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ARTICLE

# Income sources, cyclical behaviors of fiscal policy, and welfare conflict†

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**Abstract**
This paper explores the effect of fiscal policy on the welfare of heterogeneous agents with different income sources over the business cycle. Policy experiments allow the government to choose the cyclical properties of fiscal policy instruments conditional on long-run levels to separately maximize the welfare of entrepreneurs and workers. This policy choice creates welfare conflicts between the two groups, which is confirmed by empirical evidence. The government maximizes workers' welfare by choosing procyclical capital income tax rates and countercyclical labor income tax rates while it optimizes entrepreneurs' welfare with countercyclical capital income tax rates and strongly countercyclical labor income tax rates. The socially optimal policy is a tradeoff between the welfare of entrepreneurs and workers. The welfare conflict holds when the government modifies the cyclical behavior of policy in a crisis, or it adjusts the long-run level and cyclical behavior at the same time.

**Keywords:** Welfare conflict; heterogeneous agents; fiscal policy; business cycles

## 1. Introduction

In most developed countries and major developing countries, there has been increasing income and wealth inequality recently. A small proportion of population obtains a large amount of income from assets while others almost only get labor earnings. The difference in income sources implies that an aggregate shock in the economy may heterogeneously influence the income and welfare of different types of agents. Consequently, a policy that stabilizes the economy may also exert diverse impacts on different professional groups. Motivated by this idea, the current paper studies the effect of cyclical changes in fiscal policy on the welfare of heterogeneous agents with different income sources. Specifically, we explore whether responses of fiscal policy instruments to output, which are defined as the "timing choice of fiscal policy," affect the welfare of different groups of people differently.

We first present the empirical evidence that fiscal policy exerts heterogeneous effects on the welfare of different groups of people. Specifically, labor income tax only influences the welfare of workers while capital income tax affects both capital holders and workers, with a more prominent impact on the former. Since previous studies have implied that changes in the response of fiscal policy instruments to output modify the behaviors of these instruments over the business cycles, the response of income taxes has diverse impacts on the welfare of different groups.

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We develop a model that incorporates heterogeneous agents, business cycles, and a government sector that sets fiscal rules to reflect the empirical observations and explore the mechanism. Our model features two types of risk-averse agents: entrepreneurs who own assets and get asset gain and workers who only receive labor income. We allow entrepreneurs to invest in their own firms or non-state-contingent government bonds, but not in private firms held by other entrepreneurs. Private firms suffer from aggregate productivity shocks and independent and identically distributed (i.i.d.) idiosyncratic investment risks. Together with i.i.d. idiosyncratic investment risks, aggregate shocks exert an impact on the decisions of how much to consume and save (saving choice), and how much to invest in risky but productive assets and in risk-free assets (portfolio choice). Each period, all entrepreneurs make identical choices because of i.i.d. idiosyncratic investment risks and the absence of trade in capital among entrepreneurs. Hand-to-mouth workers supply labor inelastically to the labor market; their income differs because of idiosyncratic labor income risks. The government levies consumption, labor, and capital taxes and issues government bonds to collect its revenue. To modify the responses of fiscal policy, the government sets rules of fiscal policy instruments as functions of the output gap and a deviation from a fixed debt-to-output ratio.

The model matches the level and the cyclical behavior of the US' income distribution to some extent. The simulated income distribution features large income shares in top income groups and small income shares in bottom income groups, as shown in the Current Population Survey (CPS). As for the cyclical property of income distribution, the simulated result quantitatively matches the correlations with output of the income shares of the top 5%, the 80–95%, the 60–80%, and the bottom 20% in the income distribution.

We apply the model to carry out policy experiments in which the government modifies the cyclical properties of fiscal policy to maximize the welfare of entrepreneurs or workers by choosing parameter values that indicate the responses of policy instruments to output. We first calibrate these parameters to the US' fiscal policy data as the baseline. Then, we evaluate different combinations of counterfactual fiscal policy instruments. We find that the change in fiscal policy with business cycles creates welfare conflicts between entrepreneurs and workers in this heterogeneous agent model. Indeed, Bassetto (2014) studies how the government can manipulate intertemporal prices to favor some groups over others. He finds that the preferred policy by different groups is quantitatively very different. In the current paper, groups differ in their preferred responses of fiscal policy to output, even in the qualitative sense. For instance, a policy that specifies a low capital income tax rate during the recession benefits entrepreneurs, yet it harms workers' welfare. With this policy, the mean levels of bonds decreases compared with the baseline case, crowding in capital. More capital further raises the disposable wealth and mean consumption of entrepreneurs. The reduced debt induces an increase in the average consumption tax rate and labor income tax rate. Workers have a lower mean level of consumption because of higher tax burden. Meanwhile, the volatility of the consumption of entrepreneurs declines while workers' consumption fluctuates more. Entrepreneurs have more, smoother consumption; thus, they are better off. Workers are worse off because they experience lower and more volatile consumption.

The utilitarian social planner balances the welfare of both groups and chooses countercyclical labor and capital taxes. This policy leads to a 0.006 percentage-point decrease in the labor tax rate and a 0.037 percentage-point decrease in the capital tax rate compared to the baseline policy when the output drops by one percent.

We also calculate the indices of inequality with the baseline and counterfactual fiscal policies and determine the policy that minimizes the inequality. This policy requires a qualitatively different cyclical properties of income taxes from other policies that maximize the welfare of one group or all agents. However, the degree of inequality is close to the one calculated with the policy that maximizes the welfare of workers.

To examine the robustness of this welfare conflict, we conduct the aforementioned policy experiment in two scenarios: the government modifies the cyclical behavior of policy in a crisis or it adjusts the long-run level and cyclical behavior at the same time. The welfare conflict still holds.

The current paper belongs to the literature on fiscal policy. These studies usually either assume a representative agent and explore the effect of policy on the macroeconomy [e.g. Chatelain and Ralf (2022), Economides et al. (2022), and Gomes et al. (2022)], neglecting the effect on different people or consider heterogeneity but exclude aggregate risks [e.g. Aiyagari and Mcgrattan (1998) and Chen (2021)] leaving these studies unable to discuss how a government could modify fiscal policy to improve welfare over the business cycles.

The present paper is closer to a growing stream of literature that evaluates fiscal policy by considering heterogeneity and aggregate risks. Werning (2007) and Bassetto (2014) both employ complete markets to simplify the solution to the optimal policy problem. In contrast, we construct the model in incomplete markets, which is in line with empirical evidence. In addition, we focus on when the government should implement fiscal policy based on certain fiscal rules. Bhandari et al. (2017) and Desbonnet and Kankanamge (2017) highlight the behavior and the effect of fiscal policy, especially government debt, after introducing idiosyncratic risks with aggregate risks. Acharya and Dogra (2020) and Bhandari et al. (2021) discover the optimal monetary and fiscal policies with heterogeneous agents, idiosyncratic risks, and aggregate shocks. Armenter (2007) encompasses two types of agents to characterize the time-consistency of a set of Pareto efficient fiscal policies. We differ from these studies by addressing the cyclicality of taxes and government spending and evaluating fiscal policy. Angelopoulos et al. (2019) also show the welfare conflict in deciding the cyclical properties of income taxes among different income groups. In contrast, we find the welfare conflict between two professional groups who differ in their income sources. Like our research, Bachmann et al. (2020) explore the welfare costs of fiscal policy in the aggregate level and along the distribution. But we focus on responses of fiscal policy while they emphasize fiscal volatility (uncertainty).

Our paper is in line with the literature of uninsured idiosyncratic investment risks that has demonstrated that idiosyncratic investment risks influence the steady state or the stationary distribution of income or wealth. These studies find that market incompleteness may lead to the under- or over-accumulation of capital, depending on the parameterization [Angeletos and Calvet (2006), Covas (2006), Meh and Quadrini (2006), and Angeletos (2007)]. Other scholars apply the framework with idiosyncratic investment risks to explain certain phenomena of fiscal policy [Meh (2008), Meh and Terajima (2009), Angeletos and Panousi (2009), and Panousi (2010)] and international difference of growth [Angeletos and Panousi (2011)]. Since aggregate shocks are missing, their studies lack a discussion of the cyclical behavior or the welfare change over the business cycles. Our paper adds aggregate shocks to the framework of Angeletos (2007). The classic [Krusell and Smith (1998)] framework requires the entire wealth distribution as a state variable. Since wealth distribution is an infinite-dimensional object, the computation of such a model with aggregate risks is difficult. Our model provides a tractable tool to analyze the fluctuations of the aggregate economy and income distribution and to assess policies over the business cycles in a setting with heterogeneous agents because it allows for exact aggregation; or in another word, a representative agent can be found to study the behavior of all agents

The current paper contributes to the recent studies embedding aggregate shocks into the framework of idiosyncratic investment risks. Goldberg (2014) sets up a framework with idiosyncratic investment risks in the business cycle. Our paper differs from his in a few ways. First, we model incomplete markets with a unique non-state-contingent bond as the only security in the market; Goldberg provides state-contingent promises but with moral hazard. Second, he aims to build on a theoretical framework incorporating uninsured idiosyncratic investment risks, aggregate shocks, and a borrowing constraint, while we build this model to help answer questions about policy analysis. Di Tella and Hall (2021) employ idiosyncratic investment risks as an intermediary to link aggregate shocks and risk premia. Some other works study financial issues, including the research of Wang and Wen (2012), Miao et al. (2015), Nezafat and Slavik (2015), Ajello (2016), Zetlin-Jones and Shourideh (2017), Miao and Wang (2018), and Miao et al. (2022). Instead, we focus on fiscal policy. Cui (2016) discusses monetary–fiscal interactions with liquidity frictions, where agents face idiosyncratic investment risks. Our study emphasizes the welfare change because of the different responses of policy instruments over the business cycles.

Finally, our research develops the literature on simulating the cyclical behaviors of income distribution [e.g. Castañeda et al. (1998)]. At best, previous studies that consider idiosyncratic labor income risks and some special elements match the qualitative properties of the cyclical behaviors of income distribution. Our study generates quantitatively accurate cyclical behaviors in some income groups with idiosyncratic investment risks.

The rest of the current paper is organized as follows. We provide empirical evidence in Section 2. We extend the framework of Angeletos (2007) by incorporating business cycles in Section 3 and characterize the equilibrium in Section 4. In Section 5, we simulate the income shares of different income groups and their cyclical properties with our model and compare the quantitative results with the data. Section 6 carries out experiments on fiscal policy. Section 7 examines the robustness of the welfare conflict with a setting of a crisis and with simultaneous adjustments in the long-run and short-run behaviors of policy. Section 8 concludes the paper.

## 2. Empirical evidence

This section aims to answer whether the responses of fiscal policy instruments to output have different influences on the welfare of different groups in reality. We mainly employ the time series analysis to show diverse impacts of fiscal policy on the welfare of heterogeneous agents. In addition, the literature implies that the cyclical behaviors of fiscal policy instruments change with the responses to output. The empirical result thus indicates a positive answer.

It is difficult to measure the welfare of different groups of people. By convention, we choose the consumption index to express the welfare. As aforementioned, capital income basically belongs to capital holders while labor income is mainly obtained by workers, thus we can transform different people's consumption into the consumption attributed to different income types. Therefore, we use the shares of consumption attributed to different income types as the proxies of the welfare of different groups of people. From the National Income and Product Accounts (NIPA), the personal income (PI) contains three components, that is, wage and salary accruals (WSA), proprietor's income (PRI) and capital income (CI), and PI = WSA + PRI/2 + CI. Capital income share (Cap. Inc.) and labor income share (Lab. Inc.) are computed as Cap. Inc. = CI/PI and Lab. Inc. = WSA/PI, respectively. We then calculate the share of consumption attributed to capital income (Cap. Con., $c^k$) as Cap. Inc. × APC, where APC is the average propensity to consume constructed as personal consumption expenditures (PCE) divided by PI. Similarly, the share of consumption attributed to labor income (Lab. Con., $c^n$) is calculated as Lab. Inc. × APC.

