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MATHEMATICAL MODELLING OF WAIRAKEI GEOTHERMAL FIELD

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Abstract

Mathematical modelling of Wairakei geothermal field is reviewed, both lumped-parameter and distributed-parameter models. In both cases it is found that reliable predictions require five to ten years of history for calibration. With such calibration distributed-parameter models are now used for field management. A prudent model of Wairakei, constructed without such historical data, would underestimate field capacity and provide only general projections of the type of changes in surface activity and subsidence.

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1. Introduction

Wairakei geothermal field is located in the North Island of New Zealand, in the Taupo Volcanic Zone. In the late 1940s there was one geothermal field developed for electrical generation in the world, Laraderello in Italy. This example, and a looming electricity shortage, led to the decision to develop Wairakei for power generation. The first drilling showed a field markedly different from Larderello, as it was full of hot water rather than the expected steam. The subsequent development had a large element of exploration, and there was a significant scientific effort to understand the physical nature of the field. The power station was built by 1958, but research continued thereafter, and to the present day.

Part of this effort was mathematical modelling. As pressures drew down with exploitation, it was discovered that the drawdown at depth was extremely uniform across the entire field, so that a single pressure history described this drawdown. The wells produced fluid of generally very similar quality, a steam–water mixture equivalent to liquid water at around 260°C. It was natural therefore that the first models

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focused on the two time-series, flow and pressure, and revolved around lumped-parameter models to relate the discharge and pressure histories.

By the late 1970s computers had become sufficiently powerful to make distributed-parameter models possible, and reservoir simulation displaced the lumped-parameter models in the 1980s.

The following symbols are used in this paper:

A	reservoir area	V	reservoir volume
c	compressibility	W	mass flow
F	influence function	α, β	constants
g	acceleration due to gravity	ρ	density
P	pressure	τ	relaxation time
S	saturation	φ	porosity
S_M	storage capacity (mass released per unit pressure change)		

2. Lumped-parameter modelling

Summaries of the lumped-parameter modelling of Wairakei are given by Grant *et al.* [11], Sorey and Fradkin [28] and Blakeley and O'Sullivan [1].

The successive lumped-parameter models are all represented by simple differential equations, or by an influence function $F(t)$, the response to a unit flow starting at time $t = 0$:

$$P - P_0 = \int_0^t F(t - t')\, dW(t'),$$

where $W(t)$ is the mass flow.

The first model was that of Whiting and Ramey [32]. The work was done in 1966. Their model is a confined box of liquid water, connected to an external aquifer which provides transient recharge. The recharge was not significant, so the model reduces to a very simple form:

$$S_M \frac{dP}{dt} = W \quad \text{or} \quad F = \frac{-t}{S_M},$$

where $S_M = \rho\varphi V c$ is the storage coefficient of the confined reservoir. A good fit was found, with correlation around 0.98. Because the compressibility of water is very low, the resulting volume is unphysically large.

This work was rejected because of the unphysical volume, but spurred the development of other simple models. McNabb [17] proposed an alternative model, with an unconfined reservoir, that is, with a free surface. Then

$$S_M = \frac{A\varphi\Delta S}{\rho g}$$

and the area A has a physically reasonable value. Marshall [15, 16] fitted the history to the drawdown of a 1D vertical tank surrounded by a uniform aquifer.

Subsequent models (Bolton [3], Donaldson [5]) used a simple recharge function, proportional to the pressure drop:

$$S_M \frac{dP}{dt} = -W + \alpha(P_0 - P) \quad \text{or} \quad F = \frac{1}{\alpha}\left(\exp\left(\frac{-\alpha t}{S_M}\right) - 1\right).$$

These models all found good fits, with correlation at least 0.98. The relaxation time $\tau = S_M/\alpha$ is found to be several years, so that it is not possible to calibrate a model properly until the mid-1960s to mid-1970s (Blakeley and O'Sullivan [1]).

There was a cooperation agreement with the United States in the late 1970s in which Wairakei data was made publicly available (Pritchett *et al.* [25]), and a spurt of research using this data. Zais and Bodvarsson [36] derived the influence function from the pressure-drawdown history, and also found that the reservoir was unconfined. Wooding [33] fitted the response of a distant well to the line-source solution, finding acceptable values for permeability and aquifer thickness.

A further generalization of the recharge model was the "drainage model" (Donaldson [6]), which produces the equation (Fradkin *et al.* [9])

$$S_M \frac{dP}{dt} = -W + \alpha(P_0 - P) + \beta \frac{dW}{dt}.$$

The last term has an interpretation in terms of the delayed drainage of water from a partly saturated region above the water surface.

