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Taking stock

It is hoped that we have learned rather a lot about string theory in this book, and that the role of D-branes and other extended objects has been fascinating, entertaining, and instructive. It was with great pain that we had to sacrifice a tremendous amount of material in order to keep this book close to a sensible length, while retaining enough to succeed in telling a coherent story.

It is tempting to sit and reflect upon what great lessons we have learned, although it is not clear that this is a useful exercise at this stage, so we will be brief in our remarks. The main and most unambiguous lesson is that extended objects are vitally important to our understanding of string theory, and possibly (probably) whatever the final form of the fundamental quantum theory of space and time turns out to be. While extended objects are universally accepted as important, it is still (at a stretch) a matter of taste whether someone wants to go further and accept that it is unambiguously true that ‘string theory is not a theory of strings’. The author believes it to be so, but will not insist that the reader take a position, since it seems that nobody can yet say what string or M-theory actually are theories of.

Whatever the final theory turns out to be, and whether or not once it is found it turns out to be directly relevant to nature at all, it is clear that we have many new tools to work with which should keep us busy for some time to come in various areas. There are still many very specific questions that might be partly answered with the present technology which would have considerable benefits. For example, various gravity duals of increasingly complex gauge theory phenomena are being found from time to time, and a useful body of knowledge is being assembled about how such tools work in some detail. As a by-product, valuable lessons about strongly coupled gauge theory are being learned. This is despite the lack

of a viable technology for studying string theory in R–R backgrounds, (an issue discussed at the end of chapter 19) with which considerable leaps in our understanding will be likely.

A topic that we have not touched upon at all is the whole ‘*Brane World*’ discussion. This topic fruitfully borrows many ideas from the string theory constructions which we have discussed here, applying them to phenomenology. One class of models is that we simply live on a three-brane embedded in higher dimensions, to which are confined the gauge interactions which give rise to the standard model physics (so this is rather like a D-brane). Meanwhile, gravity lives in the whole spacetime and its relative weakness as compared to the other forces is apparently then attributable to the fact that it lives in more dimensions³³⁵. The other sort of scenario is again the idea that we live on a brane in higher dimensional spacetime, but that gravity is localised in the neighbourhood of the brane, due to the properties of the larger spacetime³³⁶.

These both lead to interesting toy models of our world, and may find a home one day within a fundamental theory. The efforts to move these ideas forward are often mistakenly identified with research in string theory, but it should be clear that although there is some overlap, these are entirely different endeavours. It is safe to say that at the time of writing the many models which are being studied in these genres are not anywhere near constrained enough by being embedded in the (relatively) tight framework of string or M-theory (as far as we understand the latter two). On the other hand, this might not turn out to be entirely a bad thing, but it is too early to say.

Both of the topics above (and much of the content of the body of research described in this book) rely on the fact that although we do not know the details of the theory, we can learn a lot about things by working with low energy truncations. Of course, any small child educated in the modern field theory era will rightly immediately speak up at this point and mention that this is not special to string theory, but is a foundation of quantum field theory in general. The remarkable thing that seems to be available to us in the stringy arena is the wide variety of different ways of embedding various low energy phenomena into string and M-theory. This inevitably leads to new effective and often geometrical tools for studying these low energy phenomena, and sometimes powerful dual descriptions of the same physics, as we have seen many times.

This urges us to begin looking around for more examples, and possibly applications to other fields of physics where strongly coupled phenomena and interesting effective field theories of various sorts abound, like condensed matter physics. An example of this is the recent activity in embedding the physics of the quantum hall effect into string theory³³⁴,

following on from recasting it in terms of tools (such as non-commutative geometry³³⁸) sharpened in the string context. This may not be the only class of examples, and in fact there may be a lot to gain by deliberately exploring connections. There is also of course great likelihood that some of these connections will enrich understanding of string and M-theory.

While it is all well and good to discuss the elegant tools that we have uncovered, perhaps for useful application to difficult (nearly) phenomenological questions, we must not sidestep the issue of the direct search for a definition of M-theory. It becomes apparent when preparing a book of this sort, which surveys a large portion of the field, that it is perhaps not surprising at all that we have not yet stumbled on the definition. At nearly every turn of a page there seems to be a host of interesting unexplored connections and directions which might lead to interesting new physics. To save embarrassment, no attempt will be made to list them, since the large number may simply be a result of the author's ignorance, rather than his profound insight. In any case, the reader has probably their own list to be getting on with.

There are a number of seemingly very interesting features of D-branes which apparently have very deep roots, however. Whether or not they are a signal of the right variables for a dynamical formulation of the underlying theory is quite possibly an entirely different matter, but they are intriguing. For example, it is very striking that D-branes seem to supply the right variables for many very elegant descriptions of various geometries such as that of instantons, monopoles, the moduli spaces of these objects, ALE spaces, etc. Essentially, these 'right variables' are all of the charged hypermultiplets of various sorts which constitute the D-branes' collective coordinates, together with an appropriate set of constraints and/or projections.

Further to this (and closely related of course) is the fact that the world-volume couplings of D-branes seem to be naturally written in terms of very powerful geometrical objects: characteristic classes of various sorts, which enable them to enumerate topology so naturally*. This is an awfully generous circumstance and makes one wonder whether we should look deliberately for other powerful tools by starting with some of our other favourite geometrical or topological devices (other characteristic classes, etc.) and attempting to make them dynamical, perhaps by designing a world-volume interaction around them. Of course, it is not clear what the best candidates are, and what guiding principle one should use. Furthermore, this has in many senses been tried before, but perhaps

* Part of the outcome of this was the K-theory¹¹³ description of D-branes alluded to but (sadly) not described in any detail in this book.

some of this approach could be revisited in the light of recent developments.

