DIRECT N-BODY CALCULATIONS

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ABSTRACT. The main principles for direct integration of large point-mass systems are outlined. Most particles are advanced by the Ahmad-Cohen neighbour scheme, using fourth-order force polynomials and individual time-steps. Close encounters and persistent binaries are handled by two-body regularization, whereas extreme triple and quadruple configurations are treated as unperturbed systems by special regularization techniques. As an illustration of these methods, we discuss some recent results of an isolated system containing 1000 particles of unequal mass, with special emphasis on the post-collapse phase.

1. INTRODUCTION

The simple formulation of the classical N-body problem permits individual orbits to be obtained by direct numerical methods. This approach allows the evolution of small stellar systems to be followed over significant times without introducing simplifying assumptions. However, the need to study such systems during the critical stages of core collapse for as many particles as possible poses a severe challenge for numerical techniques. Although globular clusters cannot be studied in this way, the direct approach is still very useful for a wide range of investigations.

The interest in systematic N-body studies has been rather sporadic (cf. Wielen, 1967; van Albada, 1968; Aarseth, 1974; Giannone and Molteni, 1984). In addition, open star clusters have received some attention; e.g. Wielen (1971); Hayli (1971); Aarseth (1973), and most recently through the extensive study of Terlevich (1984). Further work is desirable along several lines. Firstly, more approximate methods, among which are the Monte-Carlo, Fokker-Planck and moment method, need to be compared in some detail with direct calculations. Moreover, there is much to be learnt from numerical experiments relating to particular aspects, such as the effect of close encounters, core heating by binaries and the influence of a mass spectrum on the evolution. Realistic cluster simulations should also be explored further, especially in response to improved observational data. Hence it appears that numerical experiments

J. Goodman and P. Hut (eds.), Dynamics of Star Clusters, 251–259.

constitute an important tool for increasing our understanding of dynamics and the hope is that such work will also inspire improved theoretical developments.

2. INTEGRATION METHODS

The implementation of an efficient algorithm for large N-body systems requires careful considerations of several key aspects. In order of increasing complexity the main features can be categorized under four headings: (i) Force polynomials; (ii) Individual time-steps; (iii) Neighbour scheme; (iv) Close encounters. These central topics will be discussed briefly in the following since a complete description can be found elsewhere (Aarseth, 1985).

Since the number of interactions increases as N(N-1)/2, it is desirable to preserve past force values by using a polynomial representation. Practical considerations such as initialization, restarts, machine accuracy and increasing storage suggest that remembering four previous force evaluations is a good compromise. This is equivalent to representing the force polynomial for each particle by a set of three divided force differences which are held in store. In addition, contributions from the fourth difference are added as a corrector at the end of each time-step. This procedure permits the equations of motion to be advanced by significantly larger time-steps for a given accuracy, whereas low-order predictions can be made at intermediate times.

The mean particle separation tends to increase significantly with central distance. Since the time-scale for interactions is essentially controlled by the local density, it is advantageous to introduce individual time-steps for each particle. In this way, consistent solutions are obtained for a wide range of time-scales while maintaining the relative accuracy of the orbit. This scheme requires additional coordinate prediction at intermediate times but the net gain is still significant, bearing in mind the large range of density contrasts. Each time-step is selected from a relative accuracy criterion, involving the force as well as three derivatives. Consequently the time-step for a given particle is reduced appropriately during periods of strong interactions, followed by an increase if the motion is no longer subject to large force fluctuations. Moreover, the orbital information contained in the high-order force derivatives allows a more proper control of any errors.

The two features discussed so far combine into an efficient treatment for small systems. As the particle number increases, however, it is more cost-effective to separate the total force into two components, where the so-called irregular force is due to a relatively small number of neighbours. The neighbour force is updated on a time-scale determined by the local fluctuations, whereas the smoother regular component due to all other members need only be recalculated on a longer time-scale, with intermediate values obtained by prediction. The Ahmad-Cohen (1973) scheme has proved itself in a variety of calculations; it appears to be more efficient than a single force polynomial method for systems exceeding about 50 particles. In this way, a factor approaching 10 can

be gained since it is possible to have up to ten irregular force summations for each total force evaluation, and the latter effort is by far the dominant one for large N. This scheme requires additional procedures, particularly relating to a consistent neighbour treatment, where the effect due to membership changes inside a given sphere is updated during each regular force calculation.

