

Internal waves and Rayleigh–Taylor instability in magnetized compressible strongly coupled dusty plasmas

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This paper investigates the influence of compressibility on the internal wave modes and the density gradient-driven Rayleigh–Taylor (R–T) instability in magnetized strongly coupled dusty plasmas. The dusty magnetohydrodynamic model is formulated for compressible fluids, accounting for the effects of weakly coupled electrons/ions and strongly coupled dust particles under the influence of the gravitational field. The effect of the magnetic field on the dust dynamics has been incorporated through the magnetic force on the electrons and ions in quasineutral dusty plasmas. A dispersion relation of the R–T instability has been derived which has been modified because of the compressibility effect, dust acoustic wave speed and viscoelastic coefficients. The shear Alfvén and compressional viscoelastic wave modes become coupled in the dispersion characteristics. The modified R–T instability criterion is derived in terms of the Alfvén speed, viscoelastic effects and dust grain parameters. The graphical illustrations show that the growth rate of R–T instability has been suppressed due to compressibility, viscoelastic coefficients and dust acoustic speed. The results are useful to discuss the development of R–T instability in magnetized astrophysical dusty plasmas.

Keywords: dusty plasmas, plasma instabilities, strongly coupled plasmas

1. Introduction

The Rayleigh–Taylor (R–T) instability is a well-known hydrodynamic fluid instability that occurs at the interface between two fluids when a heavy fluid acts upon a light fluid in the presence of gravitational fields. The R–T instability plays a vital role in the structure formation in astrophysical and laboratory plasmas such as supernovae, white dwarfs, crab nebula, inertial confinement fusion (ICF), high-density matter, compact stars, etc. (Jun 1998; Cabot & Cook 2006; Wang 2011; Porth, Komissarov & Keppens 2014; Duffel & Kasen 2017). It is also ubiquitous in many crucial phenomena of solar prominence (Hillier 2018), the astrophysical disc around black holes (Papadopoulos & Contopoulos 2018) and particle acceleration in supernova remnants (Fraschetti *et al.* 2010). Interestingly, the presence of charged dust particles in plasmas profoundly influences structures, wave propagation characteristics and instabilities (Rao, Shukla & Yu 1990; Rosenberg 2000). The existence of dust grains also affects the dispersion properties (D’Angelo 1993), the

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growth rates of the R–T instability (Sen, Fukuyama & Honary 2010) and the formation of nonlinear vortices (Veeresha, Das & Sen 2005) in dusty plasmas. Avinash & Sen (2015) proposed the theoretical model of the R–T instability in incompressible dusty plasmas which has been experimentally examined by Pacha & Merlino (2020) and it has been shown that there is a possible connection between dust acoustic waves and the R–T instability at the interface between high- and low-density dust clouds.

In strongly coupled dusty plasmas (SCDP), the Coulomb coupling parameter Γ_d ($= Z_d^2 e^2 / a T_d$, where Z_d , e , a and T_d are the dust charge state, the electron charge, the inter-particle distance and the temperatures of dust species, respectively) is notably high ($\Gamma_d \gg 1$) (Kaw & Sen 1998). Such types of plasma environments are observed in many astrophysical and laboratory systems. Fluid instabilities play a vital role in such systems consisting of strongly coupled plasmas. In this direction, Das & Kaw (2014) have investigated the suppression of the R–T instability in strongly coupled plasmas with applications in laser compression techniques in ICF. Garai (2016) observed that viscosity and shear flow stabilize R–T modes within SCDP. Dolai & Prajapati (2018) have studied the effects of dust cloud rotation, shear velocity and intermediate magnetic fields, which stabilize the growth of the R–T instability. Most recently, Zhang & Duan (2024) have investigated the disparity in R–T instability between dusty and general fluids.

