Plumwood, and Brady's *Relevant Logics and Their Rivals*, Ridgeview, 1982, of E as the logic of entailment discussed. Without, then, a clear account of why a "bad" proof plan such as one which starts off assuming p in order to obtain $q \lor \neg q$, cannot still be a valid entailment, I at least found motivation for thinking that the Fitch calculus for E separates the true states of entailment from false ones, somewhat wanting.

The philosophical interpretation of E as a theory of entailment is given from part 2 and onwards. Mares starts by giving an illuminating account of both Tarski's and Kuratowski's axioms for a closure operator and in line with the Anderson–Belnap take on from-ness, opts in favor of Kuratowski's axiom that $\mathcal{C}(\emptyset) = \emptyset$. This, then, is a different theory than that of proof plans which does yield the set of logical theorems of E from the empty set. The two theories are, however, connected by way of Mares' theory of theory closure. The notion of a closure operator is then expanded to include conjunction so that, by definition, for any set of formulas $\Gamma \cup \{A, B\}$, $A, B \in \mathcal{C}(\Gamma)$ if and only if $A \land B \in \mathcal{C}(\Gamma)$. A weak logic called *Tarski Logic*, TL, is then introduced which has an entailment conditional $\rightarrow$ which reflects, in a sense introduced in Chapter 5, the rules of the a conjunctive-infused notion of a closure operator. The first proper consequence relation $\mathcal{C}_{TL}$—defined so that $B \in \mathcal{C}_{TL}(\Gamma)$ if and only if $A \rightarrow B$ is a logical theorem of TL, where A is some conjunction of the formulas in Γ—is then introduced. Although Mares' claim (p. 127) that Tarski's axioms *determine the logic TL* seems rather audacious—Tarski's axiom do not involve any connective—there is a pleasing naturalness to Mares' way of building up a theory of consequence. This is the part of the book that I found most interesting, although a discussion of similar attempts—especially that of the relevant "rival" attempt found in *Relevant Logics and Their Rivals*—would have made Mares contribution even more rewarding.

Mares main idea is that the theory of entailment is a theory of theory closure. Mares generalizes the closure operator $\mathcal{C}_{TL}$ to allow for $\mathcal{C}_a$, where a, then, is itself any theory of entailment, that is, an object which validates $A \land B$ if it validates both conjuncts, and which validates B if it validates A and $A \rightarrow B$ is a logical theorem of the logic in question. Intuitive rules governing such closure operators are then argued to yield the conjunction-implicative fragment of E. The semantics Mares ends up with advocating is a version of Kit Fine's semantics for relevant logic first presented in Fine's *Models for Entailment*, *Journal of Philosophical Logic*, vol. 3 (1974), pp. 347–372, and which has gained renewed interest in recent years. Part 3 of the book lays out this semantics showing how it can be used to interpret and justify the Fitch system for E, and with it, Mares' proof plan idea of entailment. I should hasten to note that Mares expands upon Fine's semantics in two important direction: negation is given a modal incompatibility reading so that $\neg A$ is to hold in a theory a just in case A fails to hold in every a-compatible theory. The semantics is then expanded to deal with quantifiers using the model theory developed by Mares himself and Robert Goldblatt.

The book ends with the last chapter of part 3 entitled *Entailment and Reasoning* dealing with various topic relating to theories which don't contain any entailment statements. Such theories end up being closed under the logical rules of FDE. Excluded middle and disjunctive syllogism need not hold, therefore, for such theories, and so if one is to model a classical theory, Mares advocates the use of "side conditions" and an extra "internal" conditional. The chapter reads much like a note for future work, which I think it, and indeed the book as a whole, will inspire to.

Tore Fjetland Øgaard

University of Bergen, Department of Philosophy, Postboks 7805, 5020 Bergen, Norway.
E-mail: Tore.Ogaard@uib.no.

Samuele Iaquinto and Giuliano Torrengo. *Fragmenting Reality: An Essay on Passage, Causality and Time*. Bloomsbury Academic, London, 2022, x + 208 pp.

Here is a problem for anyone who believes reality to be fundamentally tensed: time passes. Future becomes present, then past; the present itself constantly changes. Contradictions arise amongst tensed facts: what to do, then? We could cut this Gordian knot and claim that only one instant exists, and the facts that obtain in it; but what is to be made of the passage of time itself, if we never have more than one instant to account for it? (A bootstrapping metaphor is suggested by the authors: Baron Munchausen pulling himself out of a swamp by his own hair.)