Following Fernández-Villaverde et al. (2015), we construct consumption tax rate (Con. Tax, $\tau^c$), capital tax rate (Cap. Tax, $\tau^k$), average tax on labor income (Lab. Tax, $\tau^n$), and government spending to output ratio (Gov. Spend, *gshare*).[1] To investigate the relationship between tax rates, government spending, and the shares of consumption attributed to capital income and labor income, we estimate the standard vector autoregression (VAR) as follows,

$$X_t = A(L)X_{t-1} + \varepsilon_t, \tag{1}$$

where $X_t = (\tau_t^c, \tau_t^k, \tau_t^n, gshare_t, c_t^k, c_t^n)$ is a vector of endogenous variables. $A(L)$ is a matrix polynomial in the lag operator, and $L = 2$. $\varepsilon_t$ represents a vector of idiosyncratic errors.

We take the annual average of quarterly series from 1970 to 2019 and provides the summary statistics of key variables in Table 1. All endogenous variables are stationary in the first-difference. The model satisfies the stability conditions, and there is no autocorrelation at lag-order two. We are particularly interested in the responses of Cap. Con. and Lab. Con. to four policy instruments and present the impulse response functions (IRFs) in Figure 1(a)–(h), respectively.

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**Table 1.** Summary statistics of key variables

| Variables | Parameters | Obs. | Mean | Std. Dev. | Min | Max | Descriptions |
|---|---|---|---|---|---|---|---|
| Con. Tax | $\tau^c$ | 50 | 0.076 | 0.008 | 0.063 | 0.094 | Consumption tax rate |
| Cap. Tax | $\tau^k$ | 50 | 0.352 | 0.032 | 0.296 | 0.412 | Capital tax rate |
| Lab. Tax | $\tau^n$ | 50 | 0.227 | 0.018 | 0.177 | 0.257 | Labor tax rate |
| Gov. Spend | *gshare* | 50 | 0.199 | 0.016 | 0.174 | 0.235 | Government spending to output ratio |
| Cap. Con. | $c^k$ | 50 | 0.266 | 0.033 | 0.199 | 0.321 | Share of consumption attributed to capital income |
| Lab. Con. | $c^n$ | 50 | 0.630 | 0.031 | 0.564 | 0.692 | Share of consumption attributed to labor income |

Data source: NIPA and authors' calculation.

The IRF displayed in Figure 1(b) suggests that the share of consumption attributed to capital income (Cap. Con.) responds negatively and significantly to capital income tax, while Figure 1(a), (c), and (d) show that the responses to other policy instruments; that is, consumption tax, labor income tax, and government spending, are statistically insignificant. In Figure 1(g), the IRF suggests that the share of consumption attributed to labor income (Lab. Con.) responds to labor income tax positively and statistically significant at the 10% level, while Figure 1(e) and (h) show that the responses to consumption tax and government spending are also statistically insignificant. Furthermore, Figure 1(f) suggests that the direction of the impulse response of the share of consumption attributed to labor income (Lab. Con.) to capital income tax is blur and the response is not statistically significant at the 5% level. Our results are robust to alternative orderings in VAR.

The above results present the significant impacts of income taxes on the welfare. In contrast, consumption tax and government spending fail to have a welfare effect. Besides, labor income tax solely influences the welfare of workers while capital income tax affects both capital holders and workers, with a quantitatively more prominent impact on the welfare of capital holders. Since the two groups obtain income from separate sources, our results indicate diverse impacts of income taxes on the welfare of heterogeneous agents with different income sources.

The studies of Leeper et al. (2010), Bianchi (2012), and Bianchi and Ilut (2017) all imply that changes in the responses of fiscal policy instruments to output can modify the cyclical behaviors of fiscal policy. Linked with this implication, our empirical findings confirm that responses of some fiscal policy instruments to output affect the welfare of different groups of people differently.

## 3. The model

To reflect the empirical evidence we have found, we construct a model in which agents whose welfare depends on consumption are divided into two groups with two separate income sources and the government features setting fiscal rules that fiscal policy instruments respond to output. This section elaborates the details of the model.

### *3.1. Economy*

The economy is populated with two types of agents: entrepreneurs and workers. We normalize the measure of entrepreneurs to 1 and the measure of workers by $\lambda$. We interpret $\lambda$ as the worker–entrepreneur ratio. There is a continuum of individuals for each type indexed by $i$ and $j$. Each worker is endowed with $\frac{1}{\lambda}$ unit of time so that the aggregate labor endowment is 1. We denote $u_t$ as

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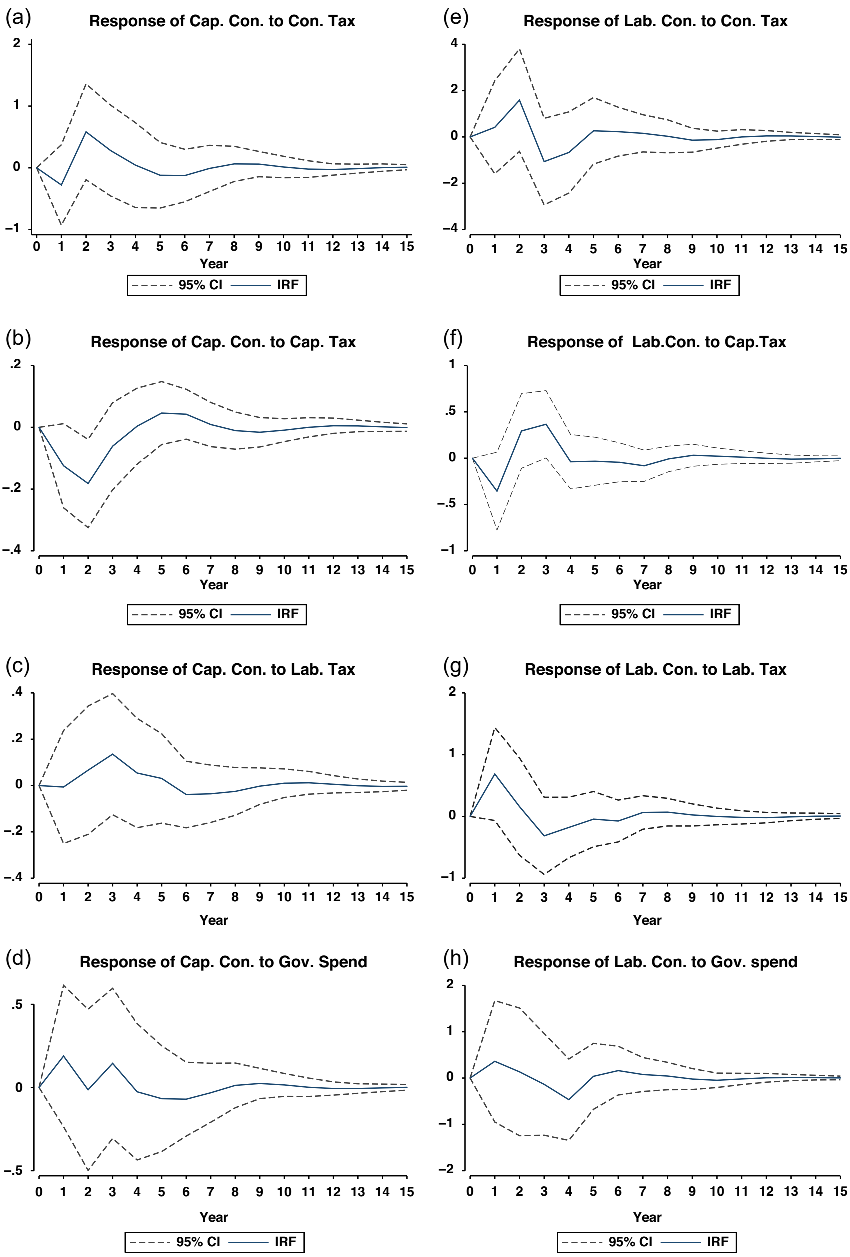


**Figure 1** Impulse responses of the shares of consumption attributed to capital and labor income to fiscal policy instruments.

instantaneous utility. Each agent maximizes her expected lifetime utility subject to her own budget constraint and borrowing constraints. The aggregate productivity, $z_t$, affects the production of each firm in the economy following an AR(1) process,

$$\log z_t = \rho_z \log z_{t-1} + \varepsilon_t^z, \tag{2}$$

where $\varepsilon_t^z$ is normally distributed, $\varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$, and the autoregressive parameter, $\rho_z \in [0, 1)$.

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### 3.2. Entrepreneurs

We assume that entrepreneurs only care about consumption. We specify a CRRA utility function $u\left(c_t^i\right) = \frac{(c_t^i)^{1-\gamma}}{1-\gamma}$, where $\gamma$ denotes the risk aversion degree. Each entrepreneur owns a private firm. At $t$, an entrepreneur can invest capital, $k_t^i$, in her own firm but not in other private firms. She can buy or sell risk-free government bonds, $b_t^i$, as an alternative financial asset. Entrepreneurs do not work and receive asset gains as their unique source of income. Let $r_t^i$ and $R_t$ denote the return on the invested capital and the interest rate of bonds, and $\tau_t^k$, the capital income tax rate. Effective wealth consists of the gross income from capital and bond holdings net of income taxation. An entrepreneur is also taxed by consumption taxes at the rate of $\tau_t^c$. The budget constraint and nonnegativity constraints are as follows:

$$(1+\tau_t^c)c_t^i + k_t^i + b_t^i = [(1-\tau_t^k)(r_t^i - \delta) + 1]k_{t-1}^i + R_{t-1}b_{t-1}^i, \tag{3}$$

$$c_t^i \geq 0 \text{ and } k_t^i \geq 0.$$

Entrepreneurs are allowed to borrow in the bond market, but the borrowing amount must fulfill the no-Ponzi condition,

$$\lim_{T\to\infty} \mathbb{E}_t \frac{b_{t-1+T}^i}{\prod_{s=0}^{T-1} R_{t-1+s}} = 0. \tag{4}$$

### 3.3. Firms

Each firm hires labor in a competitive labor market, employs its owner's capital, and produces consumption goods. We name the firm run by Entrepreneur $i$ also as $i$. We assume the neoclassical production technology,

$$y_t^i = F(k_{t-1}^i, n_t^i, A_t^i) = (A_t^i)^\alpha (k_{t-1}^i)^\alpha (n_t^i)^{1-\alpha}, \tag{5}$$

which exhibits constant returns to scale with respect to $k$ and $n$. Firm-level productivity specific to firm $i$, $A_t^i$ affects the final output, $y_t^i$. $A_t^i$ consists of two components, an idiosyncratic investment risk, $e_t^i$, and aggregate productivity, $z_t$: $\log A_t^i = \log z_t - \frac{\sigma_e^2}{2} + e_t^i$, where $e_t^i$ is independently and identically distributed among firms and across time while $z_t$ captures business cycles. The idiosyncratic risk, $e_t^i$, is modeled as a normal, $e_t^i \sim \mathcal{N}(0, \sigma_e^2)$, where $\sigma_e^2$ represents the variance of idiosyncratic investment risks. The correction term, $-\frac{\sigma_e^2}{2}$, renders the average productivity of private firms equivalent to aggregate productivity. We define the profit of Firm $i$ as the revenue net of the labor cost,

$$\pi_t^i(k_{t-1}^i, n_t^i, A_t^i) = y_t^i - w_t n_t^i, \tag{6}$$

where $w_t$ represents the wage rate at $t$. A competitive labor market ensures a universal wage rate in each period that only depends on the aggregate productivity and aggregate allocations.

The assumption of i.i.d. idiosyncratic investment risks may seem restrictive. Nevertheless, it is necessary for tractability and may not be so unrealistic given that DeBacker et al. (2012) find that business income is much less persistent than labor income and that it is characterized by higher probabilities of extreme upward or downward mobility. For instance, conditional on leaving the starting business income decile, a household faces a 52% probability of moving to either of the two immediately adjacent deciles in a year. More strikingly, households starting at the lowest decile of the business income distribution face a 12% probability of transitioning to decile 8 or higher, whereas the corresponding probability is essentially zero for labor income. Hence, business income may transition from low to high amounts within a relatively short time, making the i.i.d. shock a reasonable abstraction.