That all models produced an excellent fit to the pressure history demonstrates that quality of fit alone is not sufficient to validate a model, and the model must derive from a conceptualization of the known physical data. However, the quality of fit also implies that any of the lumped-parameter models would be valid for the purpose of providing projections of future pressure change, provided that they were fitted to a data history significantly longer than the relaxation time. This limitation excludes the model of Whiting and Ramey, but all others would be equally valid for this purpose.

The introduction of a stylized representation of the vertical flow by Fradkin *et al.* [9] shows that the lumped-parameter models had by then reached their limit of validity. A more direct way to include such physical processes is to explicitly model them in a distributed-parameter model, and by 1980 this was becoming possible.

3. Simulations

Simulation codes were developed in the late 1970s, and a code comparison using specified examples showed that all were reasonable (Sorey [29]).

Because of the limitations of computing capacity, models were limited in the number of elements and often simplified.

One-dimensional vertical models were briefly tried [1, 26]. These can provide a good pressure history match, but this is controlled by the recharge assumptions. The 1D model can explicitly model the drainage process above the water level but is as dependent as the lumped-parameter models on assumptions about lateral recharge.

Two-dimensional models were popular as an initial trial. A vertical 2D model was developed by Pritchett *et al.* [24] and later refined by Garg and Pritchett [10]. This model obtained a reasonable match to the pressure history and to its distribution across the reservoir. There was a reasonable enthalpy match except during the period of enthalpy rise in 1964–1969, when the model overstated the rise.

Mercer and Faust [19] used a code based on groundwater simulation to produce a 2D horizontal model, and matched the pressure history. All other researchers have recognized that vertical flow was important and have included the vertical dimension in models, however simplified.

The model of Blakeley and O'Sullivan [2] was a 2D model, but as a vertical-radial model provided for 3D behaviour. In particular, it is possible to correctly model lateral recharge, and to place the boundary at a sufficient distance so as to internalize most boundary assumptions. This model produced a reasonable match to the pressure history and radial pressure distribution, and to the enthalpy history. It was possible to explore the sensitivity of the model match results to parameters, in particular the average horizontal permeability.

In addition to the changes in well performance, environmental impacts were becoming important in the 1980s. Blakeley and O'Sullivan [2] used the 2D model, and Herd and McKibbin [12] used a two-layer horizontal model, to represent subsidence.

However, the pattern of subsidence is a broad area of moderate subsidence corresponding roughly to the area of drawdown, plus very localized areas of major subsidence. It is fairly easy to model the broad area, but the localized areas of intense subsidence require similarly fine geological detail to represent them.

The model of Blakeley and O'Sullivan was important as the start of the modelling by O'Sullivan and successive co-workers.

Economic restructuring in the mid-1980s meant that operation of the Wairakei project moved to a commercial basis. One consequence was that data and research results were no longer automatically published. In particular, at Wairakei there were a number of competing development proposals. The reservoir simulation was owned by Contact Energy who felt that its results were of significant value to competitors. Consequently results were published only when required by consent hearings.

The first 3D model is described by ECNZ [7, 8]; information is available only in consent documents. This model had 301 blocks, and did not include Tauhara. The surface boundary condition was atmospheric pressure and temperature but the specified fluid is liquid water. The model was expanded to 1225 blocks.

Kissling *et al.* [13] used this latter model to simulate chemical changes in the reservoir fluid, specifically changes in chloride and carbon dioxide content, obtaining a reasonable match.

The next published model is described by O'Sullivan *et al.* [14, 21]. This is a full 3D model extending to include the adjacent Tauhara field. The model grid is shown in Figure 1 with a total of 1509 blocks. Roughly, Wairakei is the area to the west (left) of the Waikato River, and Tauhara the area to the east.

A very substantial number of changes have been made. Most noticeable is that the model is now fully three-dimensional, with an irregular grid following the shape of

