(4) Each of two angles of a triangle is 60° and the included side is 4 inches. The area of the triangle, in square inches, is:

(A) $8\sqrt{3}$ (B) 8 (C) $4\sqrt{3}$ (D) 4 (E) $2\sqrt{3}$

[The letter of the correct answer, 'C', is to be written in a space on the answer sheet.]

(17) The formula $N = 8 \quad 10^8 \quad x^{-3/2}$ gives for a certain group, the number of individuals whose income exceeds x dollars. The lowest income, in dollars, of the wealthiest 800 individuals is at least:

(A) 10^4 (B) 10^6 (C) 10^8 (D) 10^{12} (E) 10^{16}.

(27) Let S be the sum of the interior angles of a polygon P for which each interior angle is $7\frac{1}{2}$ times the exterior angle at the same vertex. Then

(A) $S = 2660°$ and P may be regular (B) $S = 2660°$ and P is not regular (C) $S = 2700°$ and P is regular (D) $S = 2700°$ and P is not regular (E) $S = 2700°$ and P may or may not be regular.

(34) Two swimmers, at opposite ends of a 90 ft. pool, start to swim the length of the pool, one at the rate of 3 feet per second, the other at 2 feet per second. They swim back and forth for 12 minutes. Allowing no loss of time at the turns, find the number of times they pass each other.

(A) 24 (B) 21 (C) 20 (D) 19 (E) 18.

(39) To satisfy the equation $\dfrac{a+b}{a} = \dfrac{b}{a+b}$, a and b must be:

(A) Both rational (B) both real but not rational (C) both not real (D) One real, one not real (E) one real, one not real or both not real.

It has been suggested that some schools in this country might like to participate in this competition. I should be glad to supply further information to anyone who is interested.

Yours etc., F. R. WATSON

Manchester Grammar School

To the Editor of the *Mathematical Gazette*

DEAR SIR,

May I congratulate N. de Q. Dodds on discussing the matter of elementary division and the method of setting it out? While not sure that he has the answer as regards setting out, I am convinced that some reform is most desirable. It is extremely confusing to a poor pupil to find that sometimes the divisor is on the left, $23\overline{)4187}$, sometimes on the right, $4187 \div 23$ and sometimes underneath $\dfrac{4187}{23}$. No wonder pupils will write $4187 \div 23$ or $23 \div 4187$ indiscriminately.

Some people think that if a pupil is so poor that at the age of 13 or so he is still confused about division, then one should not bother about him (or her). But it is quite possible in this country for girls who are poor mathematically to train as primary school teachers and thus pass on their own confusion.

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Can anybody suggest for long division a method of setting out which does away with this confusion? The weakness of N. de Q. Dodds' method (p. 181) is that the divisor is placed too far away from the working.

There seem to be two lines of approach:

(1) Abolish the $\div$ sign and set down division by the fraction $\frac{18}{3}$ or the half-bracket 3)18. No change in setting out of long division would be needed, but I agree with Mr. Dodds that "divide by" is better than "divide into."

(2) Abolish the 3)18 method. A method would have to be devised for setting out long division so that the divisor is either on the left or underneath the dividend. Can anybody devise such a method?

Yours etc., Una Dromgoole

Riccarton H. S., Chch. N.Z.

To the Editor of the *Mathematical Gazette*

Dear Sir,

Dr. Easthope says in his letter published in the Mathematical Gazette for December 1960 that one cannot always impose *real* frictionless constraints appropriate to Bertrand's Theorem. Would he accept a massless structure as *real*? Since a frictionless constraint is really the idealisation of a constraint of low friction and is accepted as *real*, a massless structure, the idealisation of a structure of small mass, should, one would think, also be accepted as *real*. If massless material is allowed it is perfectly possible to provide constraints which will allow motions as close as we please to the free motion. Thus his example, the simple rod whose instantaneous centre is outside the rod, may be provided with a massless link. One end of the link is pinned to any point of the rod, the other to any point in space. By choosing the latter point at or near the centre of rotation of the final free motion one may obtain constrained motions identical with the free motion or as close to it as one pleases.

Massless constraints of this kind do no work in a small displacement of the system. They can only acquire energy by attaining infinite velocity; and this they cannot do because of the finite velocities of the massive bodies to which they are attached. Such massless constraints are in many cases—probably in all cases for which the initial state is one of rest—equivalent to constraints not involving massless bodies. For example, the massless link just described exerts the same constraint (for two-dimensional motion) as a frictionless peg attached to the rod and sliding in a circular slot cut in the plane on which the rod is resting.

The constraints considered in Bertrand's Theorem must be compatible with the initial motion. To satisfy Dr. Easthope's criterion they must also be capable of variation so that "the constrained motion differs by as little as one pleases from the motion of the free system." Since the instantaneous motion of a rigid body is simply a screwing motion about some axis, a massless constraint compatible with this may obviously be applied to any rigid body of the system. This constraining structure may then be carried as a whole on another structure which permits