### 3.4. Workers

Worker $j$ has preferences in consumption that are specified as $u\left(c_t^j\right)=\frac{(c_t^j)^{1-\gamma}}{1-\gamma}$. We assume that a worker shares the same curvature of consumption as an entrepreneur. Workers supply their labor in the competitive labor market, work in firms owned by entrepreneurs, and consume all earnings as hand-to-mouth agents. This hand-to-mouth assumption is not so unrealistic given that quite a number of households hardly own wealth other than a house. Workers provide identical working hours but differ in their labor efficiency, $e_t^j$. The labor efficiency is independently and identically distributed across workers. We assume that idiosyncratic labor efficiency follows a persistent stochastic process,

$$\log e_t^j = \rho_w \log e_{t-1}^j + \varepsilon_t^j, \tag{7}$$

where $\rho_w \in [0, 1)$ denotes the autocorrelation of labor efficiency and $\varepsilon_t^j$ is normally distributed, $\varepsilon_t^j \sim \mathcal{N}(0, \sigma_w^2)$.

A worker's personal labor income depends on the idiosyncratic labor efficiency and the wage rate. A worker is taxed by consumption taxes and labor income taxes, whose tax rates are $\tau_t^c$ and $\tau_t^n$, respectively. The assumption of hand-to-mouth implies that the aggregate consumption and income share of workers do not depend on the income distribution of workers. The budget constraint for Worker $j$ is

$$\left(1+\tau_t^c\right) c_t^j = \left(1-\tau_t^n\right) w_t \frac{1}{\lambda} e_t^j. \tag{8}$$

### 3.5. Government

Each period, the government finances government spending, $g_t$, by levying the proportional taxes $\{\tau_t^c, \tau_t^k, \tau_t^n\}$ and issuing bonds $\{B_t\}$,

$$\tau_t^c\left(\int_i c_t^i + \int_j c_t^j\right) + \int_i \tau_t^k\left(r_t^i-\delta\right) k_{t-1}^i + \int_j \tau_t^n w_t \frac{1}{\lambda} e_t^j + B_t = g_t + R_{t-1}B_{t-1}, \tag{9}$$

where $B_t$ denotes the total amount of bonds issued at $t$ and paid off at $t+1$.

The current paper intends to analyze the effect of the cyclical behavior of fiscal policy conditional on its long-run level. Thus, we assume that the government adjusts its policy over the cycles and sets rules for $\tau_t^c, \tau_t^k, \tau_t^n$, and government spending as a share of output, $gshare_t$, following Fernández-Villaverde et al. (2015):

$$\Omega_t - \overline{\Omega} = \rho_\Omega\left(\Omega_{t-1}-\overline{\Omega}\right) + m_{Y\Omega}\log\left(\frac{Y_{t-1}}{\overline{Y}}\right) + m_{B\Omega}\left(\frac{B_{t-1}}{Y_{t-1}} - \overline{Bshare}\right) + \exp\left(\sigma_{\Omega,t}\right)\varepsilon_t^\Omega, \quad \varepsilon_t^\Omega \sim \mathcal{N}(0,1),\quad t=1,2,\ldots, \tag{10}$$

where $Bshare$ represents the debt-to-output ratio and $\Omega$ denotes the four policy instruments, $\{\tau^c, \tau^k, \tau^n, gshare\}$; terms with a bar above indicate steady-state levels. This specification embraces time-varying volatility,

$$\sigma_{\Omega,t} = \left(1-\rho_{\sigma\Omega}\right)\sigma_\Omega + \rho_{\sigma\Omega}\sigma_{\Omega,t-1} + \left(1-\rho_{\sigma\Omega}^2\right)^{\frac{1}{2}}\eta_\Omega u_\Omega,\ u_\Omega \sim \mathcal{N}(0,1),\quad t=1,2,\ldots \tag{11}$$

It applies a "volatility shock" and $\eta_\Omega$ is the standard deviation of the volatility shock to the instrument.

$m_{Y\Omega}$ and $m_{B\Omega}$ denote the responses of fiscal policy instruments to output and to debt, respectively. For instance, $m_{Y\tau^k} > 0$ and $m_{B\tau^k} > 0$ when these parameters are calibrated in the US' case, which shows a cut in the capital income tax rate when facing low output and a low debt level. A change in $m_{Y\Omega}$ reflects a different degree—or even a different direction—of the responses of fiscal policy instruments over the business cycles.

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Notice that labor income tax only affects workers' income and consumption while capital income tax affects the welfare of both types of agents as indicated in the empirical result: it directly reduces the capital income of entrepreneurs and indirectly influences the wage and the consumption of workers by containing capital accumulation.

### 3.6. Timing

Every period, aggregate shocks and idiosyncratic risks hit the economy. The government then announces fiscal policy based on the current state. After observing the shocks and policy, workers supply labor while firms optimally choose the labor demand. Firms take labor and predetermined capital as inputs in the production of consumption goods. Entrepreneurs consume and save in forms of capital and bonds after they receive asset income and pay taxes. Workers pay labor income taxes and consume afterwards.

### 3.7. Stationarity

The current model would imply a nonstationary distribution of wealth and income. We add an exogenous death probability, $Pr_d$, to all entrepreneurs to guarantee that every entrepreneur has limited income. Specifically, the death shock is assumed to happen after every entrepreneur makes her own decision each time. Some entrepreneurs die, while the same number of newborns enter into the economy. Every newborn inherits the average amount of risky and risk-free assets previously owned by the dead. We will show in the later section that entrepreneurs make saving and portfolio choices independent of income distribution, making these decisions only as functions of the aggregate state. Therefore, this assumption does not change the original choices before death and facilitates computation.

## 4. Equilibrium

This section first defines the equilibrium of this model and then characterizes the equilibrium at the individual and aggregate levels. The aggregate allocations are independent of wealth distribution, only relying on the macroeconomic conditions.

### 4.1. Equilibrium Definition

Denote $K_{t-1}, B_{t-1}, z_t, \Omega_t$ as the aggregate state, $S_t$ and $k^i_{t-1}, b^i_{t-1}, e^i_t$ as the individual state for entrepreneur i, $s^i_t$.

**Definition.** *An equilibrium is a stochastic sequence of prices* $\{w_t(S_t), R_t(S_t)\}_{t=0}^{\infty}$, *a stochastic sequence of individual allocations* $\{c^i_t(s^i_t; S_t), k^i_t(s^i_t; S_t), b^i_t(s^i_t; S_t)\}_{t=0}^{\infty}, i \in [0, 1]$, *for entrepreneurs,* $\{c^j_t(e^j_t; S_t), n^j_t(e^j_t; S_t)\}_{t=0}^{\infty}, j \in (1, \lambda + 1]$, *for workers,* $\{y^i_t(s^i_t; S_t), n^i_t(s^i_t; S_t)\}_{t=0}^{\infty}$ *for firms, and aggregate allocations* $\{C^E_t(S_t), C^W_t(S_t), K_t(S_t), Y_t(S_t)\}_{t=0}^{\infty}$, *such that*

*(1) Given prices* $\{w_t(S_t), R_t(S_t)\}_{t=0}^{\infty}$, *fiscal policy* $\{\Omega_t, B_t(S_t)\}_{t=0}^{\infty}$, *and the distribution of initial assets* $k^i_{-1}$ *and* $b^i_{-1}$, *every entrepreneur i and every worker j maximize their respective lifetime utility by choosing* $\{c^i_t(s^i_t; S_t), k^i_t(s^i_t; S_t), b^i_t(s^i_t; S_t)\}_{t=0}^{\infty}$ *and* $\{c^j_t(e^j_t; S_t), n^j_t(e^j_t; S_t)\}_{t=0}^{\infty}$ *and every firm i maximizes its profit by choosing* $\{n^i_t(s^i_t; S_t), y^i_t(s^i_t; S_t)\}_{t=0}^{\infty}$.

*(2) Aggregation:* $C^E_t(S_t) = \int_i c^i_t(s^i_t; S_t)$, $Y_t(S_t) = \int_i y^i_t(s^i_t; S_t)$, $K_t(S_t) = \int_i k^i_t(s^i_t; S_t)$, *and* $C^W_t(S_t) = \int_j c^j_t(S_t)$ *for all t.*

*(3) Labor market clearing:* $\int_i n^i_t(s^i_t; S_t) = \int_j n^j_t(e^j_t; S_t)$ *for all t.*

*(4) Bond market clearing:* $\int_i b^i_t(s^i_t; S_t) = B_t(S_t)$ *for all t.*

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*(5) Goods market clearing:* $C_t^E(S_t) + C_t^W(S_t) + K_t(S_t) + g_t(S_t) = Y_t(S_t) + (1-\delta)K_{t-1}(S_{t-1})$.

*(6) The government budget constraint holds given the policy instrument specifications for all t, that is, (9), (10), and (11) hold.*

For expository purposes, we will mostly drop the state in each variable for the rest of the current paper and only use the expression with the state when necessary.

### 4.2. Individual Behavior

Because Firm $i$ chooses employment, $n_t^i$, after observing the shock and after the capital stock is determined, $n_t^i$ is the only control variable to maximize profit. By constant returns to scale, the optimal firm employment and the maximized profit are linear in capital, as in Angeletos (2007):

$$n_t^i = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} A_t^i k_{t-1}^i = n\left(A_t^i, w_t\right) k_{t-1}^i, \tag{12}$$

$$\pi_t^i = \left[\alpha \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}-1} A_t^i\right] k_{t-1}^i = r\left(A_t^i, w_t\right) k_{t-1}^i. \tag{13}$$

The firm experiences linear returns to investment by adjusting its employment according to a linear function to capital. Notice that the return to capital, $r_t^i = r(A_t^i, w_t)$, is expressed by firm-specific productivity, $A_t^i$, and the economy-wide wage, $w_t$, which can be further expressed by the aggregate productivity and total capital stock.

We denote the effective wealth of Entrepreneur $i$ in period $t$ by

$$x_t^i \equiv \left[(1-\tau_t^k)(r(A_t^i, w_t) - \delta) + 1\right] k_{t-1}^i + R_{t-1} b_{t-1}^i. \tag{14}$$

We rewrite the budget constraint as $(1+\tau_t^c)c_t^i + k_t^i + b_t^i = x_t^i$.

We characterize an entrepreneur's behavior following Angeletos (2007) in the following lemma, the proof of which is provided in Appendix A:

**Lemma.** *Given prices and fiscal policy, the optimal consumption, capital stock in the private firm, and bond holdings are linear in effective wealth,*

$$(1+\tau_t^c)c_t^i = \nu_t x_t^i, \tag{15}$$

$$k_t^i = (1-\nu_t)\phi_t x_t^i, \tag{16}$$

$$b_t^i = (1-\nu_t)(1-\phi_t) x_t^i, \tag{17}$$

*where the marginal propensity to consume out of effective wealth, $\nu_t$, and the share of private equity in the portfolio, $\phi_t$, are two variables that depend solely on the current aggregate states, $S_t$, and satisfying,*

$$\phi_t = \underset{\phi\in[0,1]}{\arg\max}\, \mathbb{CE}_t \left\{\phi_t[(1-\tau_{t+1}^k)(r(A_{t+1}^i, w_{t+1}) - \delta) + 1] + (1-\phi_t)R_t\right\}, \tag{18}$$

$$\frac{\nu_t^{-\gamma}}{(1+\tau_t^c)^{1-\gamma}} = \beta\,(1-\nu_t)^{-\gamma}\, \mathbb{E}_t \left\{\frac{\nu_{t+1}^{-\gamma}}{(1+\tau_{t+1}^c)^{1-\gamma}} \left\{\phi_t\left[\left(1-\tau_{t+1}^k\right)\left(r\left(A_{t+1}^i, w_{t+1}\right) - \delta\right) + 1\right] + (1-\phi_t)R_t\right\}^{1-\gamma}\right\}, \tag{19}$$

*where $\mathbb{CE}$ represents the certainty equivalent of an entrepreneur.*

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*Define the value function for Entrepreneur i as* $V(x_t^i)$*,*

$$V\left(x_t^i\right) = \frac{\nu_t^{-\gamma}(x_t^i)^{1-\gamma}}{(1-\gamma)(1+\tau_t^c)^{1-\gamma}}. \tag{20}$$

The entrepreneur makes the optimal portfolio choice, $\phi_t$, by maximizing risk-adjusted portfolio returns that are expressed by the certainty equivalent, $\mathbb{CE}$, of the portfolio return given the saving choice, $(1-\nu_t)$. If the return to capital is surely greater than the return to bonds, all savings are invested in capital. The uncertainty of the capital return, however, induces the entrepreneur to divide investment into both assets to ensure an interior solution to the portfolio problem. A change in the capital tax rate directly influences the portfolio choice since government bonds are free from taxation. Given the portfolio choice, the entrepreneur chooses the marginal propensity to consume according to the intertemporal condition to maximize lifetime utility. Because of i.i.d. idiosyncratic investment risks and the homogeneity of production and utility functions, entrepreneurs make saving and portfolio decisions independent of wealth distribution. Hence, the marginal propensity to consume and the share of private equity in the portfolio only depend on the aggregate state.