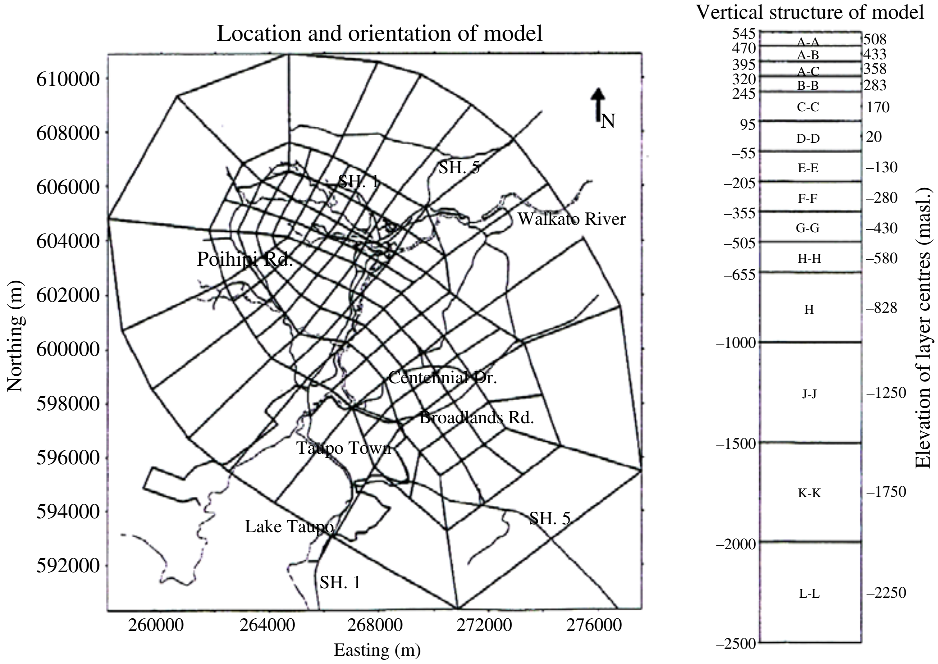

FIGURE 1. Grid layout for the Wairakei–Tauhara model, from O'Sullivan *et al.* [21].

field, and its extension to Tauhara. Gridding has become relatively fine in the upper layers in order to represent drainage and surface processes. The surface boundary condition is 1 bar with air, so that the air–water interface can be modelled. There has been extensive calibration so that observed pressure and temperature are modelled across the field, both in the natural state and production history; and discharge enthalpy is matched.

O'Sullivan *et al.* [22] describe the further development of the model, with finer gridding of the surface layers. With this it is now possible to get a reasonable match to the history of the surface heat flux. Both at Wairakei and Tauhara there was an increase in surface steam discharge, starting around 1960, followed by a slow decline. There are now 5418 blocks in the model.

Development and refinement of the model continue, with a 27886-block model under development (O'Sullivan *et al.* [23]). The developments since 2001 improve the model fit to observed data but do not add any new features or fits to any new type of data. O'Sullivan (pers. comm.) reports that as the model has grown larger in area and depth, the recharge coefficient has deceased. That is, some of the external recharge has been internalized as the model expands, and the hot recharge observed at shallower depths is in part produced by convective heat sweep at greater depth.

Tokita *et al.* [30], as a part of research on the effect of development on hot springs, developed a simulation of Wairakei–Tauhara for the purpose of exploring the effect of development on hot springs in Tauhara, this being the most distant such effect known anywhere.

A number of other models were developed for consent applications by competing parties over the period 1997–2004 (Young [35], SKM [27], Burnell [4]). All are documented only in part, and only in consent documents; there is no full publication. Compared to the models of O'Sullivan and co-workers, they are coarser, and provide fits, of the reported data, of at best comparable quality to the earlier models of similar coarseness. They are not described further. O'Sullivan *et al.* [23] give brief details.

A number of detailed models directed towards specific effects have been developed. All of them involve finer gridding in the relevant regions of the reservoir.

Subsidence in Tauhara, caused by Wairakei, became an environmental issue. Menzies and Lawless [18] developed a 2D model of Tauhara, to define the reservoir changes in Tauhara. This was followed by a 2D model of the Tauhara reservoir and subsidence, using a finite-element single-phase geomechanical simulator to model pressure and subsidence (White *et al.* [31]). Geological structure was adjusted to fit observed subsidence, on a scale much finer than well spacing, so there is no possibility of specifying this structure before subsidence has been observed. These models were of Tauhara only, pressure and temperature data being specified inputs from either Wairakei simulation outputs or measured well data, and future pressure changes being taken from Wairakei simulation output.

Yeh and O'Sullivan [34] used pressure data from the Wairakei simulation as input to a finite-element subsidence model. As with other subsidence models, finer geological detail is required to model the areas of localized intense subsidence.

Zarrouk *et al.* [37] developed a local fine-grid model of the Poihipi steam zone on the margins of the field.

A group of surface features known as the Alum Lakes ceased flowing over 1997–2001. Deep pressures had been stable since the early 1980s. Newson and O'Sullivan [20] developed a 2D model of Wairakei with much finer detail in the near-surface layers, and with that were able to roughly model the changes in these springs.

4. Predictive value of the modelling

The lumped-parameter models are fitted to the pressure and discharge history. They cannot be properly calibrated until five to ten years of history are available, and once calibrated, provide a reasonable prediction of future pressure trends.