In recent years, significant attention has been devoted to investigating the R–T instability under diverse physical conditions. Dharodi & Das (2021) have performed a numerical simulation focusing on R–T and buoyancy-driven instabilities within SCDP, highlighting the suppression of R–T instability as the coupling strength increases. Wani *et al.* (2022) have performed molecular dynamic simulation on suppressing the R–T instability in strongly coupled plasmas. They found that the growth rate of the R–T instability in a strongly coupled plasma is higher than that of the hydrodynamic viscoelastic fluid. Recently, Sun *et al.* (2023) have investigated the effects of horizontal magnetic field and electrical resistivity on R–T instability using the (magnetohydrodynamic) MHD fluid theory. Previous works on R–T instability are primarily focused on the incompressible limits of stratified unmagnetized dusty plasmas. However, the compressibility effects and magnetic fields also play an important role in the R–T instability in dusty plasmas, which has not been studied in previous work. Thus, owing to the astrophysical and laboratory significance, we have investigated the effects of compressibility on internal gravity waves and R–T instability in magnetized SCDP.

The remaining part of the paper is structured as follows: in § 2, we formulate the mathematical model equations describing the R–T instability in compressible magnetized SCDP. In § 3, the dispersion relation of the R–T instability is derived using the normal mode analysis. In § 4, the analytical dispersion relation is analysed in various cases to study the role of viscoelastic parameters, compressibility and dust acoustic wave. Finally, § 5 represents the summary of the present problem.

2. Governing fluid equations

Let us consider a three-component magnetized SCDP consisting of weakly coupled electrons and ions and strongly coupled negatively charged dust grains. The system is embedded under the influence of an external uniform magnetic field $\mathbf{B} = B_0 \hat{\mathbf{z}}$ (where $\hat{\mathbf{z}}$ is the unit vector along the direction of the magnetic field) and contributes to the elastic property of the system through tensile stress. The massive charged dust particles are under the influence of gravitational acceleration $\mathbf{g} = -g\hat{\mathbf{z}}$. The magnetic field's strength is such that the electron and ion gyro-radius are much less than the dimension of the dust cloud and, hence, they are magnetized. The electrons and ions are collisionless fluids and show

local fluid-like behaviour when they are fully magnetized. The pressures of the Boltzmann distributed electron and ion fluids are balanced with the corresponding Lorentz forces.

Using the MHD fluid theory, let us describe the dynamics of the inertialess magnetized electron and ion fluids as follows:

$$0 = -\nabla p_e - en_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}), \quad (2.1)$$

and

$$0 = -\nabla p_i + en_i (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}), \quad (2.2)$$

where $n_{e,i}$ are the number density of the electron and ion fluids, $\mathbf{u}_{e,i}$ are the electron and ion fluid velocities, $p_{e,i}$ are the electron and ion pressures and \mathbf{E} and \mathbf{B} are the electric field and magnetic field vectors.

Equations (2.1) and (2.2) together with the quasineutrality condition $en_i - en_e = eZ_d n_d$ (where n_d is the dust number density), give the following equation:

$$eZ_d n_d \mathbf{E} = \nabla(p_e + p_i) + e(n_e \mathbf{u}_e + n_i \mathbf{u}_i) \times \mathbf{B}. \quad (2.3)$$

Since the strongly coupled dust particles are massive ($m_d \gg m_{i,e}$), their gravitational effect is considered. The compressibility effects are considered through the dust pressure in the equation of motion and continuity equations. The generalized hydrodynamic (GH) model incorporating the viscoelastic coefficient is used to write the momentum transfer equation for a strongly coupled dusty fluid in the following form:

$$\begin{aligned} & \left(1 + \tau_m \frac{\partial}{\partial t}\right) \left[m_d n_d \frac{\partial \mathbf{u}_d}{\partial t} + eZ_d n_d (\mathbf{E} + \mathbf{u}_d \times \mathbf{B}) + \nabla p_d - m_d n_d \mathbf{g} \right] \\ & = \eta \nabla^2 \mathbf{u}_d + \left(\xi + \frac{\eta}{3} \right) \nabla (\nabla \cdot \mathbf{u}_d), \end{aligned} \quad (2.4)$$

where τ_m , η and ξ are the relaxation parameter and the shear and bulk viscosity coefficients. Here, m_d , $\mathbf{u}_d (= u_x \hat{x} + u_y \hat{y} + u_z \hat{z})$ and p_d are the dust mass, the dust fluid velocity and the dust fluid pressure, respectively.

