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REGULAR PAPER

Output constrained neural adaptive control for a class of KKVs with non-affine inputs and unmodeled dynamics

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Abstract
In this paper, an adaptive neural output-constrained control algorithm is proposed for a class of non-affine kinetic kill vehicle (KKV) systems. The key point is that the non-affine control law can be designed and the output of the KKV system conform to the output limit with the aid of the proposed method. Due to the aerodynamic moments, the actual control torque is non-affine, which can be addressed by introducing an integral process to the design of the controller. Besides, in order to improve the control precision, a nonlinear mapping is put forward so that the output constraint can be transformed to the constraint of the introduced dynamic signal that can be simply achieved. From the simulation results it can be concluded that the states of the KKV system can track the desired trajectories in spite of different working conditions and the control precision is higher compared with other control methods.

1.0 Introduction

In recent years, there have been a number of control techniques and strategies for uncertain nonlinear systems [1–3]. In those schemes, it is assumed that the control system is affine commonly [4, 5]. However, the control input in practice applications have nonlinear characteristics, that is to say, the system is considered to be nonaffine, such as aircraft systems [6], chemical processes [7], wind energy systems [8] and so on. Compared with affine system, there is no proportional relationship between input and control gain. Therefore, the control design of nonaffine systems becomes a challenging topic.

During the past few decades, considerable attempts have been made for the nonlinear control system with nonaffine input form. For a class of uncertain nonlinear systems, an adaptive output feedback control scheme based on linear parameterised neural network is proposed [9]. In [10], the Taylor series expansion is utilised to transform the nonaffine system into affine system. Meanwhile, the state observer is introduced to estimate the system state. Only one adaptive parameter needs to be considered in the backstepping process by using this control strategy. Then, a synthesis method is designed for a class of nonaffine systems [11]. In this work, the fixed-point and control effectiveness term assumptions are eliminated. The adaptive dynamic surface control (DSC) [12] is applied for a class of nonaffine system, so that the complexity caused by the repeated differentiation of the virtual input in back-stepping method has been alleviated. In [13], combining a low-pass filter with state transformation, a new transformation method is designed to avoid the difficulty introduced by nonaffine properties.

Practical systems often need to meet different kinds of constraints, such as precision specification [14], aircraft dynamics characteristics [15] and so on. For the purpose of satisfying such requirements, the output constrained control problem have been developed during the recent decades. In the initial stage of those process, the barrier Lyapunov function (BLF), introduced into the nonlinear system, is employed to ensure the stability of systems with output constraints [16–18]. The neural networks (NN) as a kind of approximator is utilised to estimate the system uncertainties [19, 20]. Compared with the Quadratic Lyapunov Function, it is proven that an asymmetric BLF as a generalised method designed in [21] relaxes the initial condition requirements. Furthermore, the time-varying output constraints problem has been solved by using transformation techniques [22], backstepping DSC scheme [23], time-varying BLF [24] and so on. In [25], a variable separation strategy is introduced to deal with the difficulty caused by the nonstrict-feedback structure.

As an advanced adjacent space vehicle, kinetic kill vehicle (KKV) has attracted the attention of researchers in recent years. The angle of KKV is adjusted in a wide range to search for targets [26]. Then, it is very important to control the attitude angle of KKV quickly and precisely after locating the target [27]. When it comes to the terminal guidance phase, the system can be considered as a MIMO nonlinear system with strong coupling uncertainties which leads to the control difficulties [28]. Many attempts have been conducted in this topic. In early phase, several guidance schemes are designed with large energy consumption, such as time-varying biased proportional navigation guidance (PNG) [29], elliptical arc guidance [30] and integrated PNG [31, 32]. Moreover, to meet the higher performance requirements, novel guidance laws based on optimal control theory arise at the historic moment. By using a simplified mathematical model, an energy optimal guidance law (OGL) is proposed to achieve the optimal performance of KKV [33]. In [34], combining the optimal control theory with the sliding-mode control theory, an optimal sliding-mode terminal guidance law is proposed.

In spite of the fruitful research that has been mentioned above, it should be pointed out that none of the research respect to KKV takes into consideration the nonaffine dynamics, output constraint as well as the unmodeled dynamics universally. Due to the aerodynamic moments, the actual control torque is nonaffine, which lead to difficulties in the process of the controller design. Besides, only a small deviation of the attitude angle may cause the attitude control system of KKV out of control. That's the reason why high control precision is essential in the design of attitude control systems. Motivated by the aforementioned problems, we will investigate the problem of output constrained neural adaptive control for a class of KKVs with non-affine inputs and unmodeled dynamics. The main contributions of this paper are summarised as follows:

- As the authors know, it is the first neural adaptive output-constrained control algorithm for a class of KKVs with non-affine inputs and unmodeled dynamics.
- By designing an indirect control signal as well as integrating it, the control law can be obtained in the presence of nonaffine dynamics.
- Thanks to the presented nonlinear mapping, the output constraint can be transformed to the limitation of the introduced dynamic signal which is simple to achieve. The high control precision can be guaranteed.

2.0 Problem formulation and preliminaries

2.1 Dynamic model of KKV systems

Neglecting the flexible dynamics, the dynamic model of the kinetic kill vehicle can be formulated as follows:

$$\begin{cases} \dot{\gamma} = \omega_x - \tan\vartheta\left(\omega_y \cos\gamma - \omega_z \sin\gamma\right) \\ \dot{\psi} = \dfrac{\omega_y \cos\gamma - \omega_z \sin\gamma}{\cos\vartheta} \\ \dot{\vartheta} = \omega_y \sin\gamma + \omega_z \cos\gamma \\ J_x\dot{\omega}_x + \left(J_z - J_y\right)\omega_y\omega_z = M_x + d_{M,x}(t) \\ J_y\dot{\omega}_y + \left(J_x - J_z\right)\omega_z\omega_x = M_y + d_{M,y}(t) \\ J_z\dot{\omega}_z + \left(J_y - J_x\right)\omega_x\omega_y = M_z + d_{M,z}(t) \end{cases} \tag{1}$$

where γ, ψ, ϑ represent the roll angle, yaw angle and pitch angle of KKV separately. ω_x, ω_y, ω_z represent the angular velocities respect to the inertial coordinate system. M_x, M_y, M_z denote the control torque acting on the KKV. $d_{M,x}$, $d_{M,y}$, $d_{M,z}$ are the disturbances torque generated by the aerodynamic uncertainties or environmental factors. J_x, J_y, J_z denote the rotary inertias of the KKV.

Taking into consideration that KKV is influenced by not only the control torque but also aerodynamic moments existing in the process of near space flight, the certain part of the actual control torque is non-affine. Due to the existence of the disturbances induced by the jet stream and the external flow field, the actual control torque possess uncertainties. The actual control torque can be described by

$$\begin{cases} M_{a,x}(t) = g_1(F_x) + \Delta M_x(t) \\ M_{a,y}(t) = g_2\left(F_y\right) + \Delta M_y(t) \\ M_{a,z}(t) = g_3(F_z) + \Delta M_z(t) \end{cases} \tag{2}$$

where $M_{a,x}, M_{a,y}, M_{a,z}$ are the components of actual control torque. $g_1(F_x)$, $g_2\left(F_y\right)$, $g_3(F_z)$ represent the non-affine part of the control torque. ΔM_x, ΔM_y and ΔM_z are the uncertainties of actual control torque.