The simulations fit, and provide projections, for a wider range of variables. When the models were developed, 15–40 years of history were available and the model used this for calibration. Once so calibrated, the models provide good predictions for future trends in pressure, temperature and discharge enthalpy.

A number of other physical parameters—surface heat flux, subsidence, change in surface activity—have now been modelled but all require a past history for calibration. All involve permeability structure at a level of detail much finer than well data, so it would not have been possible to develop a model providing predictions before significant events happened. Now that the model exists, it can be used to provide projections of future changes. In particular, it is useful to explore the effects of a change in reservoir management. A notable example would be the effect on subsidence of a rise in reservoir pressure caused by a switch to greater injection.

It is an interesting thought experiment to consider how good a model could be constructed with exploration data but lacking a field history; that is, the problem faced in reality when developing a new field. With an array of wells spread across the field, and interference testing, it would be possible to construct a model with permeability structure derived from this testing and natural state calibration. However, it would not be possible to determine the recharge. Wairakei is very unusual in having strong recharge of hot fluid. Compared to geothermal fields around the world, it lies at the high end in terms of permeability and convective flows, and the exploitation without much reinjection also places it at the high end in terms of net mass withdrawal.

A prudent model of a new field would take a conservative approach and assume little or no such recharge. Because of this an exploration model of Wairakei, which was otherwise accurate, would underestimate field capacity.

It would not be possible to predict the changes in heat flux, or specific surface features, or subsidence, in advance of observing significant changes which provide needed calibration data. The general pattern of changes in surface activity could be projected but not accurately. Similarly, possible subsidence could be projected but the lack of sufficiently detailed geology would preclude a reliable prediction.

References

[1] M. R. Blakeley and M. J. O'Sullivan, "Simple models of the Wairakei reservoir", *Proc. 3rd NZ Geothermal Workshop*, 1981, 131–136.

[2] M. R. Blakeley and M. J. O'Sullivan, "Modelling of production and recharge at Wairakei" *Proc. 4th NZ Geothermal Workshop*, 1982, 23–31.

[3] R. G. Bolton, Unpublished report, MWD, 1968.

[4] J. Burnell, Statement of evidence presented at the hearing on a resource consent application by Geotherm Group Limited related to the use and disposal of geothermal fluid from the Wairakei geothermal system, Taupo, May 2004.

[5] I. G. Donaldson, "Regression models of Wairakei", Unpublished report, Department of Scientific and Industrial Research, New Zealand, 1971.

[6] I. G. Donaldson, "Geothermal reservoir engineering research in New Zealand. A simplistic model and the Wairakei geothermal reservoir", *Proc. 3rd Workshop on Geothermal Reservoir Engineering*, Stanford University, 1978, 36–41.

[7] ECNZ, "Water right application and impact assessment: Wairakei geothermal power station, July 1990", Electricity Corporation of New Zealand, Wellington, New Zealand, 1990.

[8] ECNZ, "Resource consent application for reinjection: Wairakei geothermal field, January 1992". Electricity Corporation of New Zealand, Wellington, New Zealand, 1992.

[9] L. Ju. Fradkin, M. L. Sorey and A. McNabb, "On identification and validation of some geothermal models", *Wat. Resour. Res.* **17** (1981) 929–936.

[10] S. K. Garg and J. W. Pritchett, "Two-dimensional simulation of the Wairakei geothermal field", *Trans. Geothermal. Res. Council* **3** (1979) 237–240.

[11] M. A. Grant, I. G. Donaldson and P. F. Bixley, *Geothermal reservoir engineering* (Academic Press, New York, 1982).

[12] M. A. Herd and R. McKibbin, "Mathematical modelling of subsidence at Wairakei geothermal field", *Proc. 7th NZ Geothermal Workshop*, Auckland University, 1985, 139–146.

[13] W. M. Kissling, K. L. Brown, M. J. O'Sullivan, S. P. White and D. P. Bullivant, "Modelling chloride and CO_2 chemistry in the Wairakei geothermal reservoir", *Geothermics* **25** (1996) 285–305.

[14] W. I. Mannington, M. J. O'Sullivan and D. P. Bullivant, "An air–water model of the Wairakei–Tauhara geothermal system", *Proc. 2000 World Geothermal Congress*, (2000), 2713–2718.

[15] D. C. Marshall, "Preliminary theory of the Wairakei geothermal field", *New Zealand J. Sci.* **9** (1966) 651–673.

[16] D. C. Marshall, "Development of a theory of the Wairakei geothermal field by the 'simplest cases first' technique", *Geothermics* **2** (1970) 669–676.