This book offers a novel alternative: to concede that reality is not globally coherent, but parsed into separate fragments. To this goal, a new position called Flow Fragmentalism is discussed, which builds on previous literature, and ultimately Fine (2005). The crucial notion, as one can expect, is that of a fragment: an internally coherent collection of present-, past-, and future-tensed facts; most fragments will comprise the same content, but differently tensed: the difference lies in which instant a fragment considers to be present. In other words, each fragment represents the entire temporal dimension as seen from a particular perspective. (There can be overlap between fragments: if JFK is murdered in 1963, two fragments centered respectively in 1964 and 1974 will share the tensed fact [JFK was murdered]; this is key to one of the more ingenious passages of the book, where a B-like order of fragments is grounded through an internal relation of partial overlap.)

Those familiar with the literature will recognize in Flow Fragmentalism a peculiar middle ground between traditional alternatives; on the one hand, the position recognizes primitive passage/tense, but on the other it arranges the temporal dimension in a B-like ordering of fragments. In other words: there is a privileged present, but not a unique one, as each fragment is centered on a unique perspective. This multiplicity of perspectives plays a special role, and this is where we move to the "flow" part of Flow Fragmentalism: although the passage of time, as codified by tense operators, is ultimately taken as primitive, there is an explanatory effort on display here, dubbed "Fragmentalist Flow": "when a future-tensed fact obtains within a fragment F, this is so because reality will contain the corresponding present-tensed version of it – that is, because a fragment that comes 'after' F contains it" (p. 35); and the same goes for past-tensed facts. Although reality is globally incoherent, there is coherence at the local level of fragments, and even harmony in the obtainment of tensed facts across fragments.

The metaphysics-cum-semantics framework of Flow Fragmentalism, as presented in Chapter 1, is put to work for a variety of purposes, such as a treatment of the openness of the future compatible with bivalence in Chapter 2; a treatment of causation in Chapter 3—a problematically external cross-fragment relation (as to why that would be a problem, I will add something below); and in Chapter 5 a treatment of time travel, primarily backwards causation both of the Novikovian (viz. non-past-altering) and non-Novikovian variety. Chapter 4 provides a much-needed amendment of the theory that is compatible with relative simultaneity.

There is much that this book can offer to the interested reader: Flow Fragmentalism promises to be a fertile breeding ground for discussion, whose ramifications cannot exhaustively be discussed here. But the metaphysical underpinnings are particularly intriguing, and two points are worth discussing.

The first point concerns the pluralistic nature of its philosophical toolkit. Here we have two equifundamental and irreducible "reality-building" operations: constitution, which is absolute, and obtainment, which is relative to fragments (they are not free-floating concepts, however: a fact constitutes reality iff it obtains in some fragment; which excludes cross-fragment external relations such as brute before/after temporal orderings, or standard event-causation). This duplication is crucial to the theory, but it can also lead to some interesting puzzles. Consider a much-coveted feature of the position: compatibility with

(ontic) presentism. Existential facts about past entities constitute reality, but only obtain in certain fragments. Hence the question: is a question of ontology such as presentism decided by matters of constitution or obtainment? We are of course free to disambiguate presentism into two theses, "constitution-presentism" and "obtainment-presentism"; as the authors admit, the issue of the compatibility with presentism tout court becomes somewhat arbitrary at this point, or perhaps relative to more specific explanatory purposes (p. 50). However, one may claim that constitution-presentism is more deserving of the label. Look at it this way: constitution and obtainment are given different weight within the language of the theory. Obtainment, formalized as a sentential operator, is baked deep into the theory, down to the semantics of propositional connectives and tense operators; e.g., conjunction displays an abnormal behaviour both inferentially and semantically, which is why although reality is incoherent across fragments, there are no true contradictions; "$p \wedge q$" is equivalent to "$[p]$ and $[q]$ obtain together". Constitution, on the other hand, can be simply expressed with a redundant "in reality" operator; as a matter of fact, the authors avoid using any formal device to express constitution at all. This is in line with a conception of constitution as "the 'unadorned' way things are the case" (p. 25), but it would then seem unwarranted to express presentism through existential quantification prefixed by an obtainment operator rather than existential quantification tout court. Even in the rest of the book, virtues and vices of presentism often emerge as relative to constitution rather than obtainment; in Chapter 3 the authors argue that their account of causation does not suffer any presentist-related problem, given that, as per tenseless constitution, Flow Fragmentalist allows reference to past entities (p. 113). Ultimately Flow Fragmentalism presents a challenge: how to manage the interplay between constitution and obtainment vis-à-vis its metaphysical commitments, e.g., ontic presentism.