Close encounters play a dominant role in the systems studied by direct methods, which are of necessity limited to a few thosand interacting mass-points. The formation of energetic binaries add to the numerical problems, since special treatments are needed for encounters between single particles and binaries as well as binary collisions. The classical two-body regularization of Kustaanheimo and Stiefel (1965, hereafter KS) transforms the equations of motion for two colliding particles into a non-singular form, and the external perturbation is also included. This representation is especially useful for dealing with persistent binaries of high energy, since in addition to the regularized form only the tidal perturbation is required, thereby reducing considerably the numerical effort. Moreover, unperturbed centre of mass motion may be adopted in the case of relatively distant perturbers, or equivalently, small semi-major axis. The KS method has proved to be extremely powerful in cluster simulations and has made it possible to study advanced stages of evolution which would otherwise not be feasible. More recently, the treatment of critical triple encounters has been included using three-body regularization (Aarseth and Zare, 1974). Only the most compact triple configurations are selected, since for simplicity such sub-systems are assumed to be isolated but such cases are also the most difficult for the KS treatment. Following the success of this new development, an analogous four-body regularization has now been introduced. This employs the global N-body regularization of Heggie (1974), using the numerical formulation of Mikkola (1984). The implementation for colliding binaries again ignores the external perturbation, but provisional tests show that suitable cases do occur in simulated systems, especially those with an initial binary population.

It is evident from the above discussion that several powerful techniques can be successfully combined when using direct integration. Needless to say, the programming complexities are considerable but the results which can be obtained justify the exercise. As an illustration of a large-scale calculation, we present some new results which emphasize the post-collapse cluster evolution.

3. EVOLUTION BEYOND CORE COLLAPSE

The implementation of triple regularization has enabled a large N-body system to be followed through the critical stage of core collapse and for a considerable time afterwards. We study an isolated cluster with N = 1000 mass-points selected from a power-law mass function with index -2.75 and a modest maximum mass ratio of 5:1. The system is initially spherical, having isotropic velocities with a virial ratio 0.25 half-way between rest and approximate equilibrium. The cluster evolution has been followed over more than 155 initial crossing times, requiring at least 500 hours of VAX machine time.

The behaviour near core collapse may be studied in several different ways. Figure 1 shows the average central potential, based on the five most strongly bound single and centre of mass particles but excluding the internal contribution from hard binary components. There is a gradual trend towards a maximum value, reached after about 42 crossing times, with a phase of large fluctuations followed by a gradual decline. This behaviour is also reflected in the central density, determined from the maximum density within an individual neighbour sphere which usually involves about 30 core particles. Note that either of these representations do not depend on the density centre itself. In addition, an alternative description based on the innermost 1% and 4% of the total mass with respect to the density centre shows a similar behaviour. From this evidence we conclude that core collapse is essentially halted after about 42 crossing times, equivalent to less than six half-mass relaxation times, and is followed by a prolonged phase of expansion. However, there are considerable fluctuations near maximum collapse, some of which are associated with processes involving binaries.

