with more data and more dynamical interpretations. Star cluster observations are and will be one of the major contributions by HST, leaving far behind the imaging of globular cluster cores from the ground.

HST and ROSAT observations of compact binaries

Compact binaries, containing a degenerate companion, such as a white dwarf, or neutron star, have been observed in globular clusters as X-ray sources by ROSAT. See the results concerning individual sources in, e.g., NGC 6397 and 47 Tucanae, and about the general statistics of the population of such sources in galactic globular clusters in the contributions by J.E. Grindlay, F. Verbunt, and M. Shara (these proceedings).

FROM QUALITATIVE TO QUANTITATIVE ANSWERS

The full interpretation of present and future ground-based and satellite data should allow a systematic improvement in our knowledge of star clusters, by providing quantitative estimates instead of the past qualitative statements (see, e.g., the previous hints about the presence of mass segregation). Great improvements in our understanding of star clusters will come from:

- a precise knowledge of Initial Mass Function and Mass Function for young and old open clusters, respectively;
- a precise knowledge of Mass Function for globular clusters;
- a quantitative estimate of mass segregation;
- taking advantage of the age and mass variety of LMC, SMC and M31 star clusters;

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

• estimating the precise fraction of binaries, from photometric and radial velocity observations;

• measuring, for large numbers of stars (a few thousands), both radial velocities and proper motions (the latter contain the information of two out of the three components of the stellar spatial velocity);

• interpreting these data with more than one model, i.e., by constraining the observations with King-Michie, Fokker-Planck, non-parametric, and N-body codes; so far, the observations gathered in one cluster have been generally constrained by one kind of model only; the use of more than one model would bring out naturally the pros and cons of each theoretical approach, and would also point out the similarities and differences between globular clusters.

Frank Verbunt

In my view it is rather surprising that the people, well... most people, who do N-body calculations have always insisted on comparing their results with globular clusters, whereas in fact what they are calculating are open clusters. My advise is to *forget* globular clusters for a while, and to concentrate on open clusters, which have a number of stars that is actually doable.

I would of course not hasard to put forward such an advise if Douglas Heggie had not done so already in his contribution to this meeting; and if I had not known that Sverre Aarseth already is applying his code to the study of open clusters.

The main advantage of concentrating on open clusters is that the observational data of these are much more detailed than for globular clusters, so that a detailed comparison is possible between the theory and the observations. The observations are rapidly improving, and soon it will be possible also to test binary evolution scenarios is detail. I will not believe any calculation on a globular cluster untill the code used has been tested extensively against observations of open clusters. (In the same way, people who use hydrodynamical codes start by doing simple test calculations of, say, flow through a tube.)

The importance of the initial conditions should be tested. What is it that determines whether a cluster becomes open or globular?

As regards globular clusters, our knowledge of the contents of the globular clusters is becoming much better, and comparison with calculations can be made in much more detail. Measurements of the period derivatives of radio pulsars provide very good estimates of the gravitational acceleration, and thereby information on the distribution in the cluster of the total mass, i.e. including white dwarfs and neutron stars. And the HST obser-

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

vations give information on the distributions of the visible stars, including blue stragglers.

Piet Hut

Theoretical studies of dense stellar systems have recently seen rapid progress in the development and refinement of many different techniques for modeling star systems. I will briefly mention six classes of techniques, while commenting on the progress to be expected therein during the years to come, until the next IAU Symposium on star cluster dynamics.

1. *N-Body Techniques.* Until the time of this meeting, N-body techniques did not go much further than producing toy models. These were certainly interesting and promising, but fell far short of modeling realistic globular clusters. The sheer computational expense was simply too forbidding to come even close to modeling clusters on a star-by-star basis, given that most globular clusters have numbers of stars in the range $10^5 - 10^6$. All this has changed, now that the GRAPE-4 special purpose computer has come online, with its Teraflops speed (see Taiji's contribution). As detailed in Makino's paper, calculations with 3×10^4 particles have now become routine, and calculations with up to 10^5 particles are feasible with the new generation of front end computers that will become available in 1996. In addition, the GRAPE designers are now setting their eyes on an even more ambitious goal, namely the development of a machine with a speed a factor 10^3 higher than that of the current GRAPE-4. Such a Petaflops computer could become operational by the year 2000, if the necessary funding will be found, something that is being actively pursued at present.

2. *Scattering Experiments.* On the other side of the spectrum of possibilities for star-by-star modeling of processes in globular clusters, we find local treatments of the 'microphysics' of close encounters between single stars and binaries. Significant progress has been made in this area, for example through the development of fully automated scattering software. Specification of a few physical parameters is sufficient to start up this software laboratory, allowing the set-up, execution, and on-line analysis of experiments to be carried out without any human intervention. Extensions of this package to include binary-binary scattering is currently underway; see McMillan's contribution to these proceedings for further details. Other approaches to the dynamics of small-N systems include a stability analysis of hierarchical triples (see Kiselova's contribution), the addition of hydrodynamical effects (Davies' paper), and an analysis of the frequency of physical collisions during scattering events (Chernoff and Huang's contribution).

