THE EFFECT OF SUBGLACIAL CHEMICAL PROCESSES ON GLACIER SLIDING

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ABSTRACT. The glacier sliding theory of Nye is modified to include the effect of solutes in subglacial regelation waters on the sliding process. Motivation for this development stems from studies of subglacially formed chemical deposits that appear to be widespread on rock surfaces recently exposed by retreating temperate glaciers. These deposits indicate clearly that considerable local concentration of solutes commonly occurs subglacially as a result of the selective rejection of solutes into the melt during the freezing associated with regelation sliding. Because solutes accumulate where regelation waters refreeze, they tend to lower the temperature there and hence inhibit the heat transport away from these areas that is essential for regelation sliding. For a simple sinusoidal bed and solute distribution in the regelation water film, the modified theory shows that a maximum excess of solutes of, for example, several millimoles/l of dissolved $CaCO_3$ along lee surfaces relative to stoss surfaces impedes basal sliding significantly, especially if the bed roughness wavelength does not exceed about one meter.

RÉSUMÉ. *L'effet des processus chimiques sous-glaciaires sur le glissement des glaciers.* La théorie de Nye du glissement des glaciers est modifiée afin d'incorporer l'effet des solutés dans l'eau de fonte sous-glaciaire sur le glissement des glaciers. Le motif de ce développement est dérivé d'études des dépôts chimiques formés à la base des glaciers tempérés et qui apparaissent être largement répandus sur les surfaces rocheuses qui ont été récemment découvertes par le recul de ces glaciers. Ces dépôts indiquent clairement, que couramment, les solutés sont fortement concentrés à des endroits bien spécifiques à la base des glaciers, à cause du rejet sélectif des solutés dans l'eau pendant le regel associé avec le glissement. Comme les solutés s'accumulent là où les eaux de fontes regèlent, ils ont tendance à y abaisser la température et ainsi à ralentir le transport de chaleur qui est essentiel pour le glissement par fonte et regel. Avec un modèle simple consistant en un lit de profil sinusoïdal et en une distribution sinusoïdale de solutés dans le film d'eau de fonte sous-glaciaire, la théorie modifiée indique que, par exemple, si la concentration de $CaCO_3$ dissout dans l'eau à l'aval d'obstacles sur le lit rocheux dépasse celle à l'amont de quelques millimoles/l, le glissement du glacier est considérablement ralenti, surtout si la longueur des ondulations du lit ne dépasse pas un mètre.

ZUSAMMENFASSUNG. *Der Einfluss subglazialer chemischer Prozesse auf des Gleiten der Gletscher.* Die Gleittheorie für Gletscher von Nye wird so modifiziert, dass sie den Einfluss von Lösungen im subglazialen Regelationswasser auf den Gleitprozess zu berücksichtigen gestattet. Der Grund für diese Entwicklung liegt in Studien an subglazial entstandenen chemischen Ablagerungen, die anscheinend auf frischen Felsflächen, die beim Rückzug temperierter Gletscher frei werden, häufig vorkommen. Diese Ablagerungen lassen eindeutig darauf schliessen, dass eine beträchtliche lokale Konzentration von Lösungen gewöhnlich subglazial als Ergebnis des selektiven Ausfalles von Lösungen in das Schmelzwasser während des Gefrierens beim Regelationsgleiten auftritt. Weil Lösungen sich dort anhäufen, wo Regelationswasser wieder gefriert, bewirken sie dort auch einen Abfall der Temperatur und verhindern so den Wärmetransport aus diesen Bereichen, der wesentlich für das Regelationsgleiten ist. Für ein einfaches, sinusförmiges Bett und eine einfache Lösungsverteilung im Film des Regelationswassers zeigt die modifizierte Theorie, dass ein maximaler Lösungsüberschuss von beispielsweise einigen Millimolen/l gelösten $CaCO_3$ längs der Leeseite gegenüber der Stosseite das Gleiten am Untergrund merklich behindert, besonders wenn die Wellenlänge der Rauhigkeit des Bettes etwa einen Meter nicht überschreitet.