An interesting endeavour which is firmly underway at the time of writing is the revisitation of string field theory. As was clear to some people long ago (back at the time when string duality was generating a heady excitement, and hasty and unkind (but perhaps forgivable) things were said about string field theory), it may yet have its uses. One such use has turned out to be the detailed study of local portions of the potential in which the string theories which we know are special vacua. Various ideas are afloat concerning the suggestion that the decay of unstable D-branes (via ‘tachyon condensation’) takes the theory to a new but familiar place.

So for example the space-filling D25-brane of bosonic open string theory decays away leaving a closed string theory vacuum. A relatively simple string field theory computation shows that the energy difference between the starting vacuum and the ending vacuum well approximates the tension of the D25-brane, which is intriguing³³⁷. Similar suggestions for the unstable branes of other theories have been tested also, with encouraging results. This has led to renewed vigour in the matter of understanding formulations of string field theory, and the interpretation of mysterious aspects of current formulations. In the latter regard for example, still puzzling to some extent is the fact that the computation done above is within the purely open string field theory. However, since the endpoint of the process is not an open string theory at all (the D25-brane has disappeared), the appearance in that framework of the closed strings which remain (if that is what remains) is not well understood. Overall, this is certainly a very interesting area which may well sharpen our understanding of aspects of the string vacua we know and how they are connected.

There has also been recent progress in understanding the fate of tachyonic purely closed string backgrounds and the vacua to which they decay³⁴⁸. For example, supersymmetry-breaking orbifold projections of \mathbb{C}_2 can be constructed, generalising the ALE spaces of chapters 13 and 14. Tachyons arise in the twisted sectors, giving interesting models in which the supersymmetry breaking is localised at the conical singularity. The decay channels involve the spaces radiating some of their curvature to infinity, becoming less singular and settling to stable vacua with curvature, or to flat space. Even in these tachyonic models, techniques closely related to some discussed in chapter 13 show that there are fascinating connections to a rich mathematical framework of singularity theory³⁴⁹.

Included in the above are topics which incorporate the fact that we are very much at home with the idea of non-BPS D-branes, and in fact some of the outcome of that research may be to find more useful techniques for finding them and working with them. Non-BPS D-branes were not

discussed at all in this book, unfortunately, but the tachyon condensation techniques can be used to point to their existence (and sometimes stability)^{18, 21}. Furthermore, the K-theory organisation¹¹³ of D-branes in fact does not care whether they are supersymmetric or not. This is very encouraging but we need more than a classification, we need a technology. With a good handle on the properties of non-BPS D-branes we can address two large areas of research. One is the area of probing various dualities beyond the supersymmetric sector. This applies to field theory dualities, where we might learn about more realistic strongly coupled gauge theory phenomena with such tools, and also string theory dualities, where connections to non-supersymmetric string vacua can be explored. This is a major motivation (in part) for some of the endeavours mentioned above.

The whole area of 'holography'^{286, 287} has certainly only just begun to be uncovered. It is perhaps clear that the collection of ideas surrounding that topic is an intriguing part of a profound story about spacetime, quantum mechanics, gravity and field theory, but the problem so far is the lack of a constructive way of phrasing the Holographic Principle: it tells one what the count of degrees of freedom ought to be, but gives (at time of writing) no insights as to how to implement the hologram. It may be again that this is a result of working with the wrong basic objects: as a result, it seems that the few working examples of holography that we have, like the AdS/CFT correspondence and perhaps matrix theory, are too different from each other in order to teach us anything general but yet specific enough.

On the whole, it remains a very exciting area in which to be working. In fact, one has a feeling of anticipation that there is something just around the corner which will put us back into the remarkable situation we were in a few years ago. Up to late 1994 or early 1995 we used to dream about various scenarios in string theory (perturbative and non-perturbative) which we had scarcely any tools to help us realise. Branes, and particularly D-branes, came along as the sharp tools that were needed and some of those dreams were made concrete, others discarded. For a while, D-branes were so intrinsically rich with new physics that they supplied us with new scenarios that we had not dreamed of at all, and gave us further ideas about how to concretely study those new situations.

Now we are in the situation where things have become hard again. Since the Second Revolution, we now dream in technicolour: branes are still telling us that there are remarkable places to which the theory can go (eg. by their changing shape and expanding into other branes, mingling with spacetime geometry or describing it as non-commutative, etc.) but it is harder for them to take us there. In other words, it is perhaps time

for them to hand over to another tool that can take us to these new places. One has a feeling that this new tool (or tools) may well be within our grasp, perhaps just at the edge of our collective peripheral vision. It may be that, as happened with D-branes initially, the new tool has already been discovered, but has not yet been recognised. Perhaps it is time to squint more quizzically at some of the objects which lurk in our notebooks.

There are many more topics of considerable importance which we have only touched upon, or not mentioned at all. An obvious one is the emergence of non-commutative geometry in both field theory and string theory which has been a topic of much research³³⁸. We touched upon it in one of its guises earlier in section 13.6, but have left most of it unexplored. There are other topics too, such as compactifications of string theory and M-theory to four dimensions using some of the new ideas and techniques that D-branes have supplied, perhaps giving new insights into phenomenology and/or other important four dimensional physics. Any of those topics could well be the area in which the next major breakthrough occurs, which is exciting. The hope is that, regardless of where the next breakthrough might be, this book will serve as a useful guide to some of the ideas and tools which have brought us to this point, and which may well help in moving things further.