Substitution of (2.3) into (2.4) gives the total current density $\mathbf{J} = e(n_i \mathbf{u}_i - n_e \mathbf{u}_e - Z_d n_d \mathbf{u}_d)$, which is replaced in terms of the magnetic field vector using Ampere's law. Finally, we get the modified equation of motion as

$$\begin{aligned} & \left(1 + \tau_m \frac{\partial}{\partial t}\right) \left[\rho_d \frac{\partial \mathbf{u}_d}{\partial t} + \nabla p_d - \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} - \rho_d \mathbf{g} \right] \\ & = \eta \nabla^2 \mathbf{u}_d + \left(\xi + \frac{\eta}{3} \right) \nabla (\nabla \cdot \mathbf{u}_d), \end{aligned} \quad (2.5)$$

where $\rho_d = m_d n_d$ is the dust fluid density and $p_d \approx p_e + p_i + p_d$ is the total plasma pressure. The effective dust thermal pressure is related to the compressibility effects such that $p_d = \mu_d k_B T_d n_d$ (where μ_d is the compressibility parameter).

The magnetic induction equation is given by

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u}_d \times \mathbf{B}). \quad (2.6)$$

The continuity equation for a dusty fluid is given by

$$\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{u}_d) = 0. \quad (2.7)$$

In this paper, we consider the compressibility effect on the R–T instability in SCDP. The dusty fluid is assumed to be compressible and, hence, it can be divergence free ($\nabla \cdot \mathbf{u}_d \neq 0$). The compressibility effects play a crucial role in understanding the complex dynamics of SCDP. The equation of state for SCDP is given by (Garai *et al.* 2014)

$$\nabla p_d = \left(\frac{Z_d^2 n_{d0} T_e T_i}{m_d (T_e n_{i0} + T_i n_{e0})} + \frac{\gamma_d \mu_d T_d}{m_d} \right) \nabla \rho_d = c_d^2 \nabla \rho_d. \quad (2.8)$$

In this equation, the two coupled terms in the brackets have the dimension of squared velocity. This is designated as the squared characteristic speed or dust acoustic wave speed in SCDP and represented by c_d^2 . Equations (2.5)–(2.7) represent the basic governing fluid equations for the considered R–T configuration in magnetized compressible SCDP.

3. Dispersion relation of R–T instability

The hydrodynamic equilibrium of the present configuration is studied by balancing net plasma pressure with the force produced due to gravitational acceleration and the magnetic force. The static equilibrium condition demands that $\mathbf{u}_d = 0$ and $\partial/\partial t = 0$. This gives the following equilibrium arrangement:

$$-\nabla \left(p_{d0} + \frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (\mathbf{B}_0 \cdot \nabla) \mathbf{B}_0 + \rho_{d0} \mathbf{g} = 0. \quad (3.1)$$

Following the magnetostatic condition ($\nabla \cdot \mathbf{B}_0 = 0$), the magnetic field terms drop out in the case of a uniform magnetic field, and the equilibrium condition becomes $dp_{d0}/dz = -\rho_{d0}g$.

Let us now discuss the R–T stability/instability of the magnetized SCDP in the compressible limits. We are working in a basic Cartesian coordinate system. For simplicity, we assume that spatial variations occur only in the x – z plane, with the equilibrium global fluid density being a function of z alone, denoted $\rho_{d0} = \rho_{d0}(z)$. In order to conduct a local stability analysis, we express each physical quantity as a sum of its equilibrium component f_0 and a small perturbed component f_1 (where $f_1 \ll f_0$). To linearize the basic equations, let us employ the method of the Fourier transform in time and the x – z spatial coordinates. Accordingly, the perturbed quantities vary as $f_1 \propto \bar{f}_1 \exp(ik_x x + ik_z z + i\omega t)$, where \bar{f}_1 is the amplitude. Here, $k^2 = k_x^2 + k_z^2$ signifies the squared perturbation wavenumber and ω is the frequency of harmonic perturbations.

Now, we linearize the governing equations (2.5)–(2.8) of the considered system and employ the perturbations as defined above.