3. *Numerical Approximation Techniques.* Until this year, simulations of globular clusters with realistic particle numbers of 10^5 and larger could only

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

be undertaken through a variety of numerical approximation techniques. Between twenty and thirty years ago, various Monte Carlo approaches have been developed for solving the evolution of a star cluster in the Fokker-Planck approximation. Soon afterwards, direct integration techniques have been developed for solving the Fokker-Planck equation. In addition, various conducting gas sphere models have been developed. The combination of all these models, and detailed comparative studies of the results of the different techniques, have proved beneficial for our understanding of the strengths and weaknesses, as well as the limits of applicability, of the individual models. For detailed descriptions, see the talks in these proceedings by Giersz, Lee, Spurzem, and Takahashi, the posters by Drukier, Einsel and Spurzem, and McMillan and Engle, as well as the talk by Heggie, who reports on an interesting technique in which the results from many small-N runs are averaged. Significant further progress can be expected, both in the ongoing development and refinement of individual techniques, as well as in applications in connection with the new GRAPE hardware. Even though realistic N-body calculations now are feasible, allowing a star-by-star modeling of globular clusters, these calculations still remain expensive, requiring sometimes weeks and often months for a single run to reach completion. Approximate methods therefore are called upon in order to interpolate between, and perhaps even extrapolate from, the few detailed runs coming out of the GRAPE machinery.

4. *Analytic Techniques.* Although one sometimes gets the impression that current research in star cluster dynamics is almost completely a numerical enterprize, there is plenty of room left for the development of new analytic techniques. Perhaps the most promising area will be that of the development of physically inspired fitting formulas. Rather than tabulating the results of detailed numerical calculations, or applying arbitrary curve fitting to those results, it is often possible to use more physically motivated reasoning in the choice of fitting functions. This has the double advantage of allowing more physical insight in the results obtained as well as providing a measure of confidence in extrapolations beyond the regimes currently tested numerically. An example of this approach is presented in the poster paper by Heggie *et al.*. Others examples of analytical approximation techniques are given in the contribution by Mardling and in the posters by Heggie, and Heggie and Rasio.

5. *Approximate Treatments of Stellar Evolution.* In addition to the four techniques listed above, which can be applied to the purely gravitational N-body problem, a more realistic treatment of star cluster evolution requires us to go beyond the point-mass approximation. A first step in this direction is the inclusion of a time dependency of the mass of a single star, for example by taking into account the mass that is lost in later stages of

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

stellar evolution, towards the formation of a white dwarf, neutron star, or black hole. However, as soon as this step has been taken for single stars, far more complicated issues arise when we want to apply such simple recipes to the case of interacting binary stars, let alone the simultaneous interaction of three or four stars during scattering encounters. How to treat mass overflow, how to discriminate between stable and unstable cases of such overflow, what to do with common envelope phases in their evolution — all these questions require careful consideration, together with the imperative to refrain from too-fancy a type of solution. The challenge to produce a coherent set of recipes that are as simple as possible, but no simpler, across the board is a formidable one. This challenge has recently been taken up by several individuals, as can be seen from the contributions by Aarseth, Eggleton, and Portegies Zwart.

6. *Approximate Treatments of Hydrodynamics.* Another extension that is called for in realistic star cluster modeling, beyond the point-mass limit, is the inclusion of some type of hydrodynamics. Smooth Particle Hydrodynamics forms a natural candidate; see the papers by Davies and Rasio. In the not-too-distant future, it will be feasible to include local SPH calculations as an option in large N-body calculations, since the additional computational cost required will become relatively less for larger N values. The challenges in combining stellar dynamics and SPH techniques will mainly take the form of software technicalities, related to the complicated bookkeeping required for the treatment of simultaneous three-body and four-body interactions, and occasional interactions with much larger number of particles. When following 10^5 stars for a Hubble time, exceptional cases are bound to occur in which, for example, two binaries will be involved in a complex resonance encounter while at the same time encountering yet another binary. The treatment of the inclusion of stars into and escape from such a six-star interaction will be far from straightforward, and has not yet been attempted in all generality.

Other techniques could be mentioned here as well, especially when we include the modeling of dense galactic nuclei. Here the possible presence of black holes, as well as young stars and molecular clouds, create additional complications. Closer to home, the modeling of a proto-planetary nebula again sets different requirements for the physics to be included in a study of the dynamics of this type of dense stellar systems. However, the above six classes of techniques, while not being exhaustive, give at least some taste of the further developments to be expected in the near future.

Daiichiro Sugimoto

From theoretical sides I would like to discuss five points.