Define $\xi(t) = \left[\vartheta(t), \psi(t), \gamma(t)\right]^T$, $\omega(t) = \left[\omega_z(t), \omega_y(t), \omega_x(t)\right]^T$, $u(t) = \left[F_z(t), F_y(t), F_x(t)\right]^T$, then we can formulate the dynamic model of KKV system as:

$$\begin{aligned} \dot{\xi}(t) &= G(t)\,\omega(t) \\ \dot{\omega}(t) &= J^{-1} f(\xi(t), \omega(t)) + J^{-1}\bar{g}[u(t)] + J^{-1} d_0(t) \end{aligned} \tag{3}$$

where

$$G = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\dfrac{\sin\gamma}{\cos\vartheta} & \dfrac{\cos\gamma}{\cos\vartheta} & 0 \\ \tan\vartheta\sin\gamma & -\tan\vartheta\cos\gamma & 1 \end{bmatrix} \tag{4}$$

$$\begin{gathered} J = \begin{bmatrix} J_z & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_x \end{bmatrix}, f(\xi, \omega) = \begin{bmatrix} \left(J_y - J_x\right)\omega_x\omega_y \\ (J_x - J_z)\,\omega_z\omega_x \\ \left(J_z - J_y\right)\omega_y\omega_z \end{bmatrix} \\ d_0(t) = \begin{bmatrix} d_{M,z}(t) + \Delta M_z(t) \\ d_{M,y}(t) + \Delta M_y(t) \\ d_{M,x}(t) + \Delta M_x(t) \end{bmatrix}, \bar{g}[u(t)] = \begin{bmatrix} g_3(F_z) \\ g_2\left(F_y\right) \\ g_1(F_x) \end{bmatrix} \end{gathered} \tag{5}$$

Define $h(\xi, \omega) = J^{-1} f(\xi, \omega)$, $d(t) = J^{-1} d_0(t)$, $g[u(t)] = J^{-1}\bar{g}[u(t)]$, we can rewrite (3) as

$$\begin{aligned} \dot{\xi}(t) &= G(t)\,\omega(t) \\ \dot{\omega}(t) &= h(\xi(t), \omega(t)) + g[u(t)] + d(t) \end{aligned} \tag{6}$$

In the process of KKV's flight, coupled uncertainties should be noticed. Coupled uncertainties include couplings between control system channels. Define coupled uncertainty $\chi(\xi(t), \omega(t), \eta(t))$ which is affected by system states. The dynamic behaviour of the unmeasured state $\eta(t)$ can be formulated by

$$\dot{\eta}(t) = f_\eta(\eta(t), \xi(t), \omega(t)) \tag{7}$$

In practical, the fuel consumption during attitude control process and the manufacturing errors of the KKVs cannot be ignored. Considering these factors, we can rewrite (6) as

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$$
\begin{aligned}
\dot{\xi}(t) &= [G(t) + \Delta G(t)]\,\omega(t) \\
\dot{\omega}(t) &= h(\xi(t), \omega(t)) + \Delta h(\xi(t), \omega(t)) + g[u(t)] \\
&\quad + \chi(\xi(t), \omega(t), \eta(t)) + d(t) \\
\dot{\eta}(t) &= f_\eta(\eta(t), \xi(t), \omega(t)) \\
\dot{u}(t) &= v(t) \\
y(t) &= \xi(t)
\end{aligned} \tag{8}
$$

where $\Delta G(t)$, $\Delta h(\xi(t), \omega(t))$ are the uncertain parts of $G(t)$, $h(\xi(t), \omega(t))$, respectively and $v(t)$ is the indirect control signal. $y(t)$ denotes the output of the system.

Our objective is to design a dynamic control signal such that the outputs of the KKV attitude dynamic system (8) can track the desired signal $y_d(t)$ asymptotically in the presence of the nonaffine dynamics, output constraint as well as time unmodeled dynamics.

In this paper, the following assumptions are made:

Assumption 1. *The disturbances torque induced by the aerodynamic uncertainties or environmental factors are all bounded. That is to say, there exists a constant $\bar{d}$ such that $\forall t \geq 0, \|d(t)\| \leq \bar{d}$.*

Assumption 2. *In the vicinity of the equilibrium the uncertain part of G remain unchanged. In other words, it is supposed that $d\Delta G/dt \approx 0$.*

Assumption 3. *The coupled uncertainty has properties as follows*

$$\chi(\xi(t), \omega(t), \eta(t)) \leq \varphi_1(\xi(t), \omega(t)) + \varphi_2(\eta(t)) \tag{9}$$

where $\varphi_1(\cdot), \varphi_2(\cdot)$ are unknown non-negative smooth functions. Additionally, the unmeasured state $\eta(t)$ is exponentially input-to-state practically stable. That is, there exists a Lyapunov function $V_\eta(\eta(t))$ satisfying

$$
\begin{aligned}
&\alpha_1(\eta(t)) \leq V_\eta(\eta(t)) \leq \alpha_2(\eta(t)) \\
&\frac{\partial V_\eta(\eta(t))}{\partial \eta(t)} \kappa(\eta(t), \xi(t), \omega(t)) \leq -\gamma_1 V_\eta(\eta(t)) + \rho(\xi(t), \omega(t)) + \gamma_2
\end{aligned} \tag{10}
$$

where $\alpha_1(\cdot), \alpha_2(\cdot)$ are functions of class $\mathcal{K}_\infty$, γ_1, γ_2 are positive constants. $\rho(\xi(t), \omega(t)) = \xi^T(t)\,\xi(t) + \omega^T(t)\,\omega(t)$.

Remark 1. All the disturbances existing in the attitude control system of the KKV must be bounded with the upper bound $\bar{d}$. Otherwise, the disturbance will be beyond the control capability of the system and the anti-disturbance controller will be difficult to design. Besides, the structural uncertainty ΔG is mainly caused by model simplification. As a result, the uncertain part contains coupling factors and it is essential to employ **Assumption 2** to facilitate the control design process. Moreover, in order to deal with the coupled uncertainty caused by the unmodeled dynamics, **Assumption 3** is also necessary, which can be seen in many similar papers. With what has been mentioned above, the assumptions made in this paper are all proper and reasonable.

2.2 Supporting definitions and lemmas

Lemma 1 [35]. *The following inequality holds for any $\varepsilon > 0$ and $z \in \mathbb{R}$*

$$0 \leq |z| - z \tanh\left(\frac{z}{\varepsilon}\right) \leq \kappa\varepsilon \tag{11}$$

where $\kappa = 0.2785$.

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Lemma 2. *In the process of KKV's flight, there exist positive scalars* $\vartheta_{\max} < \pi/2, \psi_{\max} < \pi/2, \gamma_{\max} < \pi/2$ *that enable matrix G invertible for any* $(\vartheta, \psi, \gamma)$ *in* $A := \{(\vartheta, \psi, \gamma) \| \vartheta| < \vartheta_{\max}, |\psi| < \psi_{\max}, |\gamma| < \gamma_{\max}\}$.

Proof: It has been calculated that

$$\det(G) = -\frac{\sin^2\gamma}{\cos\vartheta} - \frac{\cos^2\gamma}{\cos\vartheta} = -\frac{1}{\cos\vartheta} \tag{12}$$

$\det(G)$ is a continuous function of ϑ. According to the properties of continuous function, for any $|\vartheta| < \vartheta_{\max}$, where $\vartheta_{\max} < \pi/2$ is a positive scalar, there exist positive scalars $\psi_{\max} < \pi/2, \gamma_{\max} < \pi/2$ which enable $\det(G) < 0$ if $|\psi| < \psi_{\max}, |\gamma| < \gamma_{\max}$. The proof is completed.

Lemma 3. *Given any constant* $\varepsilon > 0$ *and vector* $\xi \in \mathbb{R}^n$, *the following inequality holds*

$$\|\xi\| < \frac{\xi^T\xi}{\sqrt{\xi^T\xi + \varepsilon^2}} + \varepsilon \tag{13}$$

Proof: Due to $\varepsilon > 0$, it is obvious that

$$\left[\xi^T\xi + \varepsilon\sqrt{\xi^T\xi + \varepsilon^2}\right]^2 - \left[\|\xi\|\sqrt{\xi^T\xi + \varepsilon^2}\right]^2$$
$$= 2\varepsilon\xi^T\xi\sqrt{\xi^T\xi + \varepsilon^2} + \varepsilon^4 > 0 \tag{14}$$

Hence we can learn from $\sqrt{\xi^T\xi + \varepsilon^2} > 0$ that

$$\|\xi\|\sqrt{\xi^T\xi + \varepsilon^2} < \xi^T\xi + \varepsilon\sqrt{\xi^T\xi + \varepsilon^2} \tag{15}$$

Divide both sides of the inequality by $\sqrt{\xi^T\xi + \varepsilon^2}$, which completes the proof.