The hand-to-mouth worker consumes all after-tax labor income every period, implying that the consumption path is completely characterized by the budget constraint of the worker.

### 4.3. General Equilibrium

Recall that aggregate productivity, $z_t$, and idiosyncratic risks, $e_t^i$, are orthogonal. Aggregate labor demand and firm profit are given by $N_t^D = \tilde{n}(w_t, z_t)K_{t-1}$ and $\Pi_t = \tilde{r}(w_t, z_t)K_{t-1}$, where

$$\tilde{n}\left(w_t, z_t\right) \equiv \int_{-\infty}^{\infty} n\left(A_t^i, w_t\right) \, \mathrm{d}\mathcal{F}\left(e_t^i\right) = \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} z_t, \tag{21}$$

and

$$\tilde{r}\left(w_t, z_t\right) \equiv \int_{-\infty}^{\infty} r\left(A_t^i, w_t\right) \, \mathrm{d}\mathcal{F}\left(e_t^i\right) = \alpha\left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}-1} z_t. \tag{22}$$

$\mathcal{F}(\cdot)$ represents the distribution function of i.i.d. idiosyncratic investment risks. Thus, $\tilde{n}(w_t, z_t)$ denotes the average labor employed by a firm and $\tilde{r}(w_t, z_t)$, the return to capital. Let $N_t^S = \int_j \frac{1}{\lambda} e_t^j = 1$ represent the aggregate labor supply; labor market clearing requires that $N_t^D = \tilde{n}(w_t, z_t)K_{t-1} = 1$. The wage is expressed as $w_t = w(K_{t-1}, z_t) = (1-\alpha)(z_t K_{t-1})^{\alpha}$. Further algebra shows that $r_t = \tilde{r}(w_t, z_t) = \alpha z_t^{\alpha} K_{t-1}^{\alpha-1}$. We write the aggregate output as $Y_t = \Pi_t + w_t N_t^S = r_t K_{t-1} + w_t = z_t^{\alpha} K_{t-1}^{\alpha}$. Aggregate allocations are also independent of wealth distribution because consumption, private investment, and bond holdings are linear in individual wealth and because idiosyncratic risks in every period are i.i.d. across firms and periods. The general equilibrium is determined by the following equations. We denote $C_t^E$ as the aggregate consumption of entrepreneurs and $C_t^W$, of workers.

$$C_t^E + C_t^W + K_t + g_t = z_t^{\alpha} K_{t-1}^{\alpha} + (1-\delta)K_{t-1}, \tag{23}$$

$$(1+\tau_t^c)C_t^W = (1-\tau_t^n)(1-\alpha)z_t^{\alpha} K_{t-1}^{\alpha}, \tag{24}$$

$$(1+\tau_t^c)C_t^E = \nu_t\{[(1-\tau_t^k)(r_t-\delta)+1]K_{t-1} + R_{t-1}B_{t-1}\}, \tag{25}$$

$$K_t = (1-\nu_t)\phi_t\{[(1-\tau_t^k)(r_t-\delta)+1]K_{t-1} + R_{t-1}B_{t-1}\}, \tag{26}$$

$$B_t = (1-\nu_t)(1-\phi_t)\{[(1-\tau_t^k)(r_t-\delta)+1]K_{t-1} + R_{t-1}B_{t-1}\}, \tag{27}$$

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These five equations, together with (18) and (19), solve for the seven variables $C_t^E, C_t^W, K_t, B_t, R_t, \nu_t$, and $\phi_t$, given the current aggregate state and fiscal policy.

## 5. Business Cycle Statistics of Income Distribution

This section presents the level and cyclical behavior of the income distribution that the model generates. It shows that the model with aggregate shocks can simulate the level of income distribution and its cyclicality at a small computational cost. The model quantitatively matches the dynamics of the income distribution shown in the data for most quantiles.

### 5.1. Computation Process

We apply the second-order approximation to simulate a time series of equilibrium at the aggregate level over the business cycle starting from a certain state. We are able to accomplish it because every entrepreneur makes exactly identical saving and portfolio decisions regardless of their individual wealth. Thus, we can always find a representative entrepreneur to study the behavior of all entrepreneurs. Our model employs the exact aggregation as in the framework of Angeletos (2007) in the setting with aggregate risks. This modeling approach with idiosyncratic investment risks computes the aggregate variables without considering wealth distribution, making it simple to study the dynamics of the aggregate economy over the business cycles with heterogeneous agents.

The model calculates the dynamics of entrepreneurs' income distribution in a simple way. We divide aggregate allocations into the individual level. The next period's capital and bonds held by entrepreneur $i$ can be calculated as a product of saving choice, portfolio choice, and effective wealth as in (16) and (17). The first two components are obtained from the time series of the general equilibrium. We separate the last part into idiosyncratic capital returns, interest rate of bonds, and amounts of capital and bonds held from the last period. The time series of equilibrium contains values for the interest rate and the average returns to capital. From (44) in Appendix B, we get the idiosyncratic returns to capital by inputting a stochastic process of idiosyncratic risks. The amount of assets forms a recursion given the idiosyncratic risks and asset prices at the aggregate level.

We create 2000 grids for the number of entrepreneurs, and, by assumption, $2000\lambda$ is the number of workers. We equally allocate the steady-state level of capital and bonds to 2000 entrepreneurs as the initial allocation. Because individual returns to capital are idiosyncratic to entrepreneurs, lucky entrepreneurs accumulate more and more assets so that the gap of the capital income rises over time. Yet the assumption of random death prevents infinite inequality. We record the income of every agent, either an entrepreneur or a worker, order all values, and then compute quantile statistics. We run a simulation with 500 periods and drop the first 250 periods to guarantee stability.

### 5.2. Calibration

A numerical analysis is required to discuss the welfare change and evaluate fiscal policy. This subsection first describes the details of calibrating the model.

We assume that a period corresponds to a quarter. We use some common parameter values in the literature on business cycles. The capital share of output, $\alpha$, is assumed to be 0.34 and the risk aversion degree, $\gamma$, 2. We compute the Solow residuals using quarterly data from the first quarter of 1973 to the last quarter of 2011.[2] Then, we estimate the autoregressive parameter, $\rho_z$, and the standard deviation of the technology process, $\sigma_z$.

The probability of death, $Pr_d$, for entrepreneurs is intended to match the working lifetime of an entrepreneur or the lasting time of a private firm. However, the statistic is unclear and hard

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to measure. We assume the probability of death to be 0.00625 for an expected working lifetime of 40 years. We calibrate the discount rate, $\beta$, and the depreciation rate, $\delta$, to produce the annual public-holding debt-to-GDP ratio of 37.60% and capital to GDP ratio of 2.7, as in the USA. We assume the worker–entrepreneur ratio to be 5 to match the estimated proportion of self-employed or business owners in the USA as reported by Cagetti and De Nardi (2006).

It is challenging to find an exact measurement of idiosyncratic investment risks. DeBacker et al. (2012) find that the standard deviation of uninsurable idiosyncratic income risks from privately held businesses accounts for 45% of the average business income.[3] Therefore, we calibrate our model to match their finding that the cross-sectional standard deviation of an individual firm's return is 45% of the average return. Notice that a number of macroeconomic studies, such as the one of Sandri (2014), apply the volatility of firm returns as 50% of the mean return. Thus, we ensure that our calibration is close to their choice and that our results are comparable.

We set the autocorrelation of idiosyncratic labor income risks and the standard deviation equal to 0.99 and 0.0097. All parameter values are quarterly counterparts of the estimation from Storesletten et al. (2004).

We apply the same parameter values for the policy rule estimated by Fernández-Villaverde et al. (2015). All the tax rates have positive correlations with output and debt, while the government spending share is negatively correlated with output and debt.

All the above parameter values for the baseline calibration are included in Table 2.

### 5.3. Simulated income distribution

The income of Entrepreneur $i$ is defined as the asset gains, $I_t^i = (r_t^i - \delta)k_t^i + (R_t - 1)b_t^i$, while the income of Worker $j$ is the labor income, $\frac{1}{\lambda} w_t e_t^j$. For comparison, we take the US income distribution from the CPS and compute the means of income shares over time, as shown in Table 3. The next column reports the means of income shares of different income groups obtained from the simulation. Our model fits the data in the sense that it generates considerable income inequality, even though the rich obtain less income and the poor share more compared with the data.

The last two columns of Table 3 report the correlations with output of the income shares owned by income groups in the data and in our model. We choose the same data from the CPS as for the level of income inequity and choose seasonally adjusted real GDP as measured by 2009 billion dollars during 1947 and 2013. We detrend the shares, ratios, and the logarithm of real GDP by using the HP-filter and calculate the correlations. For the model, we pick the time series of income shares of the corresponding income groups and obtain the correlations with output over the business cycles.

Our model with the baseline calibration quantitatively matches the correlation with output of income shares except for the 20–40% group and the 40–60% group. Our model reflects the small procyclicality for the bottom 20% and the small countercyclicality for the 60–80% and 80–95% groups, and it precisely duplicates the cyclical behavior of the income share of the top income group. It is still very challenging to generate decent cyclical behaviors of income distribution even after the emergence of many heterogeneous agent models. Our model provides a tractable and efficient framework to simulate the realistic dynamics of income distribution.