INTRODUCTION

Thin superficial chemical deposits that formed subglacially are widespread on bedrock recently exposed by retreating temperate glaciers. The most common and best developed of these deposits consist of calcite and are generally found on limestone and other carbonate-rich rocks (Bauer, 1961; Kers, 1964; Ford and others, 1970; Hallet, 1975, in press, unpublished). Less conspicuous subglacially formed coatings of practically pure silica occurring on andesite bedrock have also been reported (Hallet, 1975). The deposits are characteristically found at the lee of bedrock protuberances, where they form thin coatings that are furrowed and fluted parallel to the direction of former ice flow. Those made of calcite often occur as small spicules pointing down-glacier, which suggests that they formed in intimate contact with the sliding ice, rather than in subglacial cavities bridged by the glacier sole. Their genesis appears

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to be closely related to the regelation–slip process, which is essential in enabling temperate glaciers to slide over their beds. In this process basal ice melts in zones of higher pressure and lower temperature along surfaces facing up-glacier, and the resulting water flows between the ice and rock toward zones of reduced pressure and higher temperature along lee surfaces, where it refreezes. The heat necessary for melting is provided by the latent heat of freezing, conducted through ice and rock from the lee to the stoss sides of the obstacles. The subglacial deposits appear to form because mineral dissolved from stoss surfaces is transported by the melt water to the lee surface and concentrated there by freezing. The water eventually becomes supersaturated and the dissolved material precipitates from solution, forming the subglacial deposits.

The concentration of solutes along lee surfaces reduces the melting temperature there. With the necessary constraint that the ice be at its melting point, the lee surface temperature is lowered, thereby reducing the temperature gradient across the bed obstacles and hence the heat flow that is critical for regelation sliding. Lliboutry (1971, p. 27–28) recognized that this potentially could inhibit sliding markedly, but concluded that instead of building up there, the solutes must either be carried away by a net flux of water through the regelation film or incorporated in the ice as brine pockets trapped by freezing. The subglacial chemical deposits indicate that considerable concentration of solutes can occur, at least to the point where the water becomes supersaturated. Once precipitation begins, further solute rejection by the freezing ice merely results in additional precipitation. The concentration in regelating waters is, therefore, limited by the solubility of the deposited material.

The effect of solutes on the regelation process has been studied quantitatively in connection with the motion of round wires through temperate ice (Drake and Shreve, 1973, p. 69–75). They found that concentration of solutes at the rear of the wire could account for the observed wire velocities being as much as two orders of magnitude slower than predicted by the simple regelation theory. Although they suggested other mechanisms possibly causing the discrepancies between observed and expected velocities, it is clear from their analysis that solutes may strongly influence the regelation process. Nye (1973[a], p. 394) emphasized the need to consider possible implications of the findings of Drake and Shreve on the analogous problem of glacier sliding. By indicating that relatively high concentrations can exist at the glacier bed, the subglacial chemical deposits offer clear motivation for modifying glacier sliding theory to take into account the effect of solutes.

MODIFIED THEORY

The bed-slip theory presented by Nye (1969, 1970) is followed in this treatment because it is the most explicit presentation of the theory in its modern form where the drag on the bed is expressed in terms of the Fourier transform of the bed surface. Because of the considerable uncertainties in predicting the subglacial solute distribution, it does not at this time seem warranted to follow the more refined treatment of Kamb (1970), which includes the effects of the non-linear flow law of ice. All references to a Nye paper without explicit citation will refer to Nye (1969).

In his model of glacial bed slip, Nye considered temperate ice sliding slowly over a rigid wavy surface by two processes: deformation and regelation. Following Nye, we consider the glacier bed to be wavy only in the direction of sliding, which is designated the x direction. It is represented by the function

$$z_0(x) = \epsilon f(x) \tag{1}$$

where ϵ is a dimensionless number small compared to unity, and $f(x)$ is a function of order unity and mean zero. Nye (1970, p. 399–401) has shown that this restriction on the amplitude of the surface undulations is unnecessary, provided they are much smaller than the ice thickness. The only constraint necessary on z_0 is that dz_0/dx be everywhere small compared to

unity. For clarity of presentation, however, the present treatment will follow Nye's earlier, more explicit paper, which uses Equation (1).