Lemma 4 [36]. *For any* $\varepsilon > 0$, *define set* $\Omega_\varepsilon = \{x \| x\| < 0.2554\varepsilon\}$. *Then, for any* $x \notin \Omega_\varepsilon$, *this inequality holds*

$$1 - 16\tanh^2\left(\frac{x}{\varepsilon}\right) \le 0 \tag{16}$$

Lemma 5 [37]. $f : \mathbb{R} \to \mathbb{R}$ *is a continuously differentiable function defined on* $[0, \infty)$. $\lim_{t\to\infty} f(t)$ *exists and is bounded. If* $\dot{f}(t)$ *is uniformly continuous in* $[0, \infty)$, *then* $\lim_{t\to\infty} \dot{f}(t) = 0$.

2.3. Neural approximation

For an arbitrary continuous function $F(z)$ defined on a compact set Ω_z, a radial basis function neural network (RBFNN) can be used to approximate it.

$$y(z) = \hat{\theta}^T\Phi(z) \tag{17}$$

where $z = [z_1, z_2, \cdots, z_n]^T \in \mathbb{R}^n$ is the input vector of the RBFNN. y is the output of the RBFNN. $\hat{\theta}^T = \left[\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_m\right] \in \mathbb{R}^m$ is the weight matrix. $\Phi(z) = [\Phi_1(z), \Phi_2(z), \cdots, \Phi_m(z)]^T \in \mathbb{R}^m$, n is the input number and m represents the node number. $\Phi_i(\cdot)$ is chosen as the Gaussian function in this paper.

$$\Phi_i(z) = \exp\left[\frac{-(z-\mu_i)^T(z-\mu_i)}{\sigma_i^2}\right], i = 1, 2, \ldots, m \tag{18}$$

where μ_i is the center of the receptive field and σ_i is the width of the Gaussian function. For continuous function $F(z)$, there exists an optimal weight matrix θ such that

$$F(z) = \theta^T\Phi(z) + \varepsilon \tag{19}$$

where ε is the approximation error that can be arbitrarily small by increasing the node number. Generally, θ is chosen as the value that minimises the distance between $F(z)$ and $y(z)$ in the sense of L_2 norm.

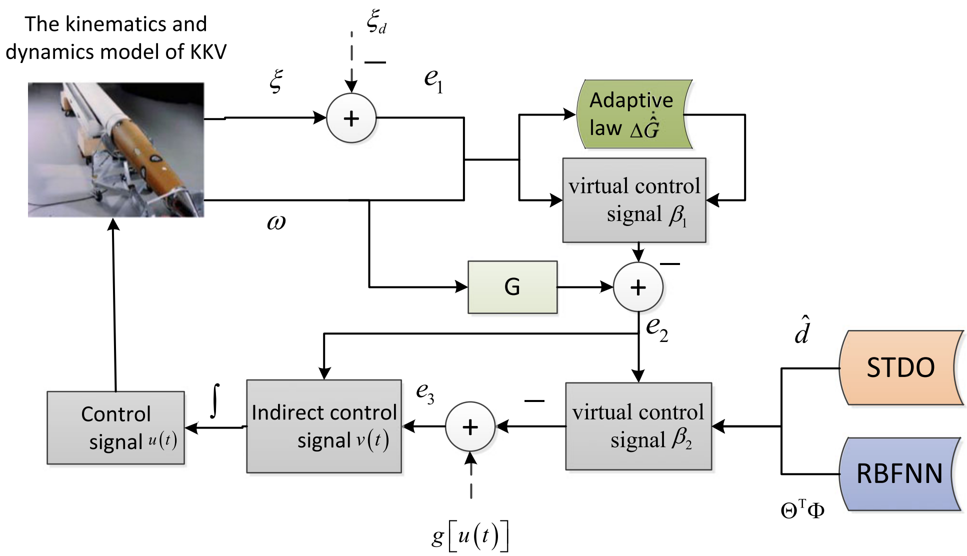

Figure 1. *The structure of the proposed control scheme for KKV.*

3.0 Main results

In this section, an adaptive neural output-constrained controller is designed. Based on the attitude angles and angular velocities of the KKV, an adaptive law is put forward to approximate the structural uncertainty and the virtual control signal of the inner loop is presented. In addition, a Super-Twisting Disturbance Observer (STDO) as well as a RBFNN are employed to attenuate the disturbance and the actuator faults respectively. As a result, the virtual control signal of the outer loop can be proposed. Last but not least, by means of introducing an integral process to the indirect control signal, the control signal can be obtained in spite of the non-affine dynamics. The block of the proposed control method can be seen in Fig. 1.

3.1 Output constraint transformation

For the purpose of guaranteeing the constraints of the system output signal, a nonlinear mapping is introduced as

$$\bar{\eta}(t) = \Xi\left(y(t)\,, y_{up}(t)\,, y_{low}(t)\right) \tag{20}$$

where $y_{up}(t)\,, y_{low}(t)$ are the constraints of the output signal. $\Xi\left(y(t)\,, y_{up}(t)\,, y_{low}(t)\right)$ is selected to satisfy that

$$\Xi^{-1}\left(\bar{\eta}(t)\,, y_{up}(t)\,, y_{low}(t)\right) = \frac{y_{up}(t) - y_{low}(t)}{2}\tanh(\bar{\eta}(t)) + \frac{y_{up}(t) + y_{low}(t)}{2} \tag{21}$$

It is obvious that

$$\begin{aligned} \lim_{\bar{\eta}(t)\to+\infty} \Xi^{-1}\left(\bar{\eta}(t)\,, y_{up}(t)\,, y_{low}(t)\right) &= y_{up}(t) \\ \lim_{\bar{\eta}(t)\to-\infty} \Xi^{-1}\left(\bar{\eta}(t)\,, y_{up}(t)\,, y_{low}(t)\right) &= y_{low}(t) \end{aligned} \tag{22}$$

Considering that $\Xi(\cdot)$ is a one to one mapping from $y(t)$ to $\bar{\eta}(t)$, we can know that if $\bar{\eta}(t)$is constrained to be bounded by any constant, then $y(t)$will be limited to stay in the time-varying open interval$\left(y_{up}(t)\,, y_{low}(t)\right)$. Hence, it is easy to know that the output constrained control problem has been transformed to the stable controller design for the dynamic equation of $\bar{\eta}(t)$.

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Define the tracking errors as $e_1(t) = \bar{\eta}(t) - \bar{\eta}_d(t)$, $e_2(t) = G(t)\,\omega(t) - \beta_1(t)$, $e_3(t) = g[u(t)] - \beta_2(t)$, where $\beta_1(t)$, $\beta_2(t)$ are the virtual control signals and

$$\bar{\eta}_d(t) = \Xi\left(y_d(t)\,,\, y_{up}(t)\,,\, y_{low}(t)\right) \tag{23}$$

Then we can get that

$$\begin{aligned}
\dot{e}_1(t) &= \frac{\partial \Xi}{\partial y}[G(t) + \Delta G]\,\omega(t) + \frac{\partial \Xi}{\partial y_{up}}\dot{y}_{up} + \frac{\partial \Xi}{\partial y_{low}}\dot{y}_{low} - \left[\frac{\partial \Xi}{\partial y}\dot{y}_d(t) + \frac{\partial \Xi}{\partial y_{up}}\dot{y}_{up} + \frac{\partial \Xi}{\partial y_{low}}\dot{y}_{low}\right] \\
&= \frac{\partial \Xi}{\partial y}\left[[G(t) + \Delta G]\,\omega(t) - \dot{y}_d(t)\right] \\
&= \Xi_y\left[e_2(t) + \beta_1(t) + \Delta G\omega(t) - \dot{y}_d(t)\right]
\end{aligned} \tag{24}$$

where $\Xi_y = \partial \Xi / \partial y$.

3.2 Adaptive neural controller design

Our objective is transformed to design $\beta_1(t)$to force $e_1(t)$ to converge. Hence, the virtual control law $\beta_1(t)$ can be designed as

$$\beta_1(t) = -k_\xi \Xi_y^{-1} e_1(t) - k_{\xi,0} \Xi_y^{-1} \int_0^t e_1(\tau)\,d\tau - \Xi_y^{-1}\Delta\hat{G}\omega(t) + \dot{y}_d(t) \tag{25}$$

where $\Delta\hat{G}$ is the estimation of ΔG.