## 6. Fiscal policy experiments

This section explores whether different groups of people may prefer different responses of fiscal policy to output because of different income sources. The government, fixing the steady-state level of fiscal policy, adjusts the cyclicality of policy instruments to maximize the welfare of entrepreneurs, workers, and all agents, respectively. We find a qualitative welfare conflict between

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**Table 2.** Baseline calibration

| Parameters | Values | Descriptions |
|---|---|---|
| $\alpha$ | 0.34 | capital share of output |
| $\beta$ | 0.99 | subjective discount rate |
| $\delta$ | 0.014 | depreciation rate |
| $\gamma$ | 2 | risk aversion degree |
| $\rho_z$ | 0.84 | autocorrelation of technology process |
| $\sigma_z$ | 0.025 | standard deviation of technology shock |
| $Pr_d$ | 0.00625 | probability of death for entrepreneurs |
| $\lambda$ | 5 | worker/entrepreneur ratio |
| $\sigma_e$ | 0.43 | standard deviation of idiosyncratic risks |
| $\rho_w$ | 0.99 | autocorrelation of labor efficiency |
| $\sigma_w$ | 0.0097 | cross-sectional standard deviation of labor income |
| $\overline{Bshare}$ | 37.60% | steady-state debt/output ratio |
| $\overline{gshare}$ | 20.23% | steady-state government consumption/output ratio |
| $\overline{Y}$ | 2.62 | steady-state output |
| $\overline{\tau}^k$ | 36.74% | steady-state capital income tax rate |
| $\overline{\tau}^n$ | 22.44% | steady-state labor income tax rate |
| $\overline{\tau}^c$ | 7.66% | steady-state consumption tax rate |
| $\rho_{gshare}$ | 0.99 | autocorrelation of government spending |
| $\rho_{\tau^k}$ | 0.98 | autocorrelation of capital income tax |
| $\rho_{\tau^n}$ | 0.99 | autocorrelation of labor income tax |
| $\rho_{\tau^c}$ | 0.99 | autocorrelation of consumption tax |
| $m_{Ygshare}$ | −0.004 | response of government spending share to output gap |
| $m_{Y\tau^k}$ | 0.043 | response of capital income tax to output gap |
| $m_{Y\tau^n}$ | 0.04 | response of labor income tax to output gap |
| $m_{Y\tau^c}$ | 0.001 | response of consumption tax to output gap |
| $m_{Bgshare}$ | −0.008 | response of government spending share to debt |
| $m_{B\tau^k}$ | 0.003 | response of capital income tax to debt |
| $m_{B\tau^n}$ | 0.003 | response of labor income tax to debt |
| $m_{B\tau^c}$ | 0.0001 | response of consumption tax to debt |
| $\sigma_g$ | −6.2 | log of average standard deviation of government spending shock |
| $\sigma_{\tau^k}$ | −4.89 | log of average standard deviation of shock on capital income tax |
| $\sigma_{\tau^n}$ | −6.01 | log of average standard deviation of shock on labor income tax |
| $\sigma_{\tau^c}$ | −7.12 | log of average standard deviation of shock on consumption tax |
| $\rho_{\sigma_{gshare}}$ | 0.92 | autocorrelation of government spending volatility shock |
| $\rho_{\sigma_{\tau^k}}$ | 0.65 | autocorrelation of capital income tax volatility shock |
| $\rho_{\sigma_{\tau^n}}$ | 0.46 | autocorrelation of labor income tax volatility shock |
| $\rho_{\sigma_{\tau^c}}$ | 0.73 | autocorrelation of labor income tax volatility shock |
| $\eta_{gshare}$ | 0.18 | standard deviation of government spending volatility shock |
| $\eta_{\tau^k}$ | 0.4 | standard deviation of capital income tax volatility shock |
| $\eta_{\tau^n}$ | 0.82 | standard deviation of labor income tax volatility shock |
| $\eta_{\tau^c}$ | 0.46 | standard deviation of consumption tax volatility shock |

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**Table 3.** Income inequality: level and cyclicality

| | Income Shares | | Correlations with Output | |
|---|---|---|---|---|
| **Quantiles** | **Data** | **Model** | **Data** | **Model** |
| bottom 20% | 4.1% | 5.95% | 0.29 | 0.22 |
| −40% | 10.3% | 20.99% | 0.30 | −0.21 |
| −60% | 16.4% | 21.92% | 0.15 | −0.21 |
| −80% | 23.9% | 22.78% | −0.19 | −0.21 |
| −95% | 27.0% | 17.99% | −0.44 | −0.24 |
| Top 5% | 18.4% | 10.37% | 0.07 | 0.07 |

Note: All the values of income shares are means computed using corresponding data from the Current Population Survey (CPS) from 1947 to 2013. The CPS provides the statistics of households only since 1967, so we pick up the income share of families from 1947 to 1966.

entrepreneurs and workers when choosing the time to tax capital and labor income. The government after balancing the welfare of different groups should tax capital less in the recession, which overturns the finding of Chari et al. (1994) and Farhi (2010). Besides, we also determine the policy minimizing the inequality, which generates a similar degree of inequality to the one with the policy maximizing workers' welfare.

### *6.1. Objective and method of policy experiments*

As the welfare criterion, we adopt the ex-ante utilitarian social utility in which the social planner sums up the lifetime individual utility across agents,[4]

$$U_0^{SP} = \int_i \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(c_t^i)^{1-\gamma}}{1-\gamma} \right] + \int_j \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(c_t^j)^{1-\gamma}}{1-\gamma} \right]$$
$$= \int_i V\left(x_0^i\right) + \int_j \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(c_t^j)^{1-\gamma}}{1-\gamma} \right]. \qquad (28)$$

We assume that the initial conditions of capital and bond holdings are identical under different policy specifications, making the value function of entrepreneurs maps one-to-one with the marginal propensity to consume. Moreover, since the risk aversion degree, $\gamma$, is larger than one in the following calibration, the lifetime utility for entrepreneurs increases with the marginal propensity to consume. In other words, more consumption conditional on certain effective wealth heightens the welfare of entrepreneurs.

The government searches for the proper direction and degree of responses of fiscal policy instruments over the business cycles by changing the responses of policy variables to output and comparing welfare in each scenario. We compute the consumption equivalent variation in the percentage of the aggregate lifetime utility across entrepreneurs and workers as the responses of capital income tax rate and labor income tax rate to output range from −0.02 to 0.08 and the responses of consumption tax rate and the government spending share range from −0.01 to 0.01 at the interval of 0.001.[5] The base to compute the consumption equivalent variation is the lifetime utility for each type of agents with the baseline calibration representing the US' actual fiscal policy. The economy starts from a fixed state at which state variables are evaluated at unconditional means with the baseline calibration. The government considers the effect of a transition from the baseline policy on welfare.

**Table 4.** Welfare comparison of the change in all fiscal policy instruments (in percentage)

| | **Policy Maximizing Workers' Welfare** ($m_{Y\tau^n}=0.01$, $m_{Y\tau^k}=-0.02$, $m_{Y\tau^c}=0.01$, $m_{Ygshare}=-0.01$) | | **Policy Maximizing Entrepreneurs' Welfare** ($m_{Y\tau^n}=0.08$, $m_{Y\tau^k}=0.08$, $m_{Y\tau^c}=0.01$, $m_{Ygshare}=-0.01$) | | **Policy Maximizing Social Welfare** ($m_{Y\tau^n}=0.006$, $m_{Y\tau^k}=0.037$, $m_{Y\tau^c}=0.01$, $m_{Ygshare}=-0.01$) | |
|---|---|---|---|---|---|---|
| **Variables** | **Mean** | **Std. Dev.** | **Mean** | **Std. Dev.** | **Mean** | **Std. Dev.** |
| Cap. Tax | 0.01 | 0.29 | 0.01 | 0.20 | −0.0002 | −0.01 |
| Bonds | 0.02 | −6.20 | −0.11 | 48.54 | 0.01 | −5.62 |
| Capital | −0.02 | 4.22 | 0.03 | 38.77 | −0.003 | −4.97 |
| Wage | −0.01 | 0.31 | 0.01 | 8.16 | −0.001 | −1.21 |
| Eff. Wealth | −0.02 | 36.31 | 0.01 | −9.09 | −0.001 | 3.80 |
| Lab. Tax | −0.001 | −0.59 | 0.02 | 1.04 | 0.002 | −0.61 |
| Con. Tax | −0.01 | 0.25 | 0.003 | 0.15 | −0.001 | 0.17 |
| Entr. Con. | −0.01 | −3.67 | 0.01 | −3.27 | 0.001 | −9.85 |
| Work. Con. | 0.01 | −13.09 | −0.02 | 47.57 | 0.01 | −13.27 |
| **Welfare Change** (Consumption Equivalent Variation) | | | | | | |
| Entrepreneurs | −0.01 | | 0.01 | | 0.0006 | |
| Workers | 0.03 | | −0.10 | | 0.02 | |
| Total | 0.01 | | −0.06 | | 0.02 | |

Note: The values are welfare changes in terms of consumption equivalent variation compared with the baseline policy combination ($m_{Y\tau^n}=0.04$, $m_{Y\tau^k}=0.043$, $m_{Y\tau^c}=0.001$, $m_{Ygshare}=-0.004$).

### *6.2. Welfare conflict between entrepreneurs and workers*

This subsection elaborates on the analysis of the welfare change of different groups as fiscal policy changes over the business cycles. The policy experiment shows that entrepreneurs and workers favor different responses of capital and labor income taxes in the qualitative sense, but this conflict fails to happen in the choice of consumption tax and government spending.

We simultaneously modify four parameter values that indicate the respective responses of capital income tax rate, labor income tax rate, consumption tax rate, and government spending to output. Table 4 shows the preferred policy mixes of workers, entrepreneurs, and all agents, respectively, and the welfare change in terms of the consumption equivalent variation.

Entrepreneurs and workers have diverse tastes in the cyclical behaviors of capital and labor income taxes but prefer the same consumption tax and government spending policies. In particular, workers maximize their welfare with a procyclical capital income tax and with a slightly countercyclical labor income tax, while the entrepreneurs' welfare is optimized by the countercyclical capital income tax and the strongly countercyclical labor income tax. When one group reaches its maximal welfare, the interest of the other group is compromised by the certain fiscal policy. Workers are even worse off by 0.1%, which is a large change if we realize that only the cyclical properties instead of the long-run level of tax rates are modified. Although we leave the detailed analysis of the mechanism to the next subsection, the results clearly indicate that because two groups have different income sources, the timing choice of income taxes matters for their welfare. Thus, the welfare conflict takes place when the government chooses the responses of income tax rates to output.

On the contrary, the timing choice of consumption tax and government spending fails to cause such a welfare conflict. These two groups favor the same policies of consumption tax and government spending, a countercyclical consumption tax, and a countercyclical government spending, because the two policy instruments affect each group in the same way.

Recall that our empirical result presents the relationship between the responses of fiscal policy instruments to output and the welfare of different groups of people. The model generates the welfare conflict that people with different income sources even have qualitatively diverse preferences in the change of fiscal policy over the business cycles, in line with the empirical finding.

### 6.3. Mechanism

This subsection explains why workers and entrepreneurs prefer different responses of fiscal policy to output. Because both groups only care about consumption, more and smoother consumption results in higher welfare. However, these two goals may not be obtained at the same time. If this situation happens, one effect will outweigh the other, leading to either a rise or drop in welfare.

According to Table 4, workers favor a procyclical capital tax policy that magnifies the effect of fluctuations and a slightly countercyclical labor tax policy that cushions productivity shocks. The average capital income tax rate rises, and the mean level of government bonds also increases when compared with the baseline. The two changes lead to less capital stock. As a result, the wage decreases, driving the labor income to diminish. Yet the consumption of workers on average increases because of the lower average labor income tax rate and consumption tax rate. Entrepreneurs own less effective wealth because the majority of effective wealth, the capital, shrinks. Thus, they consume less.

The volatility of the consumption of workers decreases by 13% compared with the baseline policy. Workers enjoy more and smoother consumption, so they are better off than in the baseline case. Entrepreneurs see a lower mean consumption and lower volatility of consumption. Because the mean effect dominates the fluctuation effect, entrepreneurs are worse off.

The policy mix where the labor and capital income tax rates are strongly positively correlated with the output maximizes the welfare of entrepreneurs. With this policy, the average capital tax rate increases and the bonds decrease. Theoretically, the former policy prevents entrepreneurs from investing in capital, while the latter promotes capital accumulation. The average capital stock increases because of the stronger crowd-in effect and the mean wage rises thanks to higher capital. The labor income tax rate and the consumption tax rate experience an increase in their means. Entrepreneurs have more consumption because of more asset income, while workers see a decrease in their consumption because of a greater tax burden. This policy results in a lower volatility of the consumption of entrepreneurs but a high volatility of consumption of workers. Therefore, entrepreneurs are better off because of more and smoother consumption, yet workers are worse off because their consumption is lower and more turbulent.

Notice that capital income tax discourages entrepreneurs from accumulating capital, which affects both the effective wealth of entrepreneurs and the wage. Thus, capital income tax influences the consumption of entrepreneurs and workers. On the contrary, labor taxation only has an effect on workers' consumption because only workers have labor income. The difference in income sources results in different consequences of income taxes on the welfare of two groups.

In contrast, agents with different income sources in our model are affected in the same way by consumption tax and government spending. These two policy instruments do not distinguish income sources, thus both entrepreneurs and workers prefer the same.