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

1) *Fundamental Concepts.* For the evolution of globular clusters the targets for the fundamental concepts were as follows. During 1975-84 the concept of Gravothermal Collapse was established which included not only the linear analysis but also the instability of finite amplitude as represented by similarity solution. From the standpoint of the linearized theory the gravothermal instability leading to the contraction can lead also to expansion as well. Thus, it led to the concept of gravothermal oscillation. During 1984-95 the gravothermal oscillation was shown to occur in gaseous and Fokker-Planck models, but it is only at this Symposium when the gravothermal oscillation is clearly shown to occur in N-body model. Commonly in these models the gravothermal oscillation takes place as a refrigeration cycle. It implies that the energy input by binary hardening only triggers the oscillation and the non-linear oscillation is maintained, not by the energy input from the binary hardenings but by the gravothermal instability itself. This will explain the post-collapse evolution how the globular clusters are able to survive even after the core collapse. What will be the fundamental concept that we should clarify in the coming 1995-2005? In this Symposium the results of very advanced observations were presented including HST observations and 3-dimensional spectroscopy. They will be used to construct detailed models of evolution of stellar systems including the effect of stellar evolution. At present, however, any central concepts to pursue are not clearly posed yet. The observational results should not simply await for more detailed models. What fundamentals could we extract from them? Or have the fundamental concepts been all clarified and are they only waiting for elaborations?

2) *Gravothermal Oscillation.* In the IAU Symposium No. 113 in 1984 people discussed against Sugimoto and Bettwieser concerning their theory of gravothermal oscillation. There were two points for this discussion. The first was whether such non-linear oscillation takes place or not even in gaseous or continuous model. The second was whether it takes place even in a discrete system such as star clusters. From the theory of self-gravitating system, which had been developed from the theory of stellar structure and evolution, it was self-evident, at least for me, that such oscillation took place as later clarified with different models. However, the second point could not help waiting for a large scale N-body calculations, because the computations with several thousand bodies can not extract any clear results because of statistical fluctuations. It became possible only recently when the dedicated computer GRAPE-4 became operational in this year. The result presented in this Symposium by Makino is based on computations with a 32,000 body model as much as 500 Gflops $\cdot$ months on GRAPE-4. The cost for constructing this dedicated machine was only 1.5 Million dollars. If the same amount of computation had been done on a commercially available general-

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

purpose machine, the machine time for this single computation would have costed ten times the cost for constructing GRAPE-4 machine. This is one of the examples telling us that new means are often needed for a breakthrough.

3) *Seed of Black Hole.* There seem some compelling evidences that there are massive black holes in the active galactic nuclei. In this respect Genzel's presentation in this Symposium was very interesting. Once a high density core containing a black hole is formed, the black hole will grow by eating the stars or by merging with another black hole in the core or in the nucleus of another colliding galaxy. Therefore, the real difficulty lies in the first formation of a seed of the massive black hole so far as we assume the standard model for the gravothermal collapse of single-mass star cluster. The time scale of the gravothermal collapse is simply too long for a large system and, in addition to it, the core formed by the gravothermal collapse is usually too small in mass. We obviously need an idea.

4) *Mass Segregation.* For a mass function of a star cluster, Salpeter's mass function is assumed in many cases which is expressed as $f(m)dm = m^{-2.35}dm$. If we express it as a mass weighted function as $mf(m)dm = m^{-a}d\ln m$, the value of a is as small as 0.35. This implies that possible uncertainties in the value of a are important in the context of evolution of the bulk of the mass contained in the cluster. Mass segregation proceeds so that massive stars contract to form a core consisting mainly of the massive stars. At the same time the less massive stars receive kinetic energies from the massive stars and are prevented from the collapse. When the massive stars explode as supernovae in the core of the cluster, the mass loss takes place from the core, which makes the outer region of the cluster expand. The less massive stars will be tidally stripped off by the external gravitational field exerted in the parent galaxy. This will modify the mass function very much and could be important in the context of producing the seed black hole. Thus, it seems important to simulate the gravothermal collapse with the mass spectrum and stellar evolution taken into account. Such simulations could be compared with young globular clusters in the Magellanic Clouds. The difference in the mass functions according to the different radial distances from the cluster center is now being observed in the Magellanic Clouds. Not only for this point but also for the relation between the flattened and possibly rotating globular clusters it is very important to study more closely the globular clusters in the Magellanic Clouds. They give unique examples with which we can construct a sequence of dynamical evolution for globular clusters.

5) *Stellar Collision.* As discussed in this Symposium the 3-D hydrodynamics of stellar collision is a challenging problem in the views that the direct collision of the stars should take place even in the nucleus of our

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press

Galaxy, and that the matter density in the sub-parsec region of the nucleus of NGC4258 is high enough for stellar collisions if it should consist of the stars instead of a massive black hole. The stellar collision is a rapid process with a large entropy production when seen in the stellar envelope. On the other hand when seen in the stellar core, it is a slow quasi-adiabatic process if the sound speed in the stellar core is higher than the relative velocity of the collision. The result will be dissipation of the stellar envelope and merging of the cores of the stars. The stellar structure just after the merging is important to discuss further evolution of the star itself and of the star cluster as well. It seems rather difficult to calculate accurately the entropy production during the collision processes which determines the fate of the colliding stars. Here, again, the theory of stellar structure will help the situation as it helped in clarifying the concepts of gravothermal collapse and oscillation. For such an approach we need to understand physical processes in general terms rather than simply to believe the results of numerical calculations.

https://doi.org/10.1017/S0074180900001662 Published online by Cambridge University Press