Substituting (25) into (24) yields

$$\dot{e}_1(t) = \Xi_y e_2(t) - k_\xi e_1(t) - k_{\xi,0}\int_0^t e_\xi(\tau)\,d\tau - \Delta\tilde{G}\omega(t) \tag{26}$$

Choose the Lyapunov function as follows:

$$V_1 = \frac{1}{2}e_0^T(t)\,e_0(t) + \frac{1}{2}e_1^T(t)\,e_1(t) + \frac{1}{2}Tr\left[\Delta\tilde{G}_G^T \eta_G^{-1}\Delta\tilde{G}_G\right] \tag{27}$$

where $\Delta\tilde{G} = \Delta\hat{G} - \Delta G$, $e_0(t) = \int_0^t e_1(\tau)\,d\tau$. $\eta_G > 0$ is the adaptive gain. $Tr(A)$ denotes the trace of matrix A. Along (26), we can take the differential of V_1 as

$$\begin{aligned}
\dot{V}_1 &= e_0^T(t)\,e_1(t) + e_1^T\Xi_y e_2 - k_\xi e_1^T(t)\,e_1(t) - k_{\xi,0}e_1^T(t)\,e_0(t) \\
&\quad - e_1^T(t)\,\Delta\tilde{G}\omega(t) + Tr\left[\Delta\tilde{G}^T\eta_G^{-1}\Delta\dot{\hat{G}}\right]
\end{aligned} \tag{28}$$

Define $E_\xi = \left[e_0^T(t)\,,\, e_1^T(t)\right]^T$. It is obvious that

$$\dot{V}_1 \le -E_\xi^T(t)\,QE_\xi(t) - e_1^T(t)\,\Delta\tilde{G}\omega(t) + e_1^T(t)\,\Xi_y e_2(t) + Tr\left[\Delta\tilde{G}^T\eta_G^{-1}\Delta\dot{\hat{G}}\right] \tag{29}$$

where

$$Q = \begin{bmatrix} 0 & -I_3 \\ k_{\xi,0}I_3 & k_\xi I_3 \end{bmatrix} \tag{30}$$

Since for any vector $x, y \in \mathbb{R}^n$, $x^Ty = Tr\left(yx^T\right)$ holds, we can rewrite (28) as

$$\dot{V}_1 = -E_\xi^T(t)\,QE_\xi(t) + e_1^T(t)\,\Xi_y e_2(t) + \frac{1}{\eta_G}Tr\left[\Delta\tilde{G}^T\left(\Delta\dot{\hat{G}} - \eta_G e_1(t)\,\omega^T(t)\right)\right] \tag{31}$$

Hence, in the framework of σ- modification, the adaptive law of $\Delta\hat{G}$ can be designed as

$$\Delta\dot{\hat{G}} = \eta_G\left(e_1(t)\,\omega^T(t) - \sigma_G\Delta\hat{G}\right) \tag{32}$$

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Substituting (32) into (31) yields that

$$\dot{V}_1 = -E_\xi^T(t)\,QE_\xi(t) + e_1^T(t)\,\Xi_y e_2(t) - \sigma_G Tr\left[\Delta\tilde{G}^T\Delta\hat{G}\right] \tag{33}$$

With the aid of

$$-Tr\left[\Delta\tilde{G}^T\Delta\hat{G}\right] \le -\frac{1}{2}Tr\left[\Delta\tilde{G}^T\Delta\tilde{G}\right] + \frac{1}{2}Tr\left[\Delta G^T\Delta G\right] \tag{34}$$

It can be obtained that

$$\dot{V}_1 = -E_\xi^T(t)\,QE_\xi(t) - \frac{\sigma_G}{2}Tr\left[\Delta\tilde{G}^T\Delta\tilde{G}\right] + e_1^T(t)\,\Xi_y e_2(t) + \frac{\sigma_G}{2}Tr\left[\Delta G^T\Delta G\right] \tag{35}$$

Introducing dynamic signal $r(t)$ which satisfies

$$\dot{r}(t) = -\gamma_0 r(t) + \rho(\xi(t), \omega(t)),\ r(0) = r_0 \tag{36}$$

where $\gamma_0 \in (0, \gamma_3)$. $r(t)$ has the following properties [38]:

$$\begin{aligned} & r(t) \ge 0, \forall t \ge 0 \\ & V_\eta(\eta(t)) \le r(t) + \varepsilon_r \end{aligned} \tag{37}$$

where $\varepsilon_r = V_\eta(\eta(0)) + \gamma_2/\gamma_1$.

According to (9), we can get that

$$e_2^T(t)\,G(t)\,\chi(\xi(t), \omega(t), \eta(t)) \le \left\|e_2^T(t)\,G(t)\right\|(\varphi_1(\xi(t), \omega(t)) + \varphi_2(\eta(t))) \tag{38}$$

According to Lemma 3, it is obvious that

$$\begin{aligned} & \left\|e_2^T(t)\,G(t)\right\|\varphi_1(\xi(t), \omega(t)) \le e_2^T(t)\,G(t)\,\bar{\varphi}_1(\xi(t), \omega(t)) + \varepsilon_{\varphi 1} \\ & \left\|e_2^T(t)\,G(t)\right\|\varphi_2(\eta(t)) \le \left\|e_2^T(t)\,G(t)\right\|\varphi_2\circ\alpha_1^{-1}(2r(t)) + \left\|e_2^T(t)\,G(t)\right\|\varphi_2\circ\alpha_1^{-1}(2\varepsilon_r) \end{aligned} \tag{39}$$

where $\varepsilon_{\varphi 1} > 0$ is any positive constant and

$$\bar{\varphi}_1(\xi(t), \omega(t)) = \frac{\varphi_1(\xi(t), \omega(t))\,e_2^T(t)\,G(t)\,\varphi_1(\xi(t), \omega(t))}{\sqrt{\left[e_2^T(t)\,G(t)\,\varphi_1(\xi(t), \omega(t))\right]^2 + \varepsilon_{\varphi 1}^2}} \tag{40}$$

With the aid of Young's inequality,

$$\left\|e_2^T(t)\,G(t)\right\|\varphi_2(\eta(t)) \le e_2^T(t)\,G(t)\,\bar{\varphi}_2(r(t)) + \varepsilon_{\varphi 2} + \frac{1}{4}e_2^T(t)\,G(t)\,G^T(t)\,e_2(t) + \varepsilon_{\varphi 3} \tag{41}$$

where $\varepsilon_{\varphi 2} > 0$ is any positive constant,

$$\begin{aligned} & \bar{\varphi}_2(e_2(t), r(t)) = \frac{\varphi_2\circ\alpha_1^{-1}(2r(t))\,e_2^T(t)\,G(t)\,\varphi_2\circ\alpha_1^{-1}(2r(t))}{\sqrt{\left[e_2^T(t)\,G\varphi_2\circ\alpha_1^{-1}(2r(t))\right]^2 + \varepsilon_{\varphi 2}^2}} \\ & \varepsilon_{\varphi 3} = \left[\varphi_2\circ\alpha_1^{-1}(2\varepsilon_r)\right]^2 \end{aligned} \tag{42}$$

Next, the virtual control law of the outer loop will be given. By using the second equation of (8), we can take the time derivative of $e_2(t)$ as

$$\dot{e}_2(t) = G(t)\begin{bmatrix} h(\xi(t), \omega(t)) + \Delta h(\xi(t), \omega(t)) + g[u(t)] \\ +\chi(\xi(t), \omega(t), \eta(t)) + d(t) \end{bmatrix} - \dot{\beta}_1(t) \tag{43}$$

A RBFNN is introduced in order to approximate the unknown nonlinearities. Apparently, it can be known that $\Theta^T\Phi(\xi(t), \omega(t), r(t)) + \varepsilon_\Theta = \bar{\varphi}_1(\xi(t), \omega(t)) + \bar{\varphi}_2(e_2(t), r(t)) + \Delta h(\xi(t), \omega(t))$. Moreover, the STDO is introduced to handle the time-varying disturbances.