The perturbed form of the continuity equation (2.7) is expressed as

$$i\omega \rho_{d1} + u_{z1} \frac{d\rho_{d0}}{dz} + \rho_{d0} (ik_x u_{x1} + ik_z u_{z1}) = 0. \quad (3.2)$$

The \hat{x} -component of the momentum transfer equation (2.5) using (2.6) and (3.2) is obtained as

$$\begin{aligned} & (1 + i\omega\tau_m) \left[(-\omega^2 + k_x^2 c_d^2 + V_A^2 k^2) u_{x1} - ik_x c_d^2 (\alpha + ik_z) u_{z1} \right] \\ & = -\frac{i\omega}{\rho_{d0}} \left(\xi + \frac{4\eta}{3} \right) k^2 u_{x1}. \end{aligned} \quad (3.3)$$

The \hat{z} -component of the momentum transfer equation (2.5) using (2.6) and (3.2) is obtained as

$$\begin{aligned} & (1 + i\omega\tau_m) [-\{\omega^2 + (g + ik_z c_d^2)(\alpha + ik_z)\}u_{z1} - i(g + ik_z c_d^2)k_x u_{x1}] \\ & = -\frac{i\omega}{\rho_{d0}} \left(\xi + \frac{4\eta}{3} \right) k^2 u_{z1}, \end{aligned} \tag{3.4}$$

where $V_A = B_0/\sqrt{\mu_0\rho_{d0}}$ is the dust Alfvén speed and $\alpha = (1/\rho_{d0})(d\rho_{d0}/dz)$ is the inverse density-gradient scale length, which indicates the characteristic length over which significant density variations occur in the system.

To get the dispersion relation of the R–T instability in compressible SCDP, we simplify (3.3) and (3.4), which gives the following equation:

$$\begin{aligned} & \omega^4 - 2i\omega^3 \frac{\left(\xi + \frac{4\eta}{3}\right)k^2}{\rho_{d0}(1 + i\omega\tau_m)} + \omega^2 \left[igk_z - \left(1 + \frac{ik_z c_d^2}{g}\right)\omega_{cl}^2 - (c_d^2 + V_A^2)k^2 \right. \\ & \left. - \left(\frac{\left(\xi + \frac{4\eta}{3}\right)k^2}{\rho_{d0}(1 + i\omega\tau_m)}\right)^2 \right] - i\omega \frac{\left(\xi + \frac{4\eta}{3}\right)k^2}{\rho_{d0}(1 + i\omega\tau_m)} \left[igk_z - \left(1 + \frac{ik_z c_d^2}{g}\right)\omega_{cl}^2 - (c_d^2 + V_A^2)k^2 \right] \\ & - V_A^2 k^2 (g + ik_z c_d^2)(\alpha + ik_z) = 0. \end{aligned} \tag{3.5}$$

This fourth-order polynomial equation represents the general dispersion relation of the R–T instability in compressible SCDP. The term $\omega_{cl}^2 = -\alpha g > 0$ is the classical frequency associated with internal waves. We can see that the dispersion relation of the R–T instability has been modified due to the strong coupling effect in the compressible limit. The compressional viscoelastic modes in the dusty plasma have been coupled with the internal wave mode and Alfvén wave mode. In the absence of corrections due to dust particles (e.g. dust acoustic speed), the above dispersion relation reduces to that of Lu & Qiu (2011) and Dolai & Prajapati (2017), neglecting quantum corrections in those cases. Their results are useful for analysing the R–T instability in strongly coupled degenerate white dwarfs. On the other hand, the present work is applicable to examining the R–T instability in SCDP environments in laboratory and astrophysical systems. This dispersion relation is very complex, and it can be discussed in some cases that limit our interests.

4. Results and discussions

The analysis of the dispersion relation governing the R–T instability involves the interpretation of the relationship between the growth rate of the instability and wavenumbers across various considered physical parameters. This includes viscoelastic coefficients and compressibility effects in the term representing the viscoelastic compressional wave speed (V_c). We now discuss the above dispersion relation (3.5) in the kinetic limit $\omega\tau_m \gg 1$, which gives

$$\begin{aligned} & \omega^4 - 2\omega^2 V_c^2 k^2 + (\omega^2 - V_c^2 k^2) \left[igk_z - \left(1 + \frac{ik_z c_d^2}{g}\right)\omega_{cl}^2 - (c_d^2 + V_A^2)k^2 \right] \\ & - V_A^2 k^2 [(g + ik_z c_d^2)(\alpha + ik_z)] + V_c^4 k^4 = 0, \end{aligned} \tag{4.1}$$

where $V_c^2 = (\xi + 4\eta/3)/\rho_{d0}\tau_m$ represents the squared wave speed of the compressional viscoelastic mode in dusty plasmas.