In addition to the variables used by Nye, it is necessary to add

$$M(x) = \epsilon M_1(x) + \epsilon^2 M_2(x) + \dots \tag{2}$$

where $M(x)$ is the deviation in the total concentration of all solutes in solution from the mean value in the subglacial water film. This expansion implies a restriction on the magnitude of $M(x)$ to ensure that it is of the right order of magnitude for the perturbation analysis to be valid. A suitable restriction may be to limit $M(x)$ to values smaller than $C\Delta p(x)/2\alpha$ where C is the pressure-melting coefficient (0.007 4 deg bar^{-1}), α is the molar freezing point constant (1.85 deg molar^{-1} for dilute solutions), and $\Delta p(x)$ is the greatest pressure difference across obstacles considered compatible with the perturbation analysis. Consider, for example, a glacier sliding at a moderately slow rate of 10 m a^{-1} over sinusoidal beds having a constant amplitude-to-roughness ratio of, say, 0.05, which satisfies the low dz_0/dx restriction required by the analysis. The unmodified theory of Nye predicts that $\Delta p(x) \approx$ (120 bars) $\beta/(1+\beta^2)$ with $\beta = \lambda_*/\lambda$, where λ is the wavelength of the bed and λ_* is the wavelength at which regelation and viscous deformation contribute equally to sliding over beds of constant roughness. λ_* is occasionally referred to as the transition wavelength and, with characteristic values for the viscosity of ice and thermal conductivities of ice and rock, its value is about 0.5 m. Assuming that these calculated values of $\Delta p(x)$ are compatible with the analysis, values of $M(x)$ reaching about 2×10^{-2} M can suitably be treated by the modified theory for $\lambda = 0.05$ m, and larger differences can be handled for longer undulations. Terms of order ϵ^2 and higher will be ignored in this analysis.

Nye first considered the basal pressure distribution associated with sliding by regelation without viscous deformation. He derived an expression for the temperature distribution such that the resulting heat transport supplies the heat of melting and removes the heat of freezing necessary for regelation sliding. The pressure at the ice–water interface, which is also the pressure in the regelation water film, is related to the temperature by the constraint that the ice be at its melting point. If solutes are present in the water film, they too will affect the temperature because they lower the freezing point of water. Incorporating this effect is the purpose of this extension of the theory of basal slip.

The temperature θ at the base of a temperate glacier, relative to the mean temperature there, is

$$\theta(x) = -Cp(x) - \alpha M(x) \tag{3}$$

where the notation is the same as that of Nye with necessary additions. For solutions with ionic strengths in excess of about 0.01, the freezing point lowering due to solutes is appreciably less than $\alpha M(x)$ because of a departure from ideality. For example in a 0.005 M CaCO$_3$ solution the deviation exceeds 9%. For more concentrated solutions, the freezing temperature must be obtained experimentally or by calculations that consider the activities of all species in solution (Hallet, 1975, unpublished). Although the ice may in general be non-hydrostatically stressed, the temperature at the ice–water interface for ice in equilibrium with water at pressure p is to first order the same as though the ice were stressed hydrostatically at a pressure p, and the higher-order terms are negligible for stresses that occur in glacier ice (Kamb, 1970, p. 685). Equation (3) is a simple extension of the analogous expression of Nye (p. 449) and is similar to equation (9a) presented by Drake and Shreve (1973, p. 70). The new expression for the Fourier transform of the pressure corresponding to equation (25) of Nye (p. 452) is

$$\bar{p}_1(k) = -\frac{[L\bar{W}(k) + 2\alpha K|k|\bar{M}_1(k)]}{2CK|k|} \tag{4}$$

where W is the velocity of the ice normal to the bed to first order, L is the latent heat of fusion per unit volume, K is the mean thermal conductivity of ice and rock, and the horizontal bars

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denote Fourier-transformed variables with k being the Fourier-transform variable, which can also be regarded as the wave-number.

Carrying through the solution exactly as done by Nye leads to

$$\langle \tau_{xz} \rangle = \frac{\langle z_0{}^2 \rangle}{\int\limits_{-\infty}^{\infty} \mathrm{d}k \, |\bar{z}_0(k)|^2} \left\{ 2\eta U k_\star{}^2 \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}k \, |\bar{z}_0(k)|^2 k^2 |k|}{k_\star{}^2 + k^2} - \frac{i\alpha}{C} \int\limits_{-\infty}^{\infty} \frac{\mathrm{d}k \, [\bar{z}_0(k)][\bar{M}^\star(k)] k^2 |k|}{k_\star{}^2 + k^2} \right\} \qquad (5)$$

which corresponds to equation (31) of Nye (p. 454). This formula relates the mean shear stress $\langle \tau_{xz} \rangle$ on the sliding ice to the Fourier-transformed bed topography $\bar{z}_0(k)$ and solute distribution $\bar{M}(k)$ at the ice–water interface of the subglacial water film. η is the effective viscosity of ice, U is the mean slip velocity, and $k_\star{}^2 = L/4CK\eta$. The variable $k_\star$ is the fundamental wave-number of glacier sliding theory that corresponds to the transition wavelength $\lambda_\star = 2\pi/k_\star$.