It is obvious that

$$\begin{aligned} \dot{e}_2(t) = {} & G(t)\,h(\xi(t), \omega(t)) + G(t)\,\Delta h(\xi(t), \omega(t)) + G(t)\,e_3 + G(t)\,\beta_2 \\ & + G(t)\,\chi(\xi(t), \omega(t), \eta(t)) + G(t)\,d(t) - \dot{\beta}_1(t) \end{aligned} \tag{44}$$

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Hence, $\beta_2(t)$ can be designed as

$$\beta_2(t) = G^{-1}(t)\left[-k_\omega e_2(t) - \Xi_y^T e_1(t) - \tfrac{3}{4}e_2 + \dot{\beta}_1(t) - \varphi_\rho(\xi(t), \omega(t), e_2(t))\right] - h(\xi(t), \omega(t)) - \hat{\Theta}^T\Phi(\xi(t), \omega(t), r(t)) - \hat{d}(t) \tag{45}$$

where $\hat{\Theta}$ are the estimate of Θ. ε_D is a design constant. $\varphi_\rho(\xi(t), \omega(t), e_2(t))$ will be mentioned below. $\hat{d}(t)$ is the output of a super-twisting disturbance observer, which can be given by:

$$\begin{aligned}
\dot{\hat{d}}(t) &= -k_d\left(\hat{d}(t) - p_1(t)\right) \\
\dot{p}_1(t) &= -k_{p,1}\frac{\hat{\omega}(t) - \omega(t)}{\left\|\hat{\omega}(t) - \omega(t)\right\|^{1/2}} + p_2(t) \\
\dot{p}_2(t) &= -k_{p,2}\frac{\hat{\omega}(t) - \omega(t)}{\left\|\hat{\omega}(t) - \omega(t)\right\|} \\
\dot{\hat{\omega}}(t) &= h(\xi(t), \omega(t)) + g[u(t)] + \hat{d}(t) \\
&\quad + \hat{\Theta}^T\Phi(\xi(t), \omega(t), r(t)) + \frac{1}{4}e_2^T(t)\, G(t)\, G^T(t)\, e_2(t)
\end{aligned} \tag{46}$$

where $k_d, k_{p,1}, k_{p,2}$ are the gains of the super-twisting disturbance observer, $p_1(t), p_2(t), \hat{\omega}(t)$ are the internal states.

For any constant vector $z(t) = [z_1(t), z_2(t), \cdots, z_m(t)]^T \in \mathbb{R}^m$, Tanh$(z)$ is defined as

$$\text{Tanh}(z(t)) = [\tanh z_1(t), \tanh z_2(t), \cdots, \tanh z_m(t)]^T \tag{47}$$

Choose the following Lyapunov function

$$V_2 = \frac{1}{2}e_2^T(t)\, e_2(t) + \frac{1}{2}Tr\left[\tilde{\Theta}^T\eta_\Theta^{-1}\tilde{\Theta}\right] + \frac{r(t)}{\Gamma_r} \tag{48}$$

Hence,

$$\begin{aligned}
\dot{V}_2 \le &-\left(k_\omega + \frac{3}{4}\right)e_2^T(t)\, e_2(t) - e_2^T(t)\, \Xi_y^T e_1(t) + e_2^T(t)\, G(t)\, e_3(t) - e_2^T(t)\, G(t)\, \tilde{d}(t) \\
&+ \sum_{i=1}^{3}\varepsilon_{\varphi i} + e_2^T(t)\, \varepsilon_\Theta - e_2^T(t)\, \tilde{\Theta}^T\Phi(\xi(t), \omega(t)) \\
&+ Tr\left[\tilde{\Theta}^T\eta_\Theta^{-1}\dot{\hat{\Theta}}\right] - e_2^T\varphi_\rho(\xi(t), \omega(t), e_2(t)) - \frac{\gamma_0}{\Gamma_r}r(t) + \frac{\rho(\xi(t), \omega(t))}{\Gamma_r}
\end{aligned} \tag{49}$$

According to Lemma 4, it is obvious that

$$\frac{\rho(\xi(t), \omega(t))}{\Gamma_r} = \frac{\rho(\xi(t), \omega(t))}{\Gamma_r}\left(1 - 16Tanh^T\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right)Tanh\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right)\right) + e_2^T(t)\, \varphi_\rho(\xi(t), \omega(t), e_2(t)) \tag{50}$$

where

$$\varphi_\rho(\xi(t), \omega(t), e_2(t)) = \frac{16e_2(t)\, \rho(\xi(t), \omega(t))}{\Gamma_r e_2^T(t)\, e_2(t)}Tanh^T\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right)Tanh\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right) \tag{51}$$

$\varphi_\rho(\xi(t), \omega(t), e_\omega(t))$ is a nonsingular functional vector. Considering (49) we can get that

$$\dot{V}_2 \leq -\left(k_\omega + \frac{3}{4}\right) e_2^T(t)\, e_2(t) - e_2^T(t)\, \varXi_y^T e_1(t) + e_2^T(t)\, G(t)\, e_3(t) - e_2^T(t)\, G(t)\, \tilde{d}(t)$$
$$+ \sum_{i=1}^{3} \varepsilon_{\varphi i} + e_2^T(t)\, \varepsilon_\Theta + Tr\left[\tilde{\Theta}^T \eta_\Theta^{-1} \dot{\hat{\Theta}}\right] - e_2^T(t)\, \tilde{\Theta}^T \Phi(\xi(t), \omega(t)) - \frac{\gamma_0}{\Gamma_r} r(t)$$
$$+ \frac{\rho(\xi(t), \omega(t))}{\Gamma_r}\left(1 - 16 Tanh^T\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right) Tanh\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right)\right) \tag{52}$$

It is obvious that

$$-e_2^T(t)\, G(t)\, \tilde{d}(t) \leq \frac{1}{2} e_2^T(t)\, e_2(t) + \frac{1}{2} \tilde{d}^T(t)\, G^T(t)\, G(t)\, \tilde{d}(t)$$
$$\leq \frac{1}{2} e_2^T(t)\, e_2(t) + \frac{1}{2} \lambda_{\max}\left(G^T(t)\, G(t)\right) \left\|\tilde{d}(t)\right\|^2 \tag{53}$$

Moreover, it can be verified that

$$e_2^T(t)\, \varepsilon_\Theta \leq \frac{1}{4} e_2^T(t)\, e_2(t) + \varepsilon_\Theta^2 \tag{54}$$

With the aid of (53) and (54), we know that

$$\dot{V}_2 \leq -k_\omega e_2^T(t)\, e_2(t) - e_2^T(t)\, \varXi_y^T e_1(t) + e_2^T(t)\, G(t)\, e_3(t) + \frac{1}{2} \lambda_{\max}\left(G^T(t)\, G(t)\right) \left\|\tilde{d}(t)\right\|^2$$
$$+ \sum_{i=1}^{3} \varepsilon_{\varphi i} + \varepsilon_\Theta^2 + Tr\left[\tilde{\Theta}^T \eta_\Theta^{-1} \dot{\hat{\Theta}}\right] - e_2^T(t)\, \tilde{\Theta}^T \Phi(\xi(t), \omega(t)) - \frac{\gamma_0}{\Gamma_r} r(t)$$
$$+ \frac{\rho(\xi(t), \omega(t))}{\Gamma_r}\left(1 - 16 Tanh^T\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right) Tanh\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right)\right) \tag{55}$$

In the framework of σ- modification, the adaptive laws are designed as

$$\dot{\hat{\Theta}} = \eta_\Theta \Phi(\xi(t), \omega(t), r(t))\, e_2^T(t) - \eta_\Theta \sigma_\Theta \hat{\Theta} \tag{56}$$

Substituting (56) into (55), we can obtain that

$$\dot{V}_2 \leq -k_\omega e_2^T(t)\, e_2(t) - e_2^T(t)\, \varXi_y^T e_1(t) + e_2^T(t)\, G(t)\, e_3(t) + \frac{1}{2} \lambda_{\max}\left(G^T(t)\, G(t)\right) \left\|\tilde{d}(t)\right\|^2$$
$$+ \sum_{i=1}^{3} \varepsilon_{\varphi i} + \varepsilon_\Theta^2 - \sigma_\Theta Tr\left[\tilde{\Theta}^T \hat{\Theta}\right] - \frac{\gamma_0}{\Gamma_r} r(t)$$
$$+ \frac{\rho(\xi(t), \omega(t))}{\Gamma_r}\left(1 - 16 Tanh^T\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right) Tanh\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right)\right) \tag{57}$$