[17] A. McNabb, "A model of the Wairakei geothermal field", Unpublished report, Department of Scientific and Industrial Research, New Zealand, 1967.

[18] A. J. Menzies and J. V. Lawless, "Two-dimensional modelling of the Tauhara geothermal field", *Trans. Geothermal Res. Council* **24** (2000) 611–616.

[19] J. W. Mercer and C. R. Faust, "Geothermal reservoir simulation 3. Application of liquid and vapor-dominated modelling techniques to Wairakei", *New Zealand Wat. Resour. Res.* **15** (1979) 653–671.

[20] J. A. Newson and M. J. O'Sullivan, "Reservoir pressure drawdown and the alum lakes, Wairakei", *Proc. 32nd Workshop on Geothermal Reservoir Engineering*, Stanford University, 2007.

[21] M. J. O'Sullivan, D. P. Bullivant, W. I. Mannington and S. E. Follows, "Modelling of the Wairakei–Tauhara geothermal system", *Proc. 20th NZ Geothermal Workshop*, Auckland University, 1998, 59–66.

[22] M. J. O'Sullivan, W. I. Mannington and D. P. Bullivant, "A three dimensional model of the Wairakei–Tauhara geothermal system. 2000 model development update including scenario for the Wairakei geothermal field". Report prepared for Contact Energy Limited, Department of Engineering Science, University of Auckland, Auckland, New Zealand (2001) 174pp.

[23] M. J. O'Sullivan, A. Yeh and W. I. Mannington, "A history of numerical modelling of the Wairakei geothermal field", *Geothermics* (2009), at press.

[24] J. W. Pritchett, S. F. Garg and D. H. Brownell, "Numerical simulation of production and subsidence at Wairakei, New Zealand", *Proc. 2nd Workshop on Geothermal Reservoir Engineering*, Stanford University, 1976, 310–324.

[25] J. W. Pritchett, L. F. Rice and S. K. Garg, "Wairakei geothermal reservoir engineering data", *Proc. 4th Workshop on Geothermal Reservoir Engineering*, Stanford University, 1978, 217.

[26] J. W. Pritchett, L. F. Rice and S. K. Garg, "Reservoir simulation studies: Wairakei geothermal field, New Zealand", *Final Report GREMP-11*, Lawrence Berkeley Laboratory, 1980.

[27] SKM, "The Wairakei low pressure steam zone", Unpublished report for Mercury Geotherm Limited, May 1998, 46pp.

[28] M. L. Sorey and L. Ju. Fradkin, "Validation and comparison of different models of the Wairakei geothermal reservoir", *Proc. 5th Workshop on Geothermal Reservoir Engineering*, Stanford University, 1979, 119–204.

[29] M. L. Sorey, "Numerical code comparison project a necessary step towards confidence in geothermal reservoir simulations", *Proc. 6th Workshop on Geothermal Reservoir Engineering*, Stanford University, 1980, 253–257.

[30] H. Tokita, H. Takagi, Y. Kiyota, K. Matsuda, H. Hatanaka, K. Shimada, H. Inuyama, R. Young, L. Bayrante and O. Jordon, "Development and verification of a method to forecast hot springs interference due to geothermal power exploitation", *Proc. 2000 World Geothermal Congress*, (2000) 725–730.

[31] P. J. White, J. V. Lawless, S. Terzaghi and W. Okada, "Advances in subsidence modelling of exploited geothermal fields", *Proc. 2005 World Geothermal Congress*, (2005) Paper 222, 10pp.

[32] R. L. Whiting and H. J. Ramey Jr, "Application of material and energy balances to geothermal steam production", *J. Petroleum Technology* **21** (1969) 893–900.

[33] R. A. Wooding, "Aquifer models of pressure drawdown in the Wairakei–Tauhara geothermal region", *Wat. Resour. Res.* **17** (1981) 83–92.

[34] A. Yeh and M. J. O'Sullivan, "Computer modelling of subsidence in geothermal fields". *Proc. 29th NZ Geothermal Workshop*, 2007, 9.

[35] R. Young, Statement of evidence and exhibits, presented at a joint hearing before Environment Waikato and Taupo District Council on consents for Contact Energy to take and inject geothermal fluid at Tauhara, September 1997.

[36] E. J. Zais and G. Bodvarsson, “Production decline analysis”, *Trans. Geo. Res. Council* **4** (1980) 381–384.

[37] S. J. Zarrouk, M. J. O’Sullivan, A. E. Croucher and W. I. Mannington, “Optimised numerical modelling of production from the Poihipi dry steam zone: Wairakei geothermal system”, *Proc. 31st Workshop on Geothermal Reservoir Engineering*, Stanford University, 2006.