MODEL FOR SOLUTE DISTRIBUTION AND BED TOPOGRAPHY

The principal difficulty in using Equation (5) to calculate the effect of solutes on basal sliding over a rough bed is to obtain a valid expression for the solute distribution. A partial differential equation that takes into account the diffusion and advection of solutes in the regelation water film can be formulated. Solutions of this equation describe the solute distribution but presently they are only available for the case of regelation motion past round wires (Drake and Shreve, 1973, p. 70) and single sine-wave beds (Morris, unpublished). Unfortunately these solutions do not take into account potential sources and sinks of solutes which result from precipitation and dissolution of the bedrock. Solute sinks, such as provided by chemical precipitation, are particularly important because their omission in the diffusion equation leads to theoretical solute distributions that are incompatible with any steady-state solution for the problem of regelation sliding (Lliboutry, 1971, p. 27; Morris, 1972, unpublished).

Another approach to the problem of determining the subglacial solute distribution consists of simply estimating the maximum and minimum values of the solute concentration by considering the concentration necessary to form the subglacial precipitate and by roughly determining the solute content of water resulting from pressure-melting basal ice. Using these extreme values, a likely solute distribution can then be assumed to obtain a rough idea of the magnitude of the effect of solutes on glacier sliding. Let us consider a glacier sliding over a sinusoidal bed. Besides being mathematically convenient, this bed geometry makes it possible to evaluate easily the effect of solutes on the transition wavelength. Moreover, for sliding over a sinusoidal bed, the water flow along the thin subglacial film is well understood (Nye, 1973[b], p. 190) and the effect of impurities on the thickness of the water film can easily be assessed as shown in the Appendix. For sliding over beds of more complicated geometry, the water flow deduced from the present Nye theory is not compatible with the subglacial pressure distribution obtained from the same theory. To eliminate this discrepancy a major reformulation would be needed, which would take into account the rate of change of the temperature drop across the regelation water film, as suggested by Nye (1973[a], p. 394).

The subglacial deposits are generally found at the lee of bedrock protuberances in the size range of tens of centimeters in length parallel to the former sliding direction, and solutional furrows are eroded into the corresponding stoss surfaces. On larger bedrock obstacles, the deposits often form a thin coating on lee surfaces, but there they tend to be restricted to the region bordering the down-glacier edge of the stoss surfaces. Occasionally, the deposits are also found on the stoss sides of large-scale bedrock features (see for example Kers, 1964), but in such cases they are invariably localized in small concavities or depressions in the lee of

bedrock lips. Based on the location of the deposits and associated solutional features, it is apparent that subglacial waters must commonly be supersaturated along lee surfaces and undersaturated along stoss surfaces of small bedrock obstacles. For larger obstacles the solute distribution is not as simple but its exact nature is not critical to this analysis because the influence of solutes on sliding past obstacles much larger than the transition wavelength (about 0.5 m) is minimal as will be shown later.

The water along stoss surfaces contains solutes present in regelation ice plus any dissolved from the bedrock or rock fragments present in the film. The apparent desorption of cations from small particles initially contained in the ice (Souchez and others, 1973, p. 456) could contribute appreciably to the solute content of the melt water along stoss surfaces, but this contribution is greatly limited because of the extremely short period of time that a given portion of melt water is exposed to particles while it is along a stoss surface. For example, for pure regelation sliding at a rate of 10 m a^{-1} past a bed with irregularities having an amplitude-to-wavelength ratio of 0.05, and with a regelation water film one micrometer thick (Nye, 1973[b], p. 190), the mean exposure time is less than 100 s, hence only limited exchange can occur and the water will remain relatively pure. On the lee sides, however, the solutes are effectively concentrated by the selective removal of water associated with the continuous growth of regelation ice. The simplest model portraying the essential features of the hypothetical solute distribution which is generally compatible with the distribution of depositional and solutional features formed subglacially, is a sinusoid with the same wavelength as the bed, varying from a minimum concentration in the center of the stoss side to a maximum at the center of the lee side. Hence, let us consider a bed whose profile parallel to the ice flow is