Furthermore, it can be derived that

$$\dot{V}_2 \leq -k_\omega e_2^T(t)\, e_2(t) - e_2^T(t)\, \varXi_y^T e_1(t) + e_2^T(t)\, G(t)\, e_3(t) - \frac{\sigma_\Theta}{2} Tr\left[\tilde{\Theta}^T \tilde{\Theta}\right]$$
$$- \frac{\gamma_0}{\Gamma_r} r(t) + \frac{\rho(\xi(t), \omega(t))}{\Gamma_r}\left(1 - 16 Tanh^T\left(\tfrac{e_\omega(t)}{\varepsilon_\rho}\right) Tanh\left(\tfrac{e_\omega(t)}{\varepsilon_\rho}\right)\right) + \varepsilon_2 \tag{58}$$

where

$$\varepsilon_2 = \frac{1}{2} \lambda_{\max}\left(G^T(t)\, G(t)\right) \left\|\tilde{d}(t)\right\|^2 + \frac{\sigma_\Theta}{2} Tr\left[\Theta^T \Theta\right] + \varepsilon_\Theta^2 + \sum_{i=1}^{3} \varepsilon_{\varphi i} \tag{59}$$

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Afterwards, the control law will be given. It is obvious that

$$\dot{e}_3(t) = \frac{dg[u(t)]}{du(t)} \cdot v(t) - \dot{\beta}_2(t) \tag{60}$$

Hence, $v(t)$ can be designed as

$$v(t) = \frac{1}{\frac{dg[u(t)]}{du(t)}} \left[-k_3 e_3(t) - G^T(t)\, e_2(t) + \dot{\beta}_2(t) \right] \tag{61}$$

Substituting (61) into the fourth equation of (8), the actual control law will be obtained

$$u(t) = \int_0^t v(\tau)\, d\tau + u(0) \tag{62}$$

Substituting (61) into (60), we can get that

$$\dot{e}_3(t) = -k_3 e_3(t) - G^T(t)\, e_2(t) \tag{63}$$

Choosing Lyapunov function as follows:

$$V_3 = \frac{1}{2} e_3^T(t)\, e_3(t) \tag{64}$$

Hence

$$\dot{V}_3 = -k_3 e_3^T(t)\, e_3(t) - e_3^T(t)\, G^T(t)\, e_2(t) \tag{65}$$

3.3 Stability analysis

Theorem 1. *Consider a kinetic kill vehicle system* (1), *the controller* (62), *the parameter update laws* (32), (56) *in the presence of disturbances and coupling uncertainties under Assumption* 1∼*Assumption* 3, *then the boundedness of all the signals can be ensured and the tracking errors can converge to zero.*

Proof: Choosing Lyapunov function as follows:

$$V = V_1 + V_2 + V_3 \tag{66}$$

According to (35), (58) and (65), we can get that

$$\begin{aligned}
\dot{V} \le & -E_\xi^T(t)\, QE_\xi(t) - \frac{\sigma_G}{2} Tr\left[\Delta\tilde{G}^T \Delta\tilde{G}\right] + \frac{\sigma_G}{2} Tr\left[\Delta G^T \Delta G\right] - k_\omega e_2^T(t)\, e_2(t) \\
& - \frac{\sigma_\Theta}{2} Tr\left[\tilde{\Theta}^T \tilde{\Theta}\right] - \frac{\gamma_0}{\Gamma_r} r(t) + \frac{\rho(\xi(t), \omega(t))}{\Gamma_r} \left(1 - 16 Tanh^T\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right) Tanh\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right)\right) + \varepsilon_2 \\
& - k_3 e_3^T(t)\, e_3(t) \\
\le & -\gamma V + \frac{\rho(\xi(t), \omega(t))}{\Gamma_r} \left(1 - 16 Tanh^T\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right) Tanh\left(\tfrac{e_2(t)}{\varepsilon_\rho}\right)\right) + \varepsilon_f
\end{aligned} \tag{67}$$

where

$$\gamma = \min\{2\lambda_{\min}(Q), \eta_G \sigma_G, 2k_\omega, \lambda_{\min}(\eta_\Theta)\, \sigma_\Theta, \gamma_0, 2k_3\}$$

$$\varepsilon_f = \frac{\sigma_G}{2} Tr\left[\Delta G^T \Delta G\right] + \frac{1}{2} \lambda_{\max}\left(G^T(t)\, G(t)\right) \left\|\tilde{d}(t)\right\|^2 + \frac{\sigma_\Theta}{2} Tr\left[\Theta^T \Theta\right] + \varepsilon_\Theta^2 + \sum_{i=1}^{3} \varepsilon_{\varphi i} \tag{68}$$

Define closed sets

$$\begin{aligned}
\Omega_f &= \left\{\xi, \omega \in \mathbb{R}^3 \,|\, V(\xi(t), \omega(t)) \le \gamma_2/\gamma_1\right\} \\
\Omega_\rho &= \left\{e_2 \,\|\, e_2\| < 0.2554\varepsilon_\rho\right\}
\end{aligned} \tag{69}$$

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Table 1. *The parameters of uncertainties and nonlinearities in simulation*

	ΔG	Δh
Case 1	$0_{3\times3}$	$\begin{bmatrix} 0.2\sin(6.28t/5) \\ 0.3\sin(6.28t/5) \\ 0.4\sin(6.28t/5) \end{bmatrix}$
Case 2	$\begin{bmatrix} 0.5 & 0 & 0 \\ 0.2 & 0.1 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}$	$\begin{bmatrix} 0.2\sin(6.28t/5) \\ 0.3\sin(6.28t/5) \\ 0.4\sin(6.28t/5) \end{bmatrix}$
Case 3	$\begin{bmatrix} 0.5 & 0 & 0 \\ 0.2 & 0.1 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}$	$\begin{bmatrix} \sin(6.28t/5) \\ 1.5\sin(6.28t/5) \\ 2\sin(6.28t/5) \end{bmatrix}$

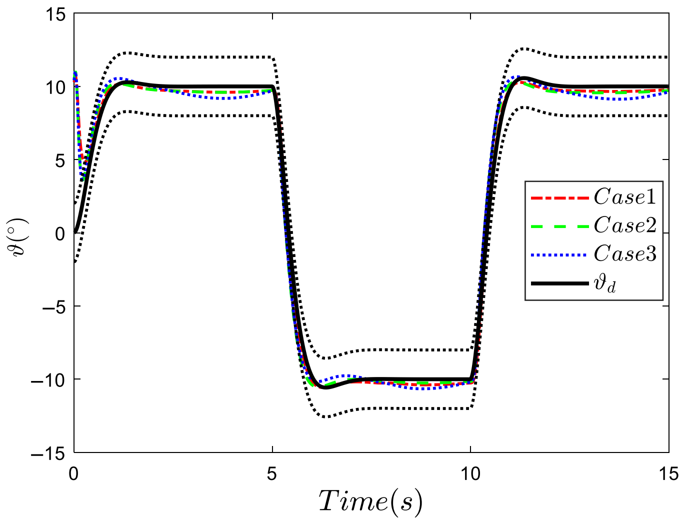

Figure 2. *The trajectory of the pitch angle of KKV under different conditions.*

According to Lemma 4, if $e_2(t) \in \Omega_f \cap \Omega_\rho$, the solutions of the KKV system $\left[e_0, e_1, e_2, e_3, \Delta\tilde{G}, \tilde{\Theta}\right]$ are all bounded. If $e_2 \notin \Omega_f \cap \Omega_\rho$, $\dot{V} < 0$ and $V(t)$ will finally converge to set $\Omega_f \cap \Omega_\rho$. In addition, due to the boundedness of $e_0(t)$, according to Lemma 5 we can get that $\lim_{t\to\infty} e_1(t) = 0$, i.e. the tracking error of the KKV system converge to zero, which completes the proof.