We assume that all the perturbations are sensitive in the horizontal x -direction, i.e. we put $k_z = 0$ and $k_x = k$ and get the modified dispersion relation of R–T instability as follows:

$$\omega^4 - \omega^2 [(2V_c^2 + c_d^2 + V_A^2)k^2 + \omega_{cl}^2] + (V_A^2 + V_c^2)k^2\omega_{cl}^2 + V_c^2k^4(c_d^2 + V_A^2 + V_c^2) = 0. \quad (4.2)$$

From the constant term of (4.2), the R–T instability criterion is derived as

$$k < k_c = \sqrt{\frac{\left(\frac{g}{\rho_{d0}}\right) \frac{d\rho_{d0}}{dz} (V_c^2 + V_A^2)}{V_c^2(c_d^2 + V_A^2 + V_c^2)}}. \quad (4.3)$$

The above condition suggests that the R–T mode is unstable for all perturbation wavenumbers $k < k_c$. If the wavenumber is greater than k_c , then no R–T instability will be observed, and the wave will behave like an internal propagating wave. This critical wavenumber depends on various physical parameters such as the compressional wave speed (V_c), Alfvén wave speed (V_A), density gradient ($d\rho_{d0}/dz$) and the dust acoustic speed (c_d). The compressibility effects and Coulomb coupling parameter exist in the expression of c_d , which determines the value of the critical wavenumber required to excite the R–T instability in SCDP. If a change occurs in the viscoelastic coefficients and compressibility, then a decrease in k_c will show stabilizing effects on the growth rate of R–T instability.

The dispersion relation (4.2) highlights the significant modification of dispersion properties in transverse propagation induced by magnetic fields. In the case of unmagnetized dusty plasmas ($V_A = 0$), the dispersion relation (4.2) simplifies to

$$(\omega^2 - V_c^2k^2)(\omega^2 - V_c^2k^2 - \omega_{cl}^2 - k^2c_d^2) = 0. \quad (4.4)$$

Equation (4.4) has two factors of which the first factor gives the purely compressional viscoelastic wave mode with phase speed $\omega/k = V_c$.

The second factor gives

$$\omega^2 = \omega_{cl}^2 + V_c^2k^2 + c_d^2k^2. \quad (4.5)$$

The above dispersion relation represents the transverse wave propagation, including the classical frequency associated with the internal waves, viscoelastic compressional wave speed and dust acoustic wave speed. The roots of the above equation will be imaginary and R–T instability observed for all the perturbation wavenumbers less than the critical values, as given by

$$k < k_{c1} = \sqrt{\frac{\left(\frac{g}{\rho_{d0}}\right) \frac{d\rho_{d0}}{dz}}{(V_c^2 + c_d^2)}}. \quad (4.6)$$

This condition shows the modified criterion for the R–T instability in an unmagnetized SCDP medium. This condition is different from that of the condition derived by Das & Kaw (2014) due to the consideration of compressibility and dusty plasma parameters. The dust acoustic speed and compressional wave speed have been coupled together to decrease the critical wavenumber, showing their stabilizing influences on the growth rate of the R–T instability. The R–T instability in SCDP is observed when the dominance of ω_{cl}^2 occurs, which includes the density-gradient term. The nature of

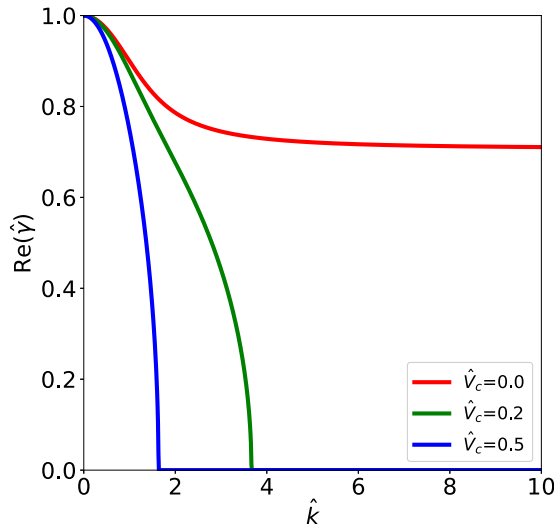


FIGURE 1. Normalized growth rate ($\text{Re } \hat{\gamma}$) is plotted vs normalized wavenumber (\hat{k}) for different values of compressional wave speed $\hat{V}_c = 0.0, 0.2$ and 0.5 .