A second foundational point concerns the explanation of the passage of time. The authors are not targeting a strawman here; a desideratum is for the resulting explanation not be reductionist, viz. one in which genuine passage is exhausted by the fact that fragments contain incompatibly tensed facts (p. 33). Something more is required. What could that be? The authors' answer is Fragmentalist Flow, but one may wonder in what sense this is an explanation of the passage of time. A good way to formulate this worry is constituted by the question: what kind of explanation is offered here? Talk of "metaphysical explanation" (p. 3) might suggest that we are dealing with some kind of constitutive grounding explanation; but we must be careful: facts of distinct fragments are distinct and irreducible existences—indeed it is crucial for the position that they are. It is troubling to think of the proposed framework as an "extensionalizing" apparatus (for lack of a better word) in which future and past tense are ultimately constituted by the present tense of other fragments. Another alternative could be one in which past-tensed facts in a fragment are determined, in a non-causal and non-constitutive yet more-than-modal way, by present facts in a 'later' fragment. One must keep in mind that tense itself is a primitive of the theory, and the proposed explanation is merely meant to illustrate the behaviour of tense-talk in certain contexts (p. 26). Perhaps this is all Fragmentalist Flow is all about; but in that case, one may wonder what more there is to it than a two-way merely modal correlation (and thus what is the difference with the converse explanation dubbed "Presentist Flow", p. 38).

Relatedly, one may wonder whether any explanation of the flow of time requires any individual tensed fact as an explanandum; a better candidate would be the succession of such facts: now the apple is ripe, and now the apple was ripe—perhaps more in the ballpark of the so-called "Priorian Flow" (p. 8). A subtle dialectical point that emerges multiple time in the book is whether, given (primitive) tense talk, there is anything more requiring an explanation in the passage of time. Unlike other tense realists, Flow Fragmentalism does not take refuge in a what-more-do-you-want attitude (pp. 27–28); although there are times where the authors must concede that too much is being asked, such as in the discussion on the so-called "update

test" (pp. 43–45). Ultimately, this is another way in which the explanandum at hand, the passage of time, is itself a moving target.

Acknowledgments. I would like to thank Ciro De Florio for many comments and discussions on the topics of this book.

[1] Fine, K. (2005). *Tense and reality*. In K. Fine, Modality and Tense (p. 261–320). Oxford University Press.

Lorenzo Azzano
Università Cattolica del Sacro Cuore, Largo Fra Agostino Gemelli, 1, 20123, Milan, Italy.
E-mail: lorenzo.az@hotmail.it

Thomas Piecha and Peter Schroeder-Heister. *Incompleteness of Intuitionistic Propositional Logic with Respect to Proof-Theoretic Semantics.* **Studia Logica**, vol. 107 (2019), no. 1, pp. 233–246.

Alexander V. Gheorghiu, Tao Gu, and David J. Pym. *Proof-Theoretic Semantics for Intuitionistic Multiplicative Linear Logic.* **Automated Reasoning with Analytic Tableaux and Related Methods,** Revantha Ramanayake and Josef Urban, Lecture Notes in Computer Science, vol. 14278, Springer, Cham, pp. 367–385.

Hermógenes Oliveira. *On Dummett's Pragmatist Justification Procedure.* **Erkenntnis**, vol. 86 (2021), no. 2, pp. 429–455.

Failed attempts to provide semantics for intuitionistic logic have a history of leaving behind useful structures. For example, Kleene's realizability and Medvedev's finite problems semantics are of theoretical interest, in spite of validating nonintuitionistic theorems

Proof-theoretic validity is another example that can be added to this list. Proof-theoretic validity first appeared in Prawitz's work in the 1970s via the following definition:

DEFINITION 1 (Schroeder-Heister). Let S be a set of inference rules containing only atomic formulas, called an *atomic base*. An argument is a proof-like structure and an argument $\mathcal{D}$ is a *valid argument* if it is S-valid for all S, where *S-valid* is defined as follows:

(1.1) If $\mathcal{D}$ is a closed argument constructed from rules in S then it is S-valid.
(1.2) If $\mathcal{D}$ is a closed argument ending in an introduction rule of the intuitionistic propositional calculus, then it is S-valid if its immediate subarguments are S-valid.
(1.3) If $\mathcal{D}$ is a closed argument which does not end in an introduction rule then it is S-valid if it reduces to an S-valid argument.
(1.4) If $\mathcal{D}$ is an open argument concluding φ with open assumptions $\varphi_0, \ldots, \varphi_n$ then it is S-valid if for all atomic systems S' extending S, and all closed S'-valid arguments $\mathcal{D}_0, \ldots, \mathcal{D}_n$ of $\varphi_0, \ldots, \varphi_n$, the following argument is S'-valid:

$$
\begin{array}{ccc}
\mathcal{D}_0 & \ldots & \mathcal{D}_n \\
\varphi_0 & \ldots & \varphi_n \\
& \mathcal{D} & \\
& \varphi &
\end{array}
$$

Prawitz's conjecture states that all and only the theorems of intuitionistic propositional logic (IPC) have valid arguments. However, which formulas have valid arguments is sensitive to the treatment of the atomic base. We can adjust what counts as an inference rule and what counts as an extension. For example adding atomic rules which allow the discharge