4.0 Simulation study

In this section, a numerical example is performed to verify the effectiveness of our algorithm. In order to illustrate the strengths of the proposed method, we compared it with disturbance observer based control (DOBC) and method without adaptive laws. Moreover, without losing generality, we investigated the performance of the proposed method under three different conditions.

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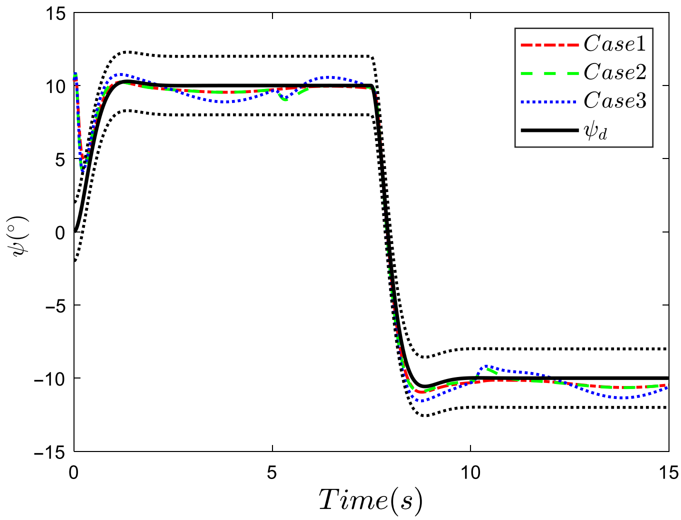

Figure 3. *The trajectory of the yaw angle of KKV under different conditions.*

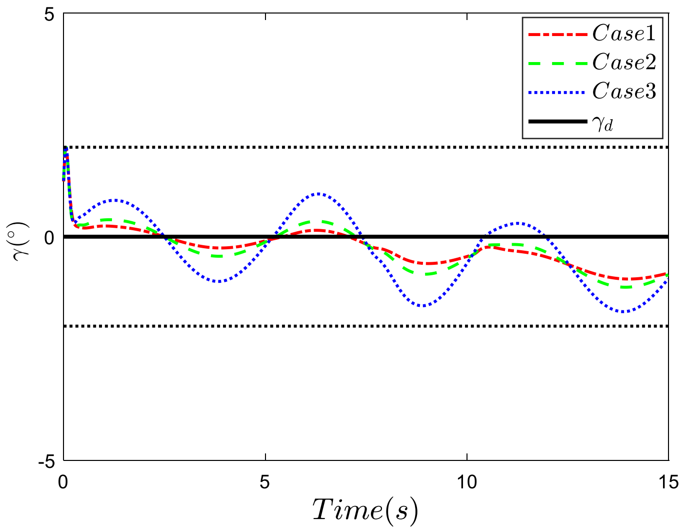

Figure 4. *The trajectory of the row angle of KKV under different conditions.*

The initial states are set as: $\begin{bmatrix}\vartheta & \psi & \gamma\end{bmatrix}^T = \begin{bmatrix}10/57.3 & 10/57.3 & 1/57.3\end{bmatrix}^T rad$, $\begin{bmatrix}\omega_z & \omega_y & \omega_x\end{bmatrix}^T = [0.5 \quad 0.5 \quad 0.5]^T rad/s$. The matrix of rotary inertias is $diag\left\{1 \quad 0.5 \quad 0.2\right\}$. The desired attitude angles are set as $\vartheta_d = 10square(0.2\pi t)$deg, $\psi_d = 10square(2\pi t/15)$deg, $\gamma_d = 0$ deg. To compute the value of $\dot{\beta}_1(t)$, $\dot{\beta}_2(t)$ in equation (45) and (61), two first-order filters are employed with the following formulation:

$$\tau\dot{\bar{\beta}}_i = -\bar{\beta}_i + \beta_i, \bar{\beta}_i(0) = \beta_i(0), i = 1, 2$$

where $\bar{\beta}_i$ is the output of the filter and $\tau > 0$ is a constant.

The disturbances are supposed to be $d_0(t) = [0.02 + 0.04\sin(6.28t/5)0.03 + 0.03\sin(6.28t/5)$ $0.04 + 0.02\sin(6.28t/5)]^T$. Some constants are set as $\Gamma_r = \varepsilon_\rho = 1, \eta_\Theta = \eta_G = 5, \sigma_\Theta = 0.9, \sigma_G = 0.1$. The unmeasured state is $\dot{\eta}(t) = -3\eta(t) + \vartheta(t)\,\psi(t)\,\gamma(t) + \omega_x(t)\,\omega_y(t)\,\omega_z(t)$, $\eta(0) = 0$ while

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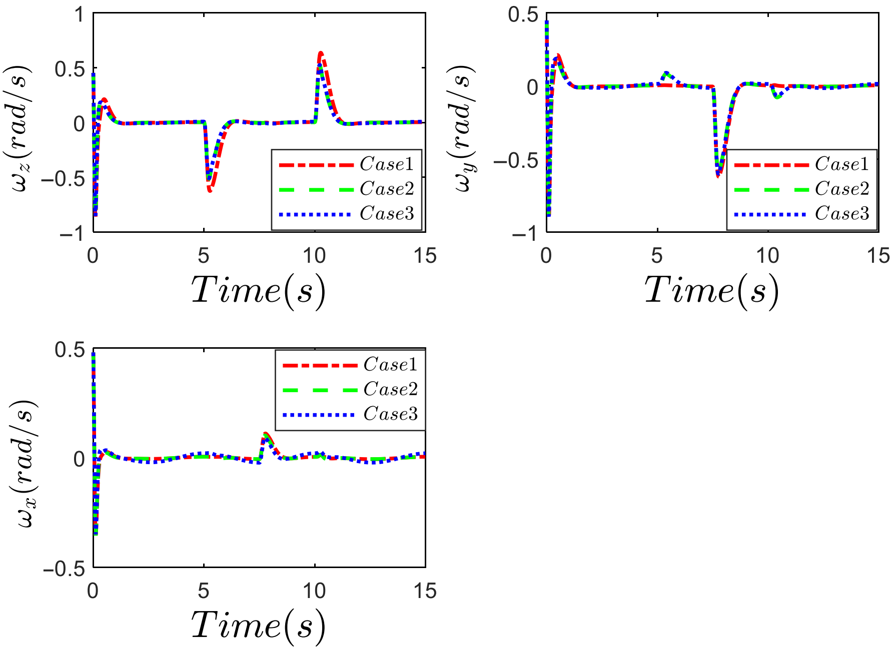

Figure 5. *The trajectories of the angular velocities of KKV under different conditions.*

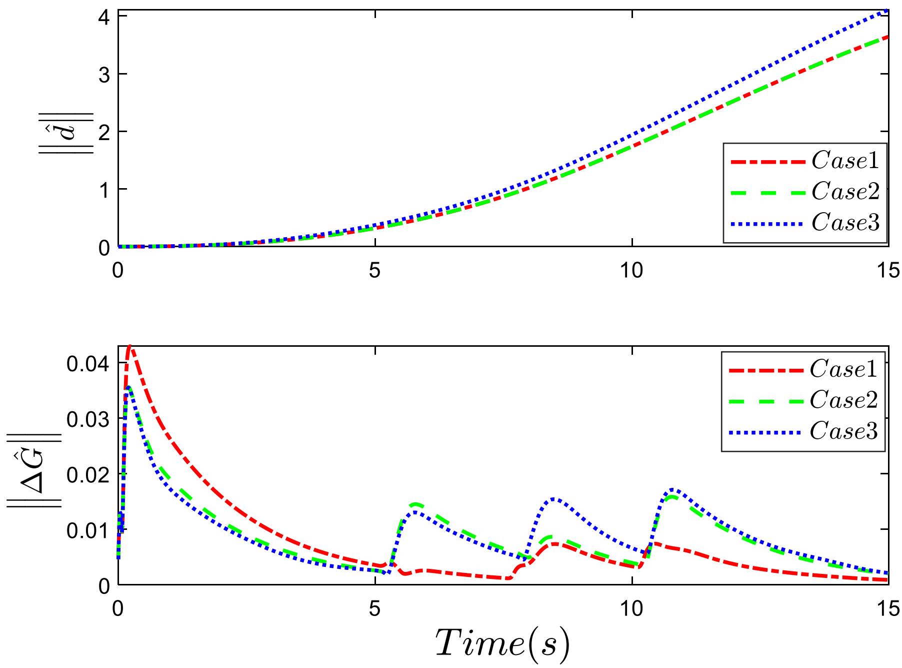

Figure 6. *The trajectories of the adaptive parameters under different conditions.*

