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Damping and the period ratio $P_1/2P_2$ of non-adiabatic slow mode

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Abstract. We investigate the combined effects of thermal conduction, compressive viscosity and optically thin radiative losses on the period ratio, $P_1/2P_2$, (P_1 is the period of the fundamental mode and P_2 is the period of its first harmonic) of a slow mode propagating one dimensionally. We obtain the dispersion relation and solve it to study the influence of non-ideal effects on the period ratio. The dependence of period ratio on thermal conductivity, compressive viscosity and radiative losses has been shown graphically. It is found that the effect of thermal conduction on the period ratio is negligible while compressive viscosity and radiation have sufficient effects for small loops and large loops respectively.

1. Introduction

Coronal loop oscillations have recently become a subject of considerable observational and theoretical interest. Since the launch of SoHo and TRACE, many examples of both standing and propagating waves have been detected in a variety of solar structures. There are observational evidences of slow modes occurring as propagating waves (DeForest & Gurman 1998; Ofman *et al.* 1997, 1999; Robbrecht *et al.* 2001; De Moortel *et al.* 2002a,b; McEwan & De Moortel 2006). The standing slow mode oscillations in solar corona has been detected (Kliem *et al.* 2002; Ofman & Wang 2002; Wang *et al.* 2003, 2004). We are interested in the detection of multiperiods in loops. Multiperiods have been first reported in standing fast waves (Verwichte *et al.* 2004; Van Doorsselaere *et al.* 2007) and now very recently in slow modes (Srivastava & Dwivedi 2010). Since higher harmonics have lower wavelengths, they carry more detailed information about a structure and are more influenced by chromospheric structure or the gravitational scale height. Andries *et al.* (2005a) studied the ratio P_1/P_2 of the fundamental oscillation period, P_1, and its first harmonic, P_2, of a kink mode oscillation, showing that this ratio falls below 2. Srivastava & Dwivedi (2010) reported the period ratio of slow mode P_1/P_2=1.54 and 1.84. The observed tendency of period ratio, $P_1/2P_2$, to be less than unity has led to a number of researchers to assess the influence of various physical effects such as longitudinal and transverse density structuring, wave dispersion and gravitational stratification on the period ratio (Andries *et al.* 2005b; McEwan *et al.* 2006, 2008).

The effect of damping on the period ratio of slow mode has been studied recently by Macnamara & Roberts (2010). They discussed the role of thermal conduction and compressive viscosity but have not included optical thin radiation to study the effects on period ratio. So in this paper, we aim at investigating the joint effects of radiation, thermal conduction and compressive viscosity on the period ratio of non-adiabatic slow modes.

2. Model Equations and Dispersion Relation

We model a single coronal loop tied at footpoints located in photosphere. We suppose that the wavelengths are much smaller than the gravitaional scale height i.e. gravitaional effects are neglected. We take the longitudinally propagating waves as purely one dimensional wave. The basic linear MHD equations describing plasma motion in 1D are (Macnamara & Roberts, 2010)

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial v_z}{\partial z} = 0, \tag{1}$$

$$\rho_0 \frac{\partial v_z}{\partial t} + \frac{\partial p}{\partial z} = \frac{4}{3}\nu \frac{\partial^2 v_z}{\partial z^2}, \tag{2}$$

$$\frac{\partial p}{\partial t} - \frac{\gamma p_0}{\rho_0}\frac{\partial \rho}{\partial t} - (\gamma-1)\kappa_\parallel \frac{\partial^2 T}{\partial z^2} + (\gamma-1)(L+\rho_0 L_\rho)\rho + (\gamma-1)\rho_0 L_T T = 0, \tag{3}$$

$$\frac{p}{p_0} = \frac{\rho}{\rho_0} + \frac{T}{T_0}. \tag{4}$$

Here ρ, p, v and T represent perturbed density, pressure, velocity and temperature respectively whereas ρ_0, p_0 and T_0 represent equilibrium density, pressure and temperature respectively. ν is the coefficient of compressive viscosity of the form $\nu = \nu_0 T^{5/2}\mathrm{kgm^{-1}s^{-1}}$ with $\nu_0 = 10^{-17}$. γ is the ratio of specific heats. Equation (3) is linearized energy equation and present form is due to non-ideal effects (radiation losses, thermal conduction and heating). We take thermal conduction to act purely along the z-axis setting $\kappa_\parallel = 10^{-11}T^{5/2}\mathrm{Wm^{-1}K^{-1}}$ as thermal conduction is strongly suppresed across a magnetic field (Spitzer, 1962). $L(\rho, T)$ is the net heat- loss function per unit mass and the time having the form $L(\rho,T) = \chi\rho T^\alpha - h$, where χ and α are piecewise continuous functions depending on the temperature (Hildner 1974; Carbonell *et al.* 2004). The heating term h is assumed fixed that maintains the equilibirium temperature without contributing to the linearized perturbation equations. L_T and L_ρ are the partial derivatives of heat-loss function with respect to temperature and density respectively i.e. $L_T = (\partial L/\partial T)_\rho, L_\rho = (\partial L/\partial \rho)_T$.

