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Orbit identification for large sets of data: preliminary results

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Abstract. We propose a strategy to attack the problems of orbit determination arising from the large number of short arcs. The method uses a solution of the linkage problem depending on the first integrals of the Keplerian motion.

Keywords. celestial mechanics, orbit determination, linkage, attributable

1. Introduction

This brief note is devoted to a discussion on one of the problems arising in orbit determination of asteroids. The identification problem consists of finding among a set of detections of celestial bodies, those belonging to the same object. The problem is of crucial importance nowadays, due to the large amount of data coming from the asteroid surveys. The Catalina Sky Survey and the Pan-STARRS project, in particular, search for moving objects in the sky every night collecting many new detections. Due to the improvements of the technology, these telescopes are able to observe many small objects, that in general are visible only during a small span of time. The AstDys website monthly computes the orbits of numbered and multiopposition asteroids whose observations are provided by the Minor Planet Center. Anyway, if the described arc is too short and the observations that are collected are few, then a single apparition orbit determination is not possible. Even if the least squares algorithm succeeded, the uncertainty would be very large. It means that the predicted portion of the sky where the next apparition should take place is larger than the field of view of the telescope and the recovery may fail. In this case we would have a lost asteroid. Hence there exists a database of designations without a good orbit that deserves investigation and we are going to concentrate on it.

Since a single arc is not enough to have a good orbit, the usual procedure is to try to find the couples of arcs that can be joined to produce a better orbit. This problem is known as linkage problem and has been considered by several authors (Granvik *et al.* (2005), Granvik & Muinonen (2008), Milani *et al.* (2004), Milani *et al.* (2005), Taff & Hall (1977)). The problem is complicated as one has to join the information on two different arcs that, in principle, may not belong to the same orbit.

There is not just one accepted method, as every proposal has some strength and weakness. In particular, the case in which the time distance between the two arcs is large is the most challenging. In Gronchi *et al.* (2010) a method based on the first integrals of Kepler's motion is proposed. It was tested on simulated data, giving good results. Subsequently, in Gronchi *et al.* (2011) the method has been improved but it has not been tested for the asteroid case.

In the following we are going to concentrate on the application of the improved method to real data. Generally, the orbit coming from a linkage procedure is not good enough to be considered reliable. Hence some work has to be done in order to test and improve it. In the next section we will recall the Keplerian integrals method and briefly describe the procedure that we are going to use in order to improve the results coming from the linkage. We will apply the procedure to a reduced database of real data and show some partial results.

2. The strategy

An observed arc is a set of observations (α_i, δ_i) of an object at time t_i. Here we indicate with α the right ascension and with δ the declination. If the arc is too short and the observations are few, the least squares algorithm (if it converges) generally gives orbits with very large uncertainty. The only information that one can get through interpolation corresponds to the values of right ascension, declination and their corresponding rates at a mean epoch of observations. The vector formed by these quantities is called attributable and is denoted as

$$\mathcal{A} = (\alpha, \delta, \dot{\alpha}, \dot{\delta}).$$

The linkage problem can be formalized as the problem of finding an orbit compatible with two given attributables $\mathcal{A}_1$ and $\mathcal{A}_2$ at time t_1 and t_2 respectively. This means to find the missing range (indicated with ρ) and range rate (indicated with $\dot{\rho}$) either at time t_1 or t_2. Among the various methods we concentrate on the method based on the Keplerian integrals. The basic idea is the following. If we suppose that $\mathcal{A}_1$ and $\mathcal{A}_2$ correspond to the same object and that the motion between times t_1 and t_2 is Keplerian, then the integrals of the two body problem must be preserved. Expressing position and velocity in spherical coordinates (ρ, α, δ) we can equate respectively the values that energy, angular momentum and Lenz vector take at t_1 and t_2. In this way we get a system of seven equations in the four unknowns $(\rho_1, \dot{\rho}_1, \rho_2, \dot{\rho}_2)$. The system is over-determined, and just four equations are enough. In Gronchi *et al.* (2010) the authors consider the angular momentum and the energy. By squaring, a polynomial system of total degree 48 is obtained. Having polynomial equations allows to compute the solutions in an efficient way. At this point we have two cases. If the system has no solutions, then the assumption that the two attributables belong to the same object is false and we discard the couple. If we have solutions, then they are tested with some compatibility conditions. These conditions are also based on the equations that we are not using to solve the system. If this control is passed we have a preliminary orbit that can act as a starting guess for the differential corrections. Note that the output orbits are endowed with covariance matrices. A test on simulated data has been performed, giving good results. Anyway, the high degree of the system could represent a problem when handling large databases. Therefore, in Gronchi *et al.* (2011), the method was improved choosing a suitable projection of the Lenz vector instead of the energy. By squaring, a polynomial system of total degree 20 was obtained.