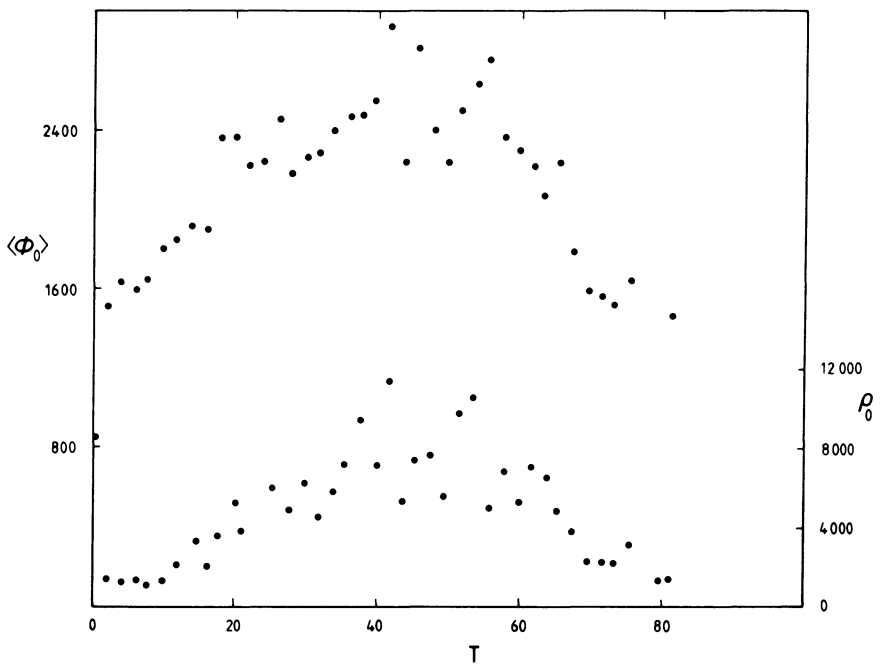

Figure 1. Illustration of core collapse. The upper data points show the average central potential based on the five most strongly bound single and c.m. particles. The lower data points show the central density as determined from the maximum neighbour density (vertical scale on the right). Time is in units of the standard crossing time, $2\langle R\rangle/\langle V\rangle$, where $\langle R\rangle$ is the half-mass radius and $\langle V\rangle$ is the equilibrium rms velocity.

https://doi.org/10.1017/S0074180900147424 Published online by Cambridge University Press

An attempt to determine the core radius by generalizing the expression for an equal-mass system does not give a meaningful result. Instead Figure 2 displays the cumulative mass as a function of radius, with the cluster divided equally into heavy and light particles, at two epochs (about 28 and 76 crossing times). Mass segregation is already quite pronounced at the early stage, in qualitative agreement with theoretical expectations. The curves for the light particles are fairly straight in the inner cluster region, indicating an approximate isothermal structure. Lack of resolution does not permit conclusions about a possible density cusp or core radius. However, the main feature of substantial core expansion indicated by Figure 1 can be seen in more detail in Figure 2. It is also significant that the compact sub-system of predominantly heavy particles, formed during the pre-collapse phase, does not show much evidence of subsequent shrinkage. This behaviour has observational implications if the most massive particles are associated with non-luminous neutron stars, since the luminous component would then only undergo a modest core collapse.

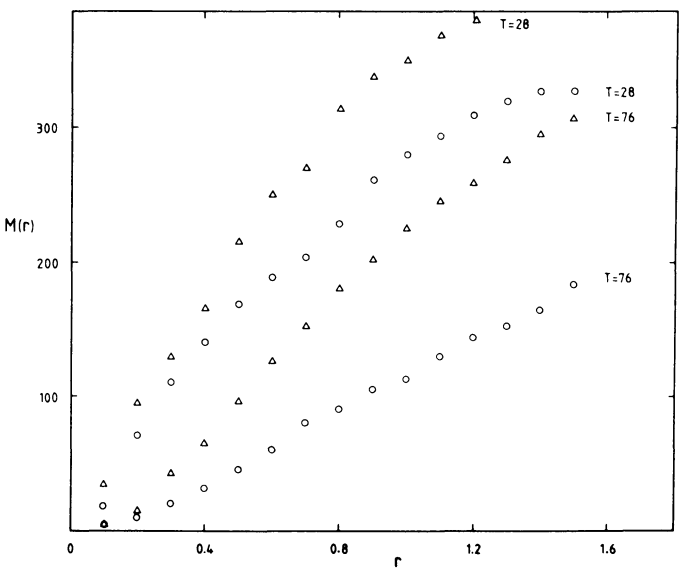

Figure 2. Cumulative mass as function of central distance. Results are shown separately for heavy and light particles (open triangles and circles) of equal total mass at early and late times (28 and 76 crossing times).

Binaries also control the later stages of core collapse in this simulation. The energy sink behaviour of central binaries in 250-body systems has been discussed in some detail before (Aarseth, 1974). Because of the smaller maximum mass ratio and larger N employed here, the first persistent hard binary does not occur until about 28 crossing times, but from then onwards an increasing amount of energy is absorbed by a few hard binaries. It is remarkable that the number of such binaries rarely exceeds two. The total energy stored in hard binaries

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is displayed in Figure 3. At the time of maximum core collapse, as determined from Figure 1, the dominant binary has acquired 7% of the total cluster energy. Detailed examination shows that energetic binaries, which tend to have massive components, are strongly concentrated towards the centre by mass segregation effects. This greatly increases the probability of binary collision and destruction of the least bound pair. On the other hand, close encounters with single particles may produce sufficient recoil to take a binary orbit outside the core, from which it subsequently spirals inwards again. With a typical semi-major axis of 0.0001 length units (initial half-mass radius) at this stage, such encounters are relatively infrequent but the increased centre of mass motion provides significant core heating via the two-body mechanism.