Fourier analysing equations (1)–(4) as exp $(i\omega t - k_z z)$, we obtain the following dispersion relation

$$\omega^3 - i\left(\frac{4}{3}\frac{\nu c_s^2}{\gamma p_0} + \frac{(\gamma-1)\kappa_\parallel T_0}{p_0}\right)k_z^2\omega^2 - \left(c_s^2 + \frac{4}{3}\frac{\nu c_s^2}{\gamma p_0}\frac{\gamma(\gamma-1)\kappa_\parallel T_0}{\gamma p_0}k_z^2\right)k_z^2\omega$$
$$+i\frac{(\gamma-1)\kappa_\parallel T_0}{\gamma p_0}c_s^2 k_z^4 + (\gamma-1)\rho_0 L_T\left(-\frac{4}{3}\frac{k_z^2\nu\omega}{p_0\rho_0} - i\frac{\omega^2}{p_0} + i\frac{k_z^2}{\rho_0}\right)T_0$$
$$-i(\gamma-1)(L+\rho_0 L_\rho)k_z^2 = 0. \tag{5}$$

If we solve dispesion relation (5) for complex freequency $\omega = \omega_r + i\omega_i$ with $k_z L = \pi/2$ and π to obtain $\omega_1 = \mathrm{real}(\omega)$ and $\omega_2 = \mathrm{real}(\omega)$ respectively, the fundamental period, P_1 can be obtained as $P_1 = 2\pi/\omega_1$ and period P_2 for first overtone as $P_2 = 2\pi/\omega_2$. So, $\frac{P_1}{2P_2} = \frac{\omega_2}{2\omega_1}$.

We now introduce the dimensionless parameters namely thermal ratio, d, radiation ratio, r (De Moortel \$ Hood, 2004) and viscosity measure ϵ as

$$d = \frac{(\gamma-1)\kappa_\parallel T_0\rho_0}{\gamma^2 p_0^2\tau} = \frac{1}{\gamma}\frac{\tau_s}{\tau_{cond}}, \quad r = \frac{(\gamma-1)\tau\rho_0^2\chi T_0^\alpha}{\gamma p_0} = \frac{\tau_s}{\tau_{rad}}, \quad \epsilon = \frac{4}{3}\nu\frac{c_s}{\gamma p_0 L}.$$

where τ_s is sound travel time. The ratios d and r are expressed in terms of time scales τ because most observed waves have a prescribed period, which we take as τ rather than a prescribed loop length. Using standard coronal values for all the variables $T_0 = 10^6 K$, $\rho_0 = 1.67 \times 10^{-12}\mathrm{kg\,m^{-}3}$, $\kappa_{\|} = 10^{-11}\mathrm{T_0^{5/2}Wm^{-1}deg^{-1}}$, $\tilde{\mu} = 0.6$, $\mathrm{R} = 8.3 \times 10^3\mathrm{m^2s^{-1}deg^{-1}}$, $\gamma = 5/3$, $\tau = 300\mathrm{s}$. gives a value $d = 0.025$ for thermal ratio and $r = 0.06$ for radiation ratio. If we set $\Omega = \omega/k_z c_s$, the dispersion relation (5) in non-dimensional parameters becomes

$$\Omega^3 - i(V + \gamma D + \alpha\gamma R)\Omega^2 - (1 + \gamma V D + \alpha\gamma V R)\Omega + iD - i(2 - \alpha)R = 0, \qquad (6)$$

where

$$V = \epsilon k_z L, \quad D = dk_z L \quad \text{and} \quad \mathrm{R} = \frac{\mathrm{r}}{\mathrm{k_z L}}.$$

When $\epsilon = d = r = 0$, then $\Omega = 0$ or $\Omega = \pm 1$. Therefore $\omega = k_z c_s$ is a solution of equation (6). The period ratio $P_1/2P_2 = \omega_2/2\omega_1 = [(\pi c_s/L)/(2\pi c_s/2L)] = 1$.