R–T instability significantly depends upon the magnetic field and properties of the dust grains. In the case of an unmagnetized, compressible viscoelastic fluid ignoring the contribution of the dust acoustic speed, the R–T instability criterion (4.6) reduces to $k < k_{c2} = \sqrt{(g/\rho_{d0})(d\rho_{d0}/dz)}/V_c$. This expression shows that the critical wavenumber k_{c2} is inversely proportional to V_c . Consequently, increased compressibility reduces the critical wavenumber and suppresses the R–T instability.

The modified dispersion relation (4.2) depends upon both the dust acoustic speed (c_d) and the compressional wave speed (V_c). In order to visualize the influence of various considered parameters graphically, we introduce dimensionless parameters as follows:

$$\hat{\omega} = \frac{\omega}{\sqrt{\alpha g}}, \quad \hat{k} = \frac{k}{\alpha}, \quad \hat{V}_c = V_c \sqrt{\frac{\alpha}{g}}, \quad \hat{c}_d = c_d \sqrt{\frac{\alpha}{g}}, \quad \text{and} \quad \hat{V}_A = V_A \sqrt{\frac{\alpha}{g}}. \quad (4.7a-e)$$

This transformation enables us to formulate the dispersion relation in a dimensionless framework. By putting ($\hat{\gamma} = i\hat{\omega}$) we get the dimensionless form of the dispersion relation (4.2) as

$$\hat{\gamma}^4 + \hat{\gamma}^2(2\hat{V}_c^2 + \hat{c}_d^2 + \hat{V}_A^2 - 1/\hat{k}^2)\hat{k}^2 - (\hat{V}_A^2 + \hat{V}_c^2)\hat{k}^2 + \hat{V}_c^2\hat{k}^4(\hat{c}_d^2 + \hat{V}_A^2 + \hat{V}_c^2) = 0. \quad (4.8)$$

The fourth-order polynomial under consideration exhibits four distinct roots: two real roots (one positive and one negative) and two complex roots. Out of these four roots, the real positive root, provides the R–T instability in the framework of the considered exponential perturbations, which is the main purpose of studying this problem. Thus, we concentrate on the real roots obtained from (4.8) for analysing the role of the compressibility and strong coupling effects on the R–T instability in magnetized SCDP. In figure 1, we have depicted the relationship between the normalized growth rate ($\text{Re } \hat{\gamma}$) and the normalized wavenumber (\hat{k}) across varying values of compressional wave speed \hat{V}_c , with fixed values of $\hat{V}_A = 0.5$ and $\hat{c}_d = 0.5$. The curves demonstrate distinct behaviours depending on the magnitude of \hat{V}_c . Notably, it is observed that the compressional wave speed (\hat{V}_c) plays a crucial role in stabilizing the R–T instability within the SCDP

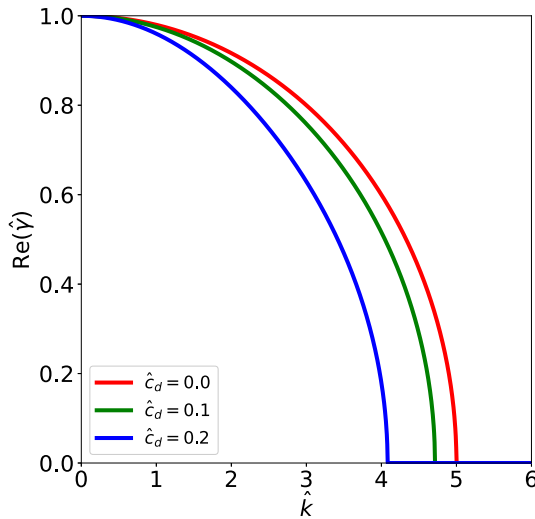


FIGURE 2. Effect of compressibility on the growth rate ($\text{Re } \hat{\gamma}$) is plotted vs normalized wavenumber (\hat{k}) for different values of dust acoustic wave speed $\hat{c}_d = 0.0, 0.1$ and 0.2 .