$$z_0(x) = A \sin k_0 x \tag{6}$$

and let the solute distribution $M(x)$ be

$$M(x) = -M_0 \cos k_0 x. \tag{7}$$

The Fourier transforms of $z_0(x)$ and $M(x)$ are

$$\bar{z}_0(k) = \frac{\pi A}{i} \{\delta(k_0-k) - \delta(k_0+k)\}, \tag{8}$$

$$\bar{M}(k) = -\pi M_0 \{\delta(k_0-k) - \delta(k_0+k)\} \tag{9}$$

where δ is the Dirac delta function. If the length of the wavy part of the bed is much longer than the wavelength of bed undulations, the mean-square amplitude of the bed is

$$\langle z_0^2 \rangle = \frac{A^2}{2}. \tag{10}$$

Substitution of Equations (8), (9) and (10) into Equation (5) gives for the mean shear stress the formula

$$\langle \tau_{xz} \rangle = \left(\frac{k_0^3}{k_\star^2 + k_0^2}\right)\left(\eta U A^2 k_\star^2 + \frac{\alpha M_0 A}{2C}\right) \tag{11}$$

which corresponds to equation (34) of Nye (p. 455). As expected, this equation indicates that a greater basal shear stress is required to produce a certain bed-slip velocity when solutes accumulate at the lee of bed obstacles than when they do not. Because regelation is the dominant process contributing to sliding at low wavelengths λ, the solutes will have the greatest effect on the sliding process for a bed characterized by short roughness elements. To investigate how their effect varies with the size of the undulations, consider a measure of the roughness R, defined as

$$R = \frac{A}{\lambda} = \frac{A k_0}{2\pi}. \tag{12}$$

Equation (11) then becomes

$$\langle \tau_{xz} \rangle = \left(\frac{1}{k_\star{}^2 + k_0{}^2} \right) \left(4\pi^2 R^2 \eta U k_0 k_\star{}^2 + \frac{\alpha M_0 \pi R k_0{}^2}{C} \right). \tag{13}$$

The product αM_0 is half of the maximum temperature difference between stoss and lee sides due to the presence of solutes. By solving Equation (13) for U and differentiating it with respect to k_0, the bed wave-number at which the basal slip velocity is minimal for a fixed shear stress can be shown to be

$$k_{0\star} = \left(\frac{C k_\star{}^2 \langle \tau_{xz} \rangle}{C \langle \tau_{xz} \rangle - \pi \alpha M_0 R} \right)^{\frac{1}{2}} \tag{14}$$

for sliding over a bed of constant roughness R. Without solutes $k_{0\star} = k_\star$. In terms of U we have

$$k_{0\star} = \frac{\alpha M_0 + [(\alpha M_0)^2 + (4\pi C \eta R^2 U k_\star)^2]^{\frac{1}{2}}}{4\pi C \eta R^2 U}. \tag{15}$$

Hence the transition wavelength is decreased by the presence of solutes.

Another difference between the conventional and modified theory is that the value of the basal shear stress no longer tends to zero, but instead, for very short wavelengths, approaches the value $\pi \alpha M_0 R / C$. In the simple theory $\langle \tau_{xz} \rangle$ is directly related to the pressure asymmetry that gives rise to a temperature gradient, which exactly allows for the heat flow necessary for the melting and refreezing associated with regelation sliding. As the obstacle size decreases, the path along which heat is conducted also decreases and hence, for a fixed regelation sliding velocity, a smaller temperature gradient suffices to provide the required heat transport. Thus, the pressure asymmetry and accordingly the mean shear stress decrease with decreasing obstacle wavelength, reaching zero when the obstacle reaches zero length. When solutes concentrate at the lee of bed obstacles, a finite pressure excess along stoss sides is necessary to maintain the sliding velocity no matter how short the obstacle is; hence, $\langle \tau_{xz} \rangle$ remains finite as $\lambda \to 0$. This extrapolation to zero wavelength is only used to provide a qualitative suggestion of the effect of solutes on sliding over very small irregularities because Equation (7), describing the solute distribution, must break down as $\lambda \to 0$, since in this limit Equation (7) would necessitate infinitely high concentration gradients. Moreover, as discussed earlier, the maximum solute difference that can suitably be treated by the analysis decreases with the wavelength for beds of constant roughness, approaching zero as $\lambda \to 0$.