Extreme interactions may even result in escape from the cluster itself. This process is illustrated in Figure 3 which shows two such events occurring after about 49 and 69 crossing times and resulting from extremely energetic triple interactions, with negligible external perturbations. These binaries account for 35% and 42% of the initial total cluster energy, respectively. In each case, the terminal centre of mass velocity, evaluated at infinity, exceeds the overall rms velocity. Consequently, the corresponding velocity of single particles escaping after such interactions can be quite substantial, with the current examples producing two terminal velocities of about eight times the rms value. Distant particles exceeding the parabolic escape velocity are removed from the calculations, leaving the cluster more bound, whereas an escaping binary also carries away negative binding

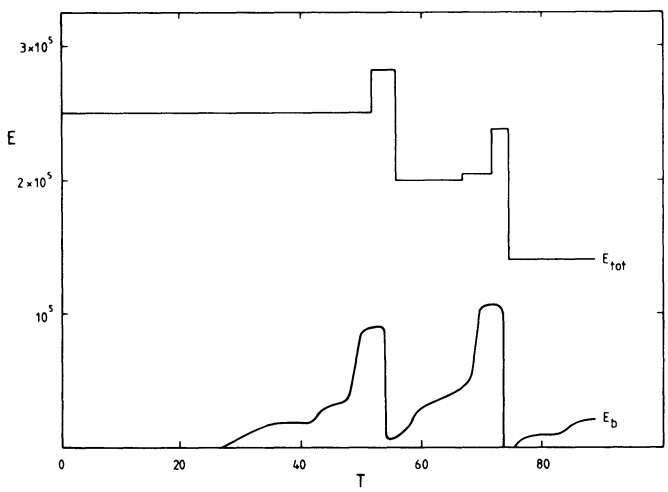

Figure 3. Binary evolution in the cluster. The lower curve shows the total energy contained in hard binaries, while the upper curve represents the total energy of all the bound particles in the cluster. Time is in units of the crossing time and energies are positive here.

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energy. The upper curve in Figure 3 shows both effects, with the extremely energetic single escaper preceding the removal of the binary. The difference between the upper and lower curve represents the energy binding the cluster itself; at the end of the phase displayed here this energy is only half the initial value. Hence the corresponding cluster expansion implies a significantly increased time-scale for further evolution.

Binaries continue to play an important role in the subsequent evolution, not included in Figure 3. Eventually, after 130 crossing times, a third energetic binary is ejected from the cluster after a strong and almost isolated triple interaction. In this case, the binding energy corresponds to 58% of the initial total energy of the cluster and the terminal escape velocity exceeds four times the rms velocity. The recoil velocity of the corresponding single escaping particle is even more extreme, exceeding ten times the rms value. In all there are 18 energetic escapers, with terminal velocity above the rms value, out of a total of about 100 escapers. Hierarchical configurations containing energetic binaries provide the most important mechanism for large energy transitions, sometimes resulting in exchange of companions and immediate escape with high velocity. The combined kinetic energy of the fast escapers represents some 70% of the initial cluster energy, with the three extreme escapers associated with the largest binary recoils accounting for over 50%. On the other hand, the three escaping binaries carry away a binding energy equivalent to 135% of the initial energy of the whole cluster. Thus the final cluster is left with a greatly reduced negative binding energy, amounting to no more than about 35% of the initial value when allowance is made for one remaining binary with nearly 2%. We may therefore conclude that binaries form and absorb energy at sufficient rates to maintain a relatively low central density and velocity dispersion, thereby effectively controlling the whole cluster evolution.

At this stage the calculations were terminated, the main objective of investigating a significant phase of post-collapse core evolution having been achieved. Only some of the more spectacular features have been discussed here, whereas a detailed analysis will be presented in due course.

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DISCUSSION

SEVERNE: To what extent is your code reversible (reproduction of the initial state upon velocity inversion)? Has it been applied to the actual study of distribution functions for the systems investigated?

AARSETH: It is possible to make a satisfactory reversal and obtain the initial conditions after an escaping particle has moved about five to ten half-mass radii outside a system. Simple distribution functions in one variable (i.e., energy, velocity) can easily be obtained at different times.

KING: These are extremely valuable calculations; but I regret that while you spend your time producing them, there is not someone else working with you to analyze them. This involves a lot of careful work and thought, and you need a collaborator who will give it the attention that it deserves.

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