We are going to consider a database of 80140 arcs of type 2. According to the definition given in Milani *et al.* (2007) these arcs show a significant curvature and can be split in at least two disjoint arcs, each without a significant curvature. A single apparition orbit determination with this kind of arcs does not give reliable results. Hence we split every arc in order to have tracklets that do not show curvature and from them we derive the corresponding attributable. In the end we are left with a database of 198947 attributables.

A brute force approach would lead to a N^2 computational complexity, where N is the total number of attributables. This implies a very long computational time. With some algorithm of sorting the complexity can be reduced to $N \log N$. Anyway as N is large, the computational time still remains too large. In order to have a reduced database we set some filters as described in Gronchi *et al.* (2010). In particular we set the minimum time span between the attributables at 100 days and the maximum at 365 days. In this way, we are not considering the identifications given by the reassembling of the previously splitted arcs.

A proposed orbit, to be considered reliable, needs to be compared with the observations. Only if we have enough observations and the residuals are sufficiently low, the orbit can be considered reliable. Hence, after the linkage, we try and find new observations that can fit the proposed orbit. Once we have more observations, the differential corrections can be performed and new residuals appear. This procedure is generally called attribution and is described in Milani *et al.* (2001).

From a list of attributables, the linkage procedure gives a new list of orbits and a left-over database of attributables. The attribution procedure will try to match orbits and attributables, improving the first ones. This latter list cannot be considered as a final result. Indeed duplicates and contradictions can be created. A method to deal with this problem has been introduced in Milani *et al.* (2005). A list is considered complete when there is no couple of identifications sharing the same attributable. The way to treat discordant identifications is crucial. We remember that two identifications are called discordant if they share some tracklets and not all the tracklets constituting one identification are contained in the other. On one side we can choose the one that, according to some quality parameters, is the best. On the other hand, we can try to merge two discordant identifications into a longer one. In this latter case we would have a new identification composed by more observations than the previous ones. The merging process could introduce new duplicates and contradictions, hence a new control has to be done, without merging, in order to have a normalized list and conclude the identification procedure.

3. Results and conclusion

The results of the application to the reduced database is a set of 580 identifications that can be summarized in the following table.

number of attributables	2	3	4	5	6	7
number of identifications	228	136	119	42	4	3

The identifications are cataloged according to the number of joined tracklets, e.g. we have 42 identifications with 5 tracklets. Those constituted by 5 or more tracklets can be considered good candidate orbits and should be submitted to a more stringent control on the residuals. The results of this control, together with an application to a larger database of tracklets will be presented in a forthcoming paper.

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References

Granvik, K., Muinonen, K., Virtanen, J., Delbò, M., Saba, L., De Sanctis, G., Morbidelli, R., Cellino, A., & Tedesco, E. 2005, In *Dynamics of Populations of Planetary Systems,* eds. Knežević, Z and Milani, A. (Cambridge University Press)

Granvik, K. & Muinonen, K. 2008, *Icarus*, 198, 130

Gronchi, G. F., Dimare, L., & Milani, A. 2010, *Cel. Mech. & Dyn. Astr.*, 107, 299
Gronchi, G. F., Farnocchia, D., & Dimare, L. 2011, *Cel. Mech. & Dyn. Astr.*, 110, 257
Milani, A., Sansaturio, M. E., & Chesley, S. R. 2001, *Icarus*, 151, 150
Milani, A., Gronchi, G. F., de' Michieli Vitturi, M., & Knežević, Z. 2004, *Cel. Mech. & Dyn. Astr.*, 90, 59
Milani, A., Gronchi, G. F., & Knežević, Z. 2007, *Earth, Moon and Planets*, 100, 83
Milani, A., Gronchi, G. F., Knežević, Z., Sansaturio, M. E., & Arratia, O. 2005, *Icarus*, 179, 350
Taff, L. G. & Hall, D. L. 1977, *Cel. Mech. & Dyn. Astr.*, 16, 481