### 6.4. Time to modify fiscal policy

Chari et al. (1994) investigate optimal fiscal policy over the business cycle. They find that the correlation of an optimal labor income tax rate is positive with a technology shock while the correlation of an optimal capital income tax rate is negative in their baseline model with uncontingent debt and state-contingent taxes. In addition, when risk aversion reaches to a high level, the labor tax

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**Table 5.** Comparison of inequality

| | Gini Index | 90/50 ratio | 50/20 ratio | Theil index within groups | Theil index among groups |
|---|---|---|---|---|---|
| **Actual Policy** | 0.2117 | 1.0766 | 1.1503 | 0.1427 | 0.0338 |
| **Policy Minimizing Inequality** | 0.2088 | 1.0766 | 1.1504 | 0.1427 | 0.0338 |
| **Policy Maximizing Workers' Welfare** | 0.2114 | 1.0766 | 1.1504 | 0.1427 | 0.0335 |
| **Policy Maximizing Entrepreneurs' Welfare** | 0.2121 | 1.0766 | 1.1502 | 0.1425 | 0.0342 |
| **Policy Maximizing Social Welfare** | 0.2115 | 1.0766 | 1.1503 | 0.1427 | 0.0338 |

Note: The policy minimizing inequality has the policy parameters as $m_{Y\tau^n} = 0.01$, $m_{Y\tau^k} = -0.02$, $m_{Y\tau^c} = 0.01$, $m_{Ygshare} = -0.01$.

rate turns to be procyclical. They argue that it is efficient for these shocks that affect the government budget constraint to be absorbed mainly by the tax on private assets. Farhi (2010) discusses the same problem of optimal policy in incomplete markets with non-state-contingent debt and non-state-contingent taxes. He also finds that capital taxes have a large positive spike following a transition from the good state to the bad state and claims that capital taxes help buffer the impact of shocks on the government budget. The labor tax rate in his model drops with a low productivity.

The policy that maximizes the social welfare in our model features a 0.006 percentage-point decrease in the labor tax rate and a 0.037 percentage-point decrease in the capital tax rate compared to the baseline policy when the output drops by one percent. Basically, the government chooses a tradeoff between both groups. The result on the capital tax rate differs from the findings of Chari et al. (1994) and Farhi (2010). The capital tax maximizing social welfare varies in the same qualitative pattern as the one favoring entrepreneurs, which matches the observation that certain benefit groups may affect the government decisions.

Since the utilitarian social utility weighs less the equality compared to many other welfare criteria, the government possibly specifies a capital tax whose rate increases in the recession if the government puts sufficient emphasis on workers.

### *6.5. Comparison of inequality*

We compare the extent of inequality among different fiscal policy instruments in this subsection. We choose three typical measurements of inequality, the Gini index, the 90/50 ratio, and the 50/20 ratio. Then, we decompose the inequality into two components—inequality within groups and among groups—by computing the Theil index. The policy that maximizes the welfare of workers reaches a level of inequality close to the lowest. But the differences of inequality among policies are small.

We modify the policy parameters in the baseline model to generate the dynamics of income distribution with different policies. The three inequality measurements are obtained from taking the mean of the corresponding indices of the generated distribution in each period and are presented in Table 5. We adopt the Gini index as the main criterion of inequality to determine the policy that results in the minimal inequality.

The policy that minimizes inequality is the combination of a procyclical labor tax, a countercyclical capital tax, a countercyclical consumption tax, and countercyclical government spending ($m_{Y\tau^n} = -0.006$, $m_{Y\tau^k} = 0.036$, $m_{Y\tau^c} = 0.01$, $m_{Ygshare} = -0.01$). This policy qualitatively differs from the actual policy and other counterfactual policies that maximize the welfare of one group or all agents. However, the level of inequality shown by the Gini index is relatively close to

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the one resulting from the policy maximizing workers' welfare. Recall that workers outnumber entrepreneurs and moreover, workers have lower average income. When the policy caters to the benefit of workers, it certainly improves the welfare of relatively poor agents, thus the income distribution becomes more equal.

The policy that maximizes the social welfare leads to a level of inequality close to the level generated by the policy maximizing workers' welfare. Since the utilitarian social utility regards every agent as equal, the policy preferred by the social planner is partial to the larger group, the group of workers. But the income distribution is still more unequal than the one resulting from the policy maximizing workers' welfare, because the planner also considers the welfare of rich entrepreneurs. Although the policy that emphasizes equity seems to contradict the one that optimizes the social welfare in the qualitative cyclicality of policy instruments, the difference of resulted income distribution is small. Thus, there is no substantial conflict between the policy maximizing the social welfare and the one minimizing inequality. The policy preferred by the social planner can guarantee the equity to a certain extent. If the government weighs more the group of workers or the poor in the social utility, then the inequality will improve, even closer to the minimal level of inequality.

## 7. Robustness check

This section examines the robustness of the welfare conflict of workers and entrepreneurs in the timing choice of fiscal policy if the government modifies the cyclical behavior of policy in a crisis or adjusts the long-run level and cyclical behavior at the same time. Different groups with different income sources still have conflicts regarding the responses of fiscal policy to the output, though the quantitative result differs from the one in the last section. Both cases witness an enlarged welfare gap between the two groups when the government implements a policy preferred by a certain group.

### *7.1. Starting from a crisis*

When a country faces a recession—or even worse, a crisis—the government will most likely adjust its fiscal policy to stimulate the economy. Because the decision is made at the time when the output is lower than the normal level (the long-run mean), the transition cost may change the above result of the welfare conflict. This subsection aims to conduct a policy experiment under a crisis to see if the welfare conflict exists. The result confirms the finding.

Economists are used to applying a decline in the output to describe a crisis. The last financial crisis witnessed the US' GDP drop from the preceding quarter by 8.9% in the third quarter of 2008. Thus, we reduce the output level by 10% from its long-run mean level to represent a large crisis as the starting point of the new simulation. Notice that the long-run targets to compute the output gap and the debt gap do not change. The base to compute the consumption equivalent variation is the lifetime utility for each type of agents with the baseline policy and the same starting point.

The result is presented in Table 6, indicating that the welfare conflict also happens in the case of low output. Workers and entrepreneurs split on preferred choices of implementing the capital income tax: workers advocate for a procyclical policy while entrepreneurs support the opposite. Both groups show qualitatively identical preferences in the time choice of the other three policy instruments. More importantly, choosing the cyclical behavior of fiscal policy at the time of the crisis results in a larger divergence between the welfare change of workers and entrepreneurs.

### *7.2. Long-run and short-run changes of fiscal policy*

We have shown that short-run changes of income taxes lead to a welfare conflict. This subsection explores how the long-run and short-run changes of fiscal policy together affect the welfare of the two groups. The results show that entrepreneurs and workers experience different welfare changes

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**Table 6.** Welfare change when a crisis happens

| | Worker-Preferred | Entrepreneur-Preferred |
|---|---|---|
| **Policy Mixes** | $(m_{Y\tau^n} = 0.066, m_{Y\tau^k} = -0.02,$ | $(m_{Y\tau^n} = 0.08, m_{Y\tau^k} = 0.08,$ |
| | $m_{Y\tau^c} = 0.01, m_{Ygshare} = -0.01)$ | $m_{Y\tau^c} = 0.01, m_{Ygshare} = -0.01)$ |
| **Welfare Change** (Consumption Equivalent Variation) | | |
| Entrepreneurs | −0.001 | 0.03 |
| Workers | 0.06 | −0.02 |

Note: The values for the welfare changes are percentage changes compared with the baseline case.

**Table 7.** Recalibration

| Parameters | Values | Descriptions |
|---|---|---|
| $\beta$ | 0.99 | subjective discount rate |
| $\delta$ | 0.021 | depreciation rate |
| $\overline{Bshare}$ | 62.68% | steady-state debt/output ratio |
| $\overline{gshare}$ | 20.11% | steady-state government consumption/output ratio |
| $\overline{Y}$ | 3.41 | steady-state output |
| $\overline{\tau}^k$ | 32.59% | steady-state capital income tax rate |
| $\overline{\tau}^n$ | 21.68% | steady-state labor income tax rate |
| $\overline{\tau}^c$ | 6.57% | steady-state consumption tax rate |

**Table 8.** Welfare change when both long-run and short-run properties change

| Groups | Worker-Preferred | Entrepreneur-Preferred |
|---|---|---|
| **Policy Mixes** | $(m_{Y\tau^n} = 0.008, m_{Y\tau^k} = -0.02,$ | $(m_{Y\tau^n} = 0.08, m_{Y\tau^k} = 0.08,$ |
| | $m_{Y\tau^c} = 0.01, m_{Ygshare} = -0.01)$ | $m_{Y\tau^c} = 0.01, m_{Ygshare} = -0.01)$ |
| **Welfare Change** (Consumption Equivalent Variation) | | |
| Entrepreneurs | −0.02 | 0.02 |
| Workers | 0.02 | −0.08 |

Note: The values for the welfare changes are percentage changes compared with the baseline case.

when the government modifies the responses of fiscal policy to the output; besides, the change in the long-run levels aggravates the conflict in choosing the fiscal policy responses to the output.

From the beginning of the last financial crisis (the third quarter of 2008), the average effective capital and labor income tax rates and the consumption tax rate in the USA have decreased compared with the average levels before the crisis. The government spending share almost remains unchanged, while the debt rises dramatically. Accordingly, we recalibrate the model to capture the change in the long-run level of fiscal policy. We set the discount rate, $\beta$, and the depreciation rate, $\delta$, to produce the annual public-holding debt-to-GDP ratio, 62.68% and capital to GDP ratio, 2.7. Then, we compute the steady-state level of the output in this case. We list the parameter values that have been adjusted in Table 7.

We input the recalibrated parameters into the model and repeat the policy experiment in the last section. The base to compute the consumption equivalent variation is the lifetime utility for each type of agents with the newly calibrated parameters.

Table 8 confirms that the welfare conflict remains if the government changes the long-run and short-run properties of fiscal policy simultaneously. Workers favor a slightly countercyclical labor income tax and a procyclical capital income tax. With this policy combination, entrepreneurs suffer a welfare loss of −0.02%, more than two times greater than the loss in the baseline case. Entrepreneurs prefer a strongly countercyclical labor income tax and a countercyclical capital income tax, with which workers also experience a welfare loss. In addition, the welfare gap between these two groups increases in each scenario compared with the experiment with only a change in the cyclical behavior.

## 8. Conclusion

The current paper focuses on how a change in fiscal policy over the business cycles, instead of long-run levels, affects the welfare of heterogeneous agents. We provide empirical evidence that the responses of fiscal policy instruments to output exert different impacts on the welfare of agents with different income sources. Then, we develop a framework to mirror the empirical findings with heterogeneous agents, business cycles, fiscal policy rules, and particularly, idiosyncratic investment risks. Idiosyncratic investment risks, albeit having a larger standard deviation than idiosyncratic labor income risks, draw less attention and are underdeveloped, especially in the literature on business cycles. Our study explores merging idiosyncratic investment risks into the business cycle framework, innovating macroeconomic research with heterogeneous agents.

The model provides a tractable tool to analyze the dynamics of aggregate variables and income distribution, discuss the change in welfare, and evaluate fiscal policy over the business cycles in the context of heterogeneous agents. The tractability results from the assumptions of no trade in capital and i.i.d. idiosyncratic investment risks that render saving and portfolio choices independent of wealth distribution. The identical behavior of all entrepreneurs conditional on their wealth leads to exact aggregation.

The model matches the level and cyclical dynamics of income inequality, as in the US data. The simulated income distribution features large income shares in top income groups and small income shares in bottom income groups, as shown in the CPS. As for the cyclical property of income distribution, the simulated result quantitatively matches most of the correlations of income shares with output.

The policy experiment emphasizes a welfare conflict between entrepreneurs and workers and is confirmed by empirical findings. The government maximizes workers' welfare by choosing capital income tax rates that are negatively correlated with output and labor income tax rates that are positively correlated with output. On the contrary, the government optimizes entrepreneurs' welfare with capital income tax rates that are positively correlated with output and labor income tax rates that are strongly positively correlated with output. Two groups' different income sources lead to diverse cyclical changes in the mean and volatility of consumption of entrepreneurs and workers with fiscal policy, which further causes the welfare conflict.