system. When viscoelastic effects are absent in the system, i.e. ($\hat{V}_c = 0$), the growth rate experiences a gradual decrease, indicating the inherent instability of the system. After a certain value of the wavenumber, the growth rate becomes almost constant. However, a notable change occurs with the introduction of the compressional wave speed (\hat{V}_c) in the system. The growth rate undergoes a significant decline, particularly around specific normalized wavenumbers, approximately $k_c \sim 1.75$, and at 3.75 it becomes zero. Thus, the viscoelastic effects play a significant role in suppressing the growth rate of the R–T instability in magnetized compressible SCDP.

In the expression of c_d , the dust compressibility parameter exists thus, by varying the values of c_d , one can directly observe the role of compressibility in the growth rate of the R–T instability. In figure 2, we explore the influence of compressibility in terms of the impact of the dust acoustic speed (\hat{c}_d) on the normalized growth rate ($\text{Re } \hat{\gamma}$), while maintaining constant values of $\hat{V}_A = \hat{V}_c = 0.2$. This figure also represents the effects of dust temperature and dust density parameters (which are present in the expression of c_d) on the growth rate of the R–T instability. The plotted curves exhibit distinct trends as \hat{c}_d varies across different magnitudes, giving insights into its impact on the R–T instability within the system. As the dust acoustic speed (\hat{c}_d) increases, we observe a systematic decrease in the growth rate. This inverse relationship between \hat{c}_d and the growth rate suggests that higher values of dust acoustic speed lead to greater stability within the system. In the absence of \hat{c}_d , the growth rate is notably larger, indicating a less stable state of the system. This stabilizing influence of dust acoustic speed and compressibility on the growth rate of R–T instability shows that \hat{c}_d acts as a dampening factor, restraining the growth of instability and promoting the system's stability.

The dust acoustic speed c_d is influenced by the density fluctuation, which also includes the effect of the finite temperature of the dust, ions and electrons (T_d , T_i and T_e) and dust number density n_d . Higher temperatures will lead to increased thermal motion of the particles. As a result, variations in the temperature of electrons and ions will directly affect the dust acoustic wave speed. Also, the increases in the dust temperature and compressibility enhance the value of the dust acoustic speed. Similarly, the adiabatic

index (γ_d) influences the speed of the acoustic mode within the dusty plasma. The higher values of dust temperature and adiabatic index will correspond to higher dust acoustic speed, which stabilizes the growth rate of R–T instability. These physical conditions are relevant in laboratory and astrophysical plasmas consisting of charged dust grains in a strongly coupled state. Hence, the present results are useful to explore the possibility of R–T instability in such strongly coupled dusty plasma environments.

5. Conclusions

In this paper, we have investigated the influence of compressibility and the presence of dust particles on the R–T instability in magnetized SCDP. We have considered the equation of state for a system consisting of strongly correlated dust grains and weakly coupled electrons and ions. Our study involved the formulation of the GH model for the considered configuration, incorporating the behaviour of strongly correlated dust particles along with weakly coupled ions and electrons. Using normal mode analysis, we have analytically derived the dispersion relation governing the R–T instability. Our investigations revealed that both compressibility and dust-driven acoustic modes significantly alter the dispersion properties of internal waves and onset criteria for the R–T instability. Specifically, we found that the compressional viscoelastic wave speed and the dust acoustic speed stabilize the growth rate of the R–T instability. The increase in compressibility and dust temperature parameters increases the dust acoustic speed, suppressing the R–T instability's growth rate.

The present work represents a comprehensive study of the internal wave propagation and the R–T instability within SCDP systems. The compressibility effect significantly influences the stability of dusty plasma systems, particularly under the influence of gravitational fields. The presence of a magnetic field induces the Alfvén modes, which are modified due to the viscoelastic and dust acoustic speeds. These results contribute to a broad understanding of the complex dynamics inherent in magnetized SCDP systems, shedding light on the impact of the interplay between compressibility, dust particles and gravitational acceleration on the behaviour of the R–T instability.

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Declaration of interests

The authors report no conflict of interest.

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