3. Results and Discussion

We solve dispersion relation (6) to find out the period ratio $P_1/2P_2$ of non-adiabatic slow mode in order to discuss how thermal conductivity, viscosity and radiative losses bring about a shift in the period ratio from unity. In the absence of thermal conduction and compressive viscosity, the variation of period ratio $P_1/2P_2$ with radiation parameter r is shown in figure 1(a). Figure 1(a) depicts the behaviour of period ratio with radiation

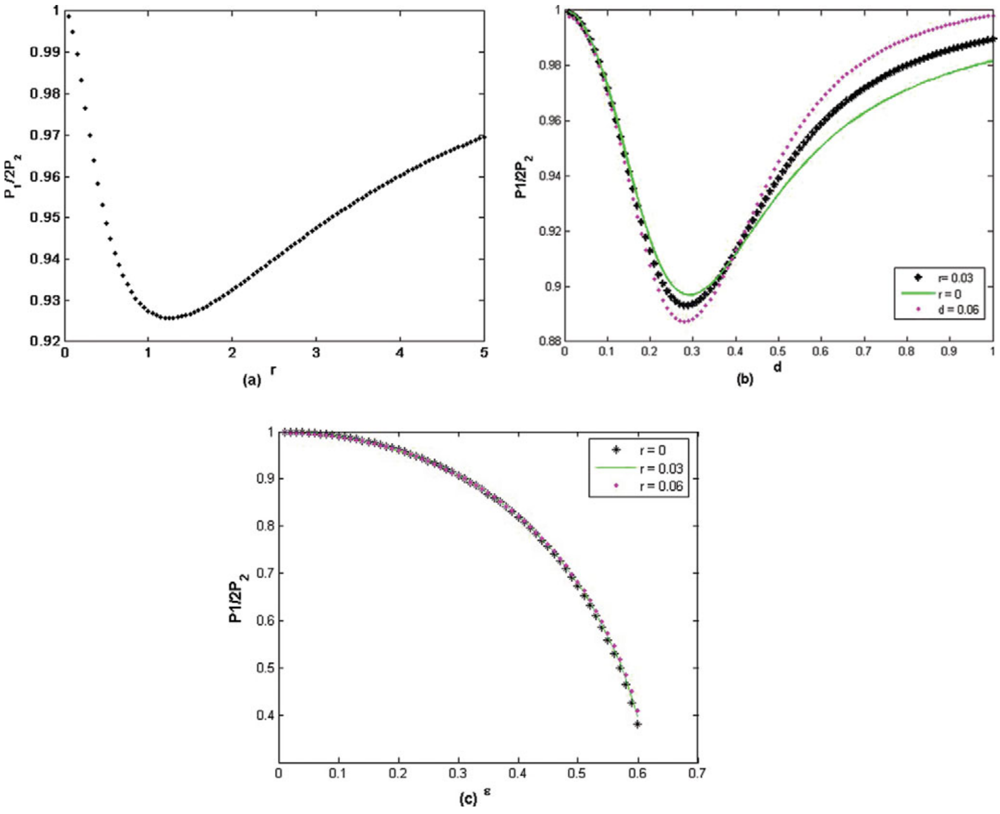

Figure 1. Period ratio as a function of (a) radiation parameter r, (b) thermal conduction parameter d for different values of radiation parameter r, and (c) compressive viscosity parameter ϵ for different values of radiation parameter r.

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ratio. The period ratio is unity when $r = 0$ as expected and decreases uniformly to a minimum value 0.9257 at $r = 1.25$. Thereafter the period ratio increases and moves towards unity for sufficiently large values of r.

Figure 1(b) shows the variation of period ratio with thermal conduction parameter d for three different values of radiation parameter $r = 0.0, 0.03$ and 0.06. The period ratio decreases to minimum values 0.8669, 0.8832 with the increaes in d up to 0.29 for $r = 0$ and $r = 0.03$ respectively. For $r = 0.06$, the minimum value of peroiod ratio 0.8873 is found at $d = 0.28$. After attaining minimum value, period ratio tends to increase and returns to unity for sufficiently large d. It is interesting to note that the period ratio is almost same at $d = 0.41$ for all values of radiation parameter. For radiation parameter $r = 0.06$, the period ratio is less than the period ratios for r=0.0 and 0.03 upto the values of thermal measure d from 0.0 to 0.41 whereas the period ratio is higher than those for $r = 0.0$ and 0.03 for the values of d from 0.41 onwards.

Figure 1(c) depicts the departure of period ratio from unity as a function of compressive viscosity for different values of radiation parameter. It is observed that when we consider the compressive viscosity together with radiation ratio, the effect of small values of radiation parameter does not influence the departure of period ratio from unity.

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