Two extra experiments are conducted to check the robustness of the result of the welfare conflict. The government may change the fiscal policy facing the recession or adjust the long-run level and cyclical behavior of policy simultaneously. The welfare conflict in the timing choice of fiscal policy holds in both cases. Furthermore, the welfare gap between entrepreneurs and workers rises when a certain policy is chosen.

Governments commonly use fiscal policy as a tool to buffer aggregate shocks over the business cycles. Typically, governments choose countercyclical fiscal policy that features cutting taxes and raising government spending in recessions. Based on our simulation, however, this policy prescription lowers the lifetime utility of workers although it benefits entrepreneurs. Governments should balance the welfare of different groups of people whilst stimulating the economy.

Our study provides a possible solution. Governments should cut income taxes and subsidize workers simultaneously in recessions. Tax cuts promote the economic recovery and heighten the welfare of entrepreneurs. Subsidies can be used to compensate for the harm to workers.

Specifically, these subsidies for workers can be substituted by part of other government spending. In this way, governments are possible to achieve both goals of efficiency and equity.

## Notes

**1** Fernández-Villaverde et al. (2015) use the time series data from 1970.Q1 to 2014.Q2. We obtain exactly the same data as in Fernández-Villaverde et al. (2015) and the period is extended up to 2019.Q4.
**2** The quarterly capital stock is constructed from annual data with the second method developed by Levy and Chen (1994) that exploits the capital accumulation relationship between capital stock and the corresponding investment series to estimate the quarterly depreciation rates that vary over time.
**3** They employ the data of individual income tax returns from the Internal Revenue Service over 23 years, compute for each household the time series standard deviation of its business income normalized by the household's average total income over time, and then combine those business income "coefficients of variation" into one cross-sectional average.
**4** We compute $\int_j \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{(c^j{}_t)^{1-\gamma}}{1-\gamma} \right]$ in Appendix C.
**5** The limits of the response parameter of each policy instrument are far beyond what the realistic policy can reach.

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## Appendix A: Proof of Lemma

We start from the characterization of allocations at the individual level.

The Euler equations derived from the entrepreneur optimization problem are as follows:

$$\frac{(c_t^i)^{-\gamma}}{1+\tau_t^c} = \beta \mathbb{E}_t \left\{ \frac{(c_{t+1}^i)^{-\gamma}}{1+\tau_{t+1}^c} \left[ \left(1-\tau_{t+1}^k\right) \left(r\left(A_{t+1}^i, w_{t+1}\right) - \delta\right) + 1 \right] \right\}, \tag{29}$$

$$\frac{(c_t^i)^{-\gamma}}{1+\tau_t^c} = \beta \mathbb{E}_t \left[ \frac{(c_{t+1}^i)^{-\gamma}}{1+\tau_{t+1}^c} R_t \right]. \tag{30}$$

We guess that the solutions to the entrepreneur optimization problem are as follows:

$(1+\tau_t^c)c_t^i = \nu_t x_t^i$, and $k_t^i = (1-\nu_t)\phi_t x_t^i$, so $b_t^i = (1-\nu_t)(1-\phi_t)x_t^i$ from the budget constraint of the entrepreneur. We will determine later the coefficients, $\nu_t$ and $\phi_t$, which only depend on the current aggregate state. To simplify the notation, we define the aggregate state at $t$, $S_t = (K_{t-1}, B_{t-1}, z_t, \Omega_t)$. Then, we write $\nu_t = \nu_t(S_t)$ and $\phi_t = \phi_t(S_t)$. With some algebra,

$$\begin{aligned} x^i_{t+1} &= \left[(1-\tau^k_{t+1})(r(A^i_{t+1}, w_{t+1}) - \delta) + 1\right] k^i_t + R_t b^i_t \\ &= \left\{\phi_t(S_t)[(1-\tau^k_{t+1})(r(A^i_{t+1}, w_{t+1}) - \delta) + 1] + (1-\phi_t(S_t))R_t\right\} (1-\nu_t(S_t))x^i_t. \end{aligned} \tag{31}$$

Then, the first Euler equation becomes:

$$\begin{aligned} \left(\frac{\nu_t(S_t)x^i_t}{1+\tau^c_t}\right)^{-\gamma} &= \beta \mathbb{E}_t \left\{\left(\frac{\nu_{t+1}(S_{t+1})x^i_{t+1}}{1+\tau^c_{t+1}}\right)^{-\gamma} \left[\left(1-\tau^k_{t+1}\right)\left(r\left(A^i_{t+1}, w_{t+1}\right) - \delta\right) + 1\right]\right\} \\ &= \beta\left(1-\nu_t\left(S_t\right)\right)^{-\gamma} \mathbb{E}_t \left\{\left(\frac{\nu_{t+1}(S_{t+1})}{1+\tau^c_{t+1}}\right)^{-\gamma} \left[\left(1-\tau^k_{t+1}\right)\left(r\left(A^i_{t+1}, w_{t+1}\right) - \delta\right) + 1\right] \right. \\ &\quad \left. \left\{\phi_t(S_t)[(1-\tau^k_{t+1})(r(A^i_{t+1}, w_{t+1}) - \delta) + 1] + (1-\phi_t(S_t))R_t\right\}^{-\gamma}\right\} (x^i_t)^{-\gamma}. \end{aligned} \tag{32}$$

We cross out the same factors from both handsides, obtaining:

$$\begin{aligned} \frac{\nu_t(S_t)^{-\gamma}}{\left(1+\tau^c_t\right)^{-\gamma}} &= \beta\left(1-\nu_t\left(S_t\right)\right)^{-\gamma} \mathbb{E}_t \left\{\frac{\nu_{t+1}(S_{t+1})^{-\gamma}}{\left(1+\tau^c_{t+1}\right)^{-\gamma}} \left[\left(1-\tau^k_{t+1}\right)\left(r\left(A^i_{t+1}, w_{t+1}\right) - \delta\right) + 1\right] \right. \\ &\quad \left. \left\{\phi_t(S_t)[(1-\tau^k_{t+1})(r(A^i_{t+1}, w_{t+1}) - \delta) + 1] + (1-\phi_t(S_t))R_t\right\}^{-\gamma}\right\}. \end{aligned} \tag{33}$$

Likewise, the second Euler equation can be transformed as:

$$\begin{aligned} \frac{\nu_t(S_t)^{-\gamma}}{\left(1+\tau^c_t\right)^{-\gamma}} &= \beta\left(1-\nu_t\left(S_t\right)\right)^{-\gamma} \mathbb{E}_t \left\{\frac{\nu_{t+1}(S_{t+1})^{-\gamma}}{\left(1+\tau^c_{t+1}\right)^{-\gamma}} R_t \right. \\ &\quad \left. \left\{\phi_t(S_t)[(1-\tau^k_{t+1})(r(A^i_{t+1}, w_{t+1}) - \delta) + 1] + (1-\phi_t(S_t))R_t\right\}^{-\gamma}\right\}. \end{aligned} \tag{34}$$

Combining these two equations, we obtain the equality between the gross returns on risky and risk-free assets:

$$\begin{aligned} &\mathbb{E}_t \left\{\frac{\nu_{t+1}(S_{t+1})^{-\gamma}}{\left(1+\tau^c_{t+1}\right)^{-\gamma}} \left[\left(1-\tau^k_{t+1}\right)\left(r\left(A^i_{t+1}, w_{t+1}\right) - \delta\right) + 1 - R_t\right] \right. \\ &\quad \left. \left\{\phi_t(S_t)[(1-\tau^k_{t+1})(r(A^i_{t+1}, w_{t+1}) - \delta) + 1] + (1-\phi_t(S_t))R_t\right\}^{-\gamma}\right\} \\ &= \int_0^\infty \left(\int_0^\infty \frac{\nu_{t+1}(S_{t+1})^{-\gamma}}{\left(1+\tau^c_{t+1}\right)^{-\gamma}} \left[\left(1-\tau^k_{t+1}\right)\left(r\left(e^i_{t+1}, \varepsilon^z_{t+1}, w\left(\varepsilon^z_{t+1}\right); z_t\right) - \delta\right) + 1 \right.\right. \\ &\quad \left. -R_t\right] \left\{\phi_t(s_t)[(1-\tau^k_{t+1})(r(e^i_{t+1}, \varepsilon^z_{t+1}, w(\varepsilon^z_{t+1}); z_t) - \delta) + 1] \right. \\ &\quad \left.\left. +(1-\phi_t(S_t))R_t\right\}^{-\gamma} \, \mathrm{d}\mathcal{F}(e^i_{t+1})\right) \mathrm{d}\mathcal{F}(\varepsilon^z_{t+1}) = 0. \end{aligned} \tag{35}$$

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Multiply (33) and (34) with $\phi_t$ and $(1-\phi_t)$, respectively, and sum up to get:

$$\frac{\nu_t(S_t)^{-\gamma}}{\left(1+\tau_t^c\right)^{-\gamma}} = \beta\left(1-\nu_t\left(S_t\right)\right)^{-\gamma}\mathbb{E}_t\left\{\frac{\nu_{t+1}(S_{t+1})^{-\gamma}}{\left(1+\tau_{t+1}^c\right)^{-\gamma}}\left\{\phi_t\left(S_t\right)\left[\left(1-\tau_{t+1}^k\right)\right.\right.\right.$$
$$\left.\left.\left.(r(A_{t+1}^i,w_{t+1})-\delta)+1\right]+(1-\phi_t(S_t))R_t\right\}^{1-\gamma}\right\}. \tag{36}$$

Define

$$\phi_t = \arg\max_{\phi\in[0,1]} \mathbb{CE}_t\left\{\phi[(1-\tau_{t+1}^k)(r(A_{t+1}^i,w_{t+1})-\delta)+1]+(1-\phi)R_t\right\}, \tag{37}$$

where

$$\mathbb{CE}_t\left\{\phi_t[(1-\tau_{t+1}^k)(r(A_{t+1}^i,w_{t+1})-\delta)+1]+(1-\phi_t)R_t\right\}$$
$$=\left[\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty}\left\{\phi_t(S_t)[(1-\tau_{t+1}^k)(r(e_{t+1}^i,\varepsilon_{t+1}^z,w(\varepsilon_{t+1}^z);z_t)-\delta)+1]\right.\right.\right.$$
$$\left.\left.\left.+(1-\phi_t(S_t))R_t\right\}^{1-\gamma}\,\mathrm{d}\mathcal{F}(e_{t+1}^i)\right)\,\mathrm{d}\mathcal{F}\left(\varepsilon_{t+1}^z\right)\right]^{\frac{1}{1-\gamma}}. \tag{38}$$

The process requires numerical solutions. The preceding integral demonstrates that the optimal saving and portfolio choices depend only on the aggregate state. We drop the notation of aggregate state from onwards.

Given $\phi_t$, $\tau_t^c$, $\tau_{t+1}^c$, and $\tau_{t+1}^k$, $\nu_t$ can be computed by (36).

Next, we derive the functional form of the value function for entrepreneurs. We first guess that the value function $V(x_t^i)=\psi_t u(x_t^i)$. From the envelope theorem, $V'\left(x_t^i\right)=\dfrac{u'(c_t^i)}{1+\tau_t^c}$, which is,

$$\psi_t\left(x_t^i\right)^{-\gamma}=\frac{(c_t^i)^{-\gamma}}{1+\tau_t^c}=\frac{(\nu_t x_t^i)^{-\gamma}}{(1+\tau_t^c)^{1-\gamma}}. \tag{39}$$

We simplify the above equation by crossing out the common factor so that

$$\psi_t=\frac{\nu_t^{-\gamma}}{(1+\tau_t^c)^{1-\gamma}}. \tag{40}$$

Hence, $V\left(x_t^i\right)=\dfrac{\nu_t^{-\gamma}(x_t^i)^{1-\gamma}}{(1-\gamma)(1+\tau_t^c)^{1-\gamma}}$.