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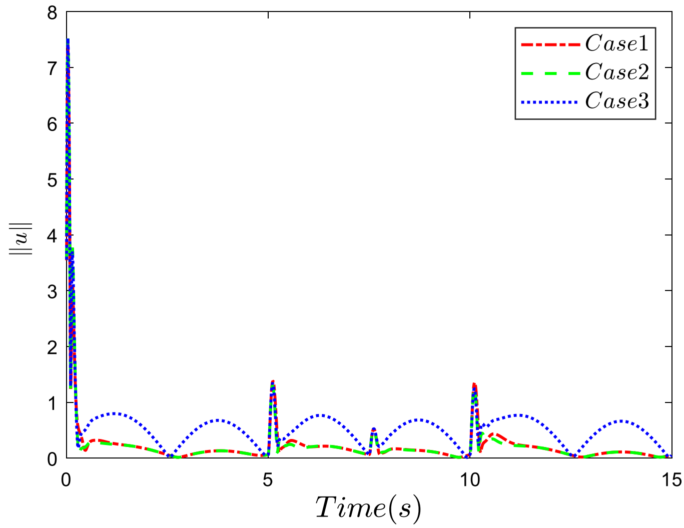

Figure 7. *The trajectories of the norm of the control signal under different conditions.*

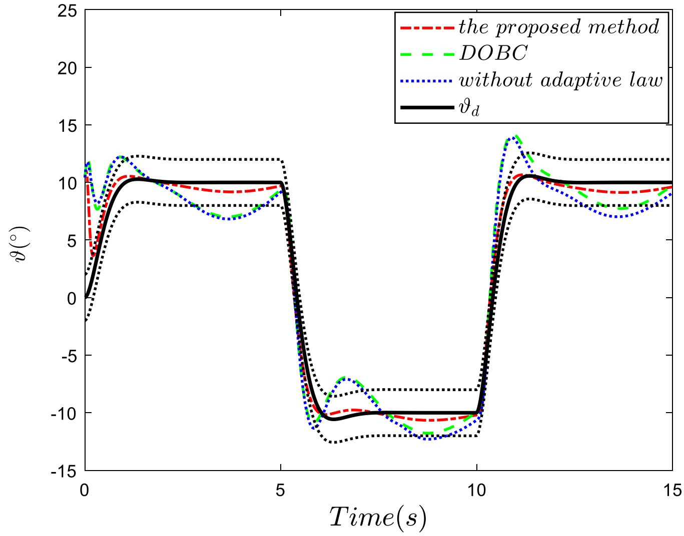

Figure 8. *The trajectory of the pitch angle of KKV with different methods.*

the dynamic signal is $\dot{r}(t) = -5r(t) + \xi^T(t)\,\xi(t) + \omega^T(t)\,\omega(t)\,, r(0) = 0$. The uncoupled uncertainties are $\chi(\xi(t)\,,\omega(t)\,,\eta(t)) = 0.5\xi(t)\sin(t) + \eta(t)(\xi(t) + \omega(t))$. The output constraints are $y_{up} = \xi_d + [2, 2, 2]^T$, $y_{low} = \xi_d - \begin{bmatrix} 2 & 2 & 2 \end{bmatrix}^T$. The non-affine input is set as $g[u(t)] = g \cdot u(t)$, $g = diag(\tanh(u_1), \tanh(u_2), \tanh(u_3))$. The parameters of uncertainties and nonlinearities are shown in Table 1.

The gains of the controller are $k_\xi = 5, k_{\xi 0} = 1, k_\omega = 20, k_3 = 30$ and the gains of STDO are set as $k_d = k_{p1} = k_{p2} = 0.1$.

The simulation results of the proposed method under different conditions are showed in Figs. 2–7. Figures 2–4 show the trajectories of the roll angle, yaw angle and pitch angle separately, from which we can see that the system states track the desired signals properly in the presence of output constraint as

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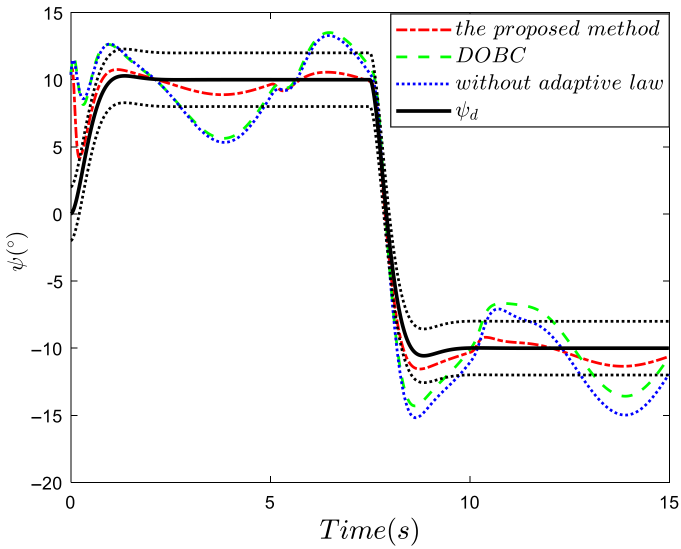

Figure 9. *The trajectory of the yaw angle of KKV with different methods.*

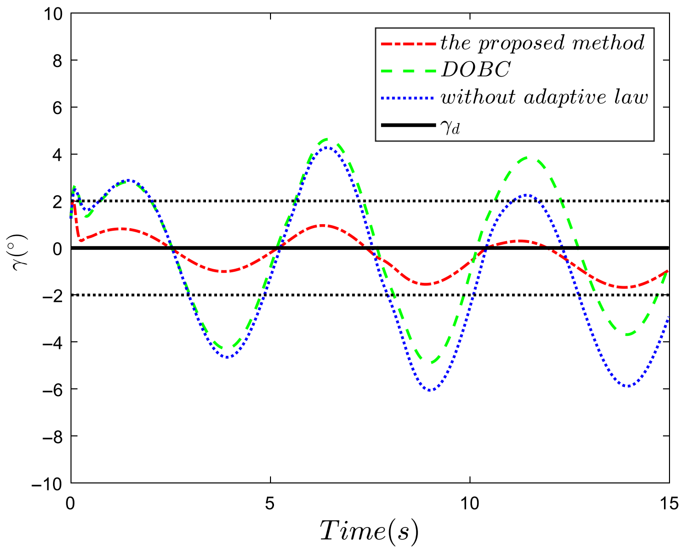

Figure 10. *The trajectory of the row angle of KKV with different methods.*

well as non-affine dynamics and the output of the system satisfy the output constraint all the time. The stability and robustness of Fig. 5 displays the trajectories of the angular velocities while Fig. 6 exhibits the trajectories of the adaptive parameters. The trajectories of the control signal can be seen in Fig. 7. In conclusion, the KKV system performs appropriately with the aid of the proposed method in spite of the non-affine dynamics as well as the output constraint. Thanks to the proposed algorithm, the tracking errors of the attitude angles can be limited within 2 deg and the disturbances are well suppressed. The stability and robustness of the system can be seen explicitly.

The simulation results of the comparison between the proposed method and two other methods are displayed in Figs. 8–10. It can be concluded that without the designed control approach, the disturbances

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and uncertainties cannot be suppressed well so that the system states miss the desired signals to some extent. Moreover, other control algorithms cannot guarantee the restriction of output signals. The high efficiency of the proposed method can be seen clearly.

5.0 Conclusions

In order to solve the control problem of a class of non-affine KKV systems with output constraint suffering from unmodeled dynamics, this paper proposes an adaptive neural output-constrained control algorithm. By introducing an integral process to the design of the controller, the non-affine input signal can be obtained. Moreover, benefiting from the presented nonlinear mapping, the requirement of output constraint can be met. From the simulation results we can tell that the system states can track the desired signals under different conditions with the proposed method and the algorithm has remarkable advantages compared with other algorithms.

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