Now, we verify whether the value function obtained fits the entrepreneur's optimization problem. Notice that $V\left(x_t^i\right)=\dfrac{(c_t^i)^{1-\gamma}}{1-\gamma}+\beta\mathbb{E}_t\left[V\left(x_{t+1}^i\right)\right]$. We insert the expressions of the value function and consumption, (15), into the equation:

$$\frac{\nu_t^{-\gamma}(x_t^i)^{1-\gamma}}{(1-\gamma)(1+\tau_t^c)^{1-\gamma}}=\frac{(\nu_t x_t^i)^{1-\gamma}}{(1-\gamma)(1+\tau_t^c)^{1-\gamma}}+\beta\mathbb{E}_t\left[\frac{\nu_{t+1}^{-\gamma}(x_{t+1}^i)^{1-\gamma}}{(1-\gamma)(1+\tau_{t+1}^c)^{1-\gamma}}\right]. \tag{41}$$

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We multiply both handsides by $(1-\gamma)$, remove the first term on the right handside to the left, and combine the two terms on the left handside:

$$\frac{(1-\nu_t)\nu_t^{-\gamma}(x_t^i)^{1-\gamma}}{(1+\tau_t^c)^{1-\gamma}} = \beta\mathbb{E}_t\left[\frac{\nu_{t+1}^{-\gamma}(x_{t+1}^i)^{1-\gamma}}{(1+\tau_{t+1}^c)^{1-\gamma}}\right]$$

$$= \beta\mathbb{E}_t\left\{\frac{\nu_{t+1}^{-\gamma}}{(1+\tau_{t+1}^c)^{1-\gamma}}\left\{\left\{\phi_t\left(S_t\right)\left[\left(1-\tau_{t+1}^k\right)\left(r\left(A_{t+1}^i, w_{t+1}\right)-\delta\right)+1\right]+\right.\right.\right.$$

$$\left.\left.\left.+(1-\phi_t(S_t))R_{t+1}\right\}(1-\nu_t)x_t^i\right\}^{1-\gamma}\right\}. \tag{42}$$

Then, we eliminate the common factors, $(1-\nu_t)$ and $(x_t^i)^{1-\gamma}$, from both handsides, obtaining:

$$\frac{\nu_t^{-\gamma}}{(1+\tau_t^c)^{1-\gamma}} = \beta\left(1-\nu_t\right)^{-\gamma}\mathbb{E}_t\left\{\frac{\nu_{t+1}^{-\gamma}}{\left(1+\tau_{t+1}^c\right)^{-\gamma}}\left\{\phi_t\left[\left(1-\tau_{t+1}^k\right)\right.\right.\right.$$
$$\left.\left.\left.(r(A_{t+1}^i, w_{t+1})-\delta)+1\right]+(1-\phi_t)R_t\right\}^{1-\gamma}\right\}, \tag{43}$$

which is validated by (36). ■

## B. Effect of idiosyncratic investment risks

This section shows how idiosyncratic investment risks affect the portfolio choice in steady state.

From the wage determination, $w_t=(1-\alpha)(z_tK_t)^{\alpha}$, and the specification of risks, we rewrite the expression of individual capital investment return as:

$$\begin{aligned} r_{t+1}^i &= \alpha\left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1}{\alpha}-1}A_{t+1}^i \\ &= \alpha z_{t+1}^{\alpha-1}K_t^{\alpha-1}A_{t+1}^i \\ &= \alpha z_{t+1}^{\alpha}K_t^{\alpha-1}\exp\left[\frac{-\sigma_e^2}{2}+e_{t+1}^i\right] \\ &= \bar{\bar{r}}_{t+1}\exp\left(e_{t+1}^i\right), \end{aligned} \tag{44}$$

where $\bar{\bar{r}}_{t+1}$ denotes $\alpha z_{t+1}^{\alpha}K_t^{\alpha-1}\exp\left[\frac{-\sigma_e^2}{2}\right]$, showing risk-adjusted average return to capital in the economy.

We use the second-order Taylor series with respect to $e_{t+1}^i$ around 0 to approximate the certainty equivalent.

$$\mathbb{CE}_t\left\{\phi_t\left[\left(1-\tau_{t+1}^k\right)(r_{t+1}^i-\delta)+1\right]+(1-\phi_t)R_t\right\}$$
$$\approx\left(\int_{-\infty}^{\infty}\left\{\phi_t\left[\left(1-\tau_{t+1}^k\right)\left(\bar{\bar{r}}_{t+1}-\delta\right)+1\right]+(1-\phi_t)\,R_t\right\}^{1-\gamma}\right.$$
$$+\frac{1-\gamma}{2}\sigma_e^2\left\{-\gamma\left\{\phi_t\left[\left(1-\tau_{t+1}^k\right)\left(\bar{\bar{r}}_{t+1}-\delta\right)+1\right]+(1-\phi_t)\,R_t\right\}^{-\gamma-1}\right.$$

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$$\left[\phi_t\left(1-\tau^k_{t+1}\right)\bar{\bar{r}}_{t+1}\right]^2+\left\{\phi_t\left[\left(1-\tau^k_{t+1}\right)\left(\bar{\bar{r}}_{t+1}-\delta\right)+1\right]+\left(1-\phi_t\right)R_t\right\}^{-\gamma} \tag{45}$$

$$\phi_t\left(1-\tau^k_{t+1}\right)\bar{\bar{r}}_{t+1}\Bigg\}\,\mathrm{d}F\left(\varepsilon^z_{t+1}\right)\Bigg)^{\frac{1}{1-\gamma}}$$

We take the first-order condition with respect to $\phi_t$ to pin down the optimal portfolio choice:

$$\begin{aligned}&\int_{-\infty}^{\infty}\Bigg\{\Bigg\{\left[\left(1-\tau^k_{t+1}\right)\left(\bar{\bar{r}}_{t+1}-\delta\right)+1-R_t\right]^3+\frac{1-\gamma}{2}\sigma_e^2\left\{\left(1-\tau^k_{t+1}\right)\bar{\bar{r}}_{t+1}\right.\\&\left[1-\delta\left(1-\tau^k_{t+1}\right)-R_t\right]+(1-\gamma)\left[\left(1-\tau^k_{t+1}\right)\bar{\bar{r}}_{t+1}\right]^2\Bigg\}\left[\left(1-\tau^k_{t+1}\right)\left(\bar{\bar{r}}_{t+1}-\delta\right)\right.\\&\left.+1-R_t\right]\Bigg\}\phi_t^2+\Bigg\{2\left[\left(1-\tau^k_{t+1}\right)\left(\bar{\bar{r}}_{t+1}-\delta\right)+1-R_t\right]^2+\frac{\sigma_e^2}{2}\Bigg\{(2-3\gamma)\\&\left[\left(1-\tau^k_{t+1}\right)\bar{\bar{r}}_{t+1}\right]^2+(2-\gamma)\left(1-\tau^k_{t+1}\right)\bar{\bar{r}}_{t+1}\left[1-\delta\left(1-\tau^k_{t+1}\right)-R_t\right]\Bigg\}\Bigg\}R_t\phi_t\\&+R_t^2\left[\left(1+\frac{\sigma_e^2}{2}\right)\left(1-\tau^k_{t+1}\right)\bar{\bar{r}}_{t+1}+1-\delta\left(1-\tau^k_{t+1}\right)-R_t\right]\Bigg\}\,\mathrm{d}F\left(\varepsilon^z_{t+1}\right)=0.\end{aligned} \tag{46}$$

The deterministic steady state rules out the aggregate shocks. We list the certainty equivalent and its derivative with respect to $\phi_t$ in steady state as follows:

$$\begin{aligned}\mathbb{CE}\approx&\Bigg(\left\{\phi\left[\left(1-\tau^k\right)\left(\bar{\bar{r}}-\delta\right)+1\right]+(1-\phi)R\right\}^{1-\gamma}\\&+\frac{1-\gamma}{2}\sigma_e^2\Bigg\{-\gamma\left\{\phi\left[\left(1-\tau^k\right)\left(\bar{\bar{r}}-\delta\right)+1\right]+(1-\phi)R\right\}^{-\gamma-1}\left[\phi\left(1-\tau^k\right)\bar{\bar{r}}\right]^2\\&+\left\{\phi\left[\left(1-\tau^k\right)\left(\bar{\bar{r}}-\delta\right)+1\right]+(1-\phi)R\right\}^{-\gamma}\phi\left(1-\tau^k\right)\bar{\bar{r}}\Bigg\}\Bigg)^{\frac{1}{1-\gamma}},\end{aligned} \tag{47}$$

$$\begin{aligned}&\Bigg\{\left[\left(1-\tau^k\right)\left(\bar{\bar{r}}-\delta\right)+1-R\right]^3+\frac{1-\gamma}{2}\sigma_e^2\Bigg\{(1-\gamma)\left[\left(1-\tau^k\right)\bar{\bar{r}}\right]^2+\left(1-\tau^k\right)\\&\bar{\bar{r}}\left[1-\delta\left(1-\tau^k\right)-R\right]\Bigg\}\left[\left(1-\tau^k\right)\left(\bar{\bar{r}}-\delta\right)+1-R\right]\Bigg\}\phi^2+\Bigg\{2\Big[\left(1-\tau^k\right)\\&\left(\bar{\bar{r}}-\delta\right)+1-R\Big]^2+\frac{\sigma_e^2}{2}\Bigg\{(2-3\gamma)\left[\left(1-\tau^k\right)\bar{\bar{r}}\right]^2+(2-\gamma)\left(1-\tau^k\right)\bar{\bar{r}}\\&\left[1-\delta\left(1-\tau^k\right)-R\right]\Bigg\}\Bigg\}R\phi+R^2\Bigg[\left(1+\frac{\sigma_e^2}{2}\right)\left(1-\tau^k\right)\bar{\bar{r}}+1-\delta\left(1-\tau^k\right)\\&-R\Bigg]=0.\end{aligned} \tag{48}$$

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## C. Welfare for Hand-to-mouth Workers

From (7),

$$\log e_{t+1}^{j} = \rho_w \log e_t^{j} + \varepsilon_{t+1}^{j}, \tag{49}$$

we have that

$$\mathbb{E}_0\left(\log e_{j,t}\right) = \mathbb{E}_0\left(\rho_w \log e_{j,t-1}\right) + \mathbb{E}_0\left(\varepsilon_{j,t}\right) = 0; \tag{50}$$

$$\mathrm{Var}_0\left(\log e_{j,t}\right) = \frac{\sigma_w^2}{1-\rho_w^2}; \tag{51}$$

$$\mathbb{E}_0\left(e_{j,t}\right) = \exp\left(\frac{\sigma_w^2}{2(1-\rho_w^2)}\right), \forall t; \tag{52}$$

and furthermore,

$$\mathbb{E}_0\left(e_{j,t}^{1-\gamma}\right) = \exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right), \forall t. \tag{53}$$

Then, the summed unconditional mean of utility at $t$ across workers is

$$\begin{aligned}\int_j \mathbb{E}_0\left\{\frac{(c_t^j)^{1-\gamma}}{1-\gamma}\right\} &= \int_j \mathbb{E}_0\left\{\frac{\left[\left(1-\tau_t^n\right)\frac{1}{\lambda}w_t e_{j,t}\right]^{1-\gamma}}{1-\gamma}\right\} \\ &= \lambda^{\gamma}\mathbb{E}_0\left\{\frac{[(1-\tau_t^n)w_t]^{1-\gamma}}{1-\gamma}\right\}\exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right).\end{aligned} \tag{54}$$

At last, the value function of workers is expressed as

$$\begin{aligned}\int_j V\left(x_0^j\right) &= \int_j \mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t \frac{(c_t^j)^{1-\gamma}}{1-\gamma} \\ &= \sum_{t=0}^{\infty}\beta^t \int_j \mathbb{E}_0\left\{\frac{(c_t^j)^{1-\gamma}}{1-\gamma}\right\} \\ &= \lambda^{\gamma}\sum_{t=0}^{\infty}\beta^t \mathbb{E}_0\left\{\frac{[(1-\tau_t^n)w_t]^{1-\gamma}}{1-\gamma}\right\}\exp\left(\frac{\sigma_w^2(1-\gamma)^2}{2(1-\rho_w^2)}\right).\end{aligned} \tag{55}$$

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