EVALUATION OF SOLUTE DISTRIBUTION

The maximum difference in concentration between the stoss and lee sides of bed obstacles is the critical parameter determining the magnitude of the effect of solutes on the sliding process.

The temperature and composition of regelating subglacial melt water, from which solutes are slowly precipitating, have to be close to their eutectic values because the eutectic point for the system in question is the only point at which the solution is simultaneously in quasi-equilibrium with both the ice and the precipitate. When calcite forms subglacially the eutectic temperature and composition are unique for a fixed pCO_2. The water at the lee of bed obstacles can be supersaturated in gases, because bubbles must initially form there, inasmuch as they are observed in regelation ice (Kamb and LaChapelle, 1964, p. 163). If a gas phase is present, its pressure can be approximately estimated from the weight of the overlying ice, and the partial pressure of its constituents then estimated from its composition. Assuming that the gaseous composition of basal ice is similar to that of surface ice (Weiss and others, 1972, p. 182) and that gas bubbles form at the freezing interface, the effective pCO_2 is likely to range from 0.1 to 0.5 bar (1×10^4 to 5×10^4 Pa) along lee surfaces at the base of a

glacier where the pressure is, say, 10 bar (10^6 Pa), corresponding to a typical ice thickness of roughly 100 m. Owing to active sliding, the pressure at the lee of bed obstacles may be considerably lower than the mean overburden pressure, so that the chosen pressure of 10 bar (10^6 Pa) may often correspond to a thicker glacier. At the eutectic point a $p\mathrm{CO_2}$ of 0.25 bar (2.5×10^4 Pa) corresponds to a calcium ion concentration of 8.1×10^{-3} M, a total dissolved $\mathrm{CaCO_3}$ and $\mathrm{CO_2}$ concentration of 7.0×10^{-2} M and a temperature of $-0.11°\mathrm{C}$ (Hallet, 1975, in press, unpublished). These values should be representative of those expected in regelating subglacial water at the lee of bed obstacles where $\mathrm{CaCO_3}$ is precipitating.

Along stoss surfaces the solute content of subglacial waters is equal to that of the regelation ice formed behind obstacles up-stream in addition to any contribution due to dissolution of the bedrock and particles contained in the melting ice, and to liberation of ions apparently adsorbed on particles (Souchez and others, 1973, p. 456). Although the latter process could in general substantially increase the solute concentration of regelation melt water, it is not likely to play an important role for glaciers underlain by carbonate bedrock because the surfaces of carbonate fragments, unlike those of many silicates, supply almost no sites for potential ion adsorption. To elucidate the behavior of dissolved carbonates in regelating waters, $\mathrm{CaCO_3}$ solutions were frozen experimentally at rates and with freezing front morphologies that are believed comparable with those expected in connection with regelation sliding (Hallet, 1975, in press, unpublished). These experiments indicate that ice, growing slowly, with no convection in the melt, has a solute content 50 to 100 times smaller than that of the melt. This solute partitioning may be used to estimate the composition of ice formed by regelation, but regelation freezing may be accompanied by quantitatively different solute redistribution because of the vastly different volumes and flow conditions in the melt between the experimental and regelation cases. Despite the uncertainty of the exact distribution coefficient, the water derived from melting basal ice will be relatively pure, as was observed by Souchez and others (1973, p. 458). This water will tend to dissolve the bedrock and suspended particles until the solution reaches the saturation point, which is principally dependent on the $\mathrm{CO_2}$ content of the basal ice. It is independent of the pressure, because the water along stoss sides is likely to be highly undersaturated with respect to dissolved gases. Assuming again that the $\mathrm{CO_2}$ content of basal ice is similar to that of surface temperate glacial ice (Weiss and others, 1972, p. 182), an effective $p\mathrm{CO_2}$ between 4×10^{-5} and 4×10^{-4} bar (4 and 40 Pa) can be expected in water along stoss surfaces. With this range of $p\mathrm{CO_2}$, the total solute concentration is likely to be considerably less than the saturation value of 2.4×10^{-3} M, because the flowing water probably does not have sufficient contact time with the bed even to approach saturation. Moreover, the concentration at the melting interface, which controls the temperature along the stoss surface, is probably considerably less than the overall bulk concentration of the melt water. Thus the concentration of dissolved $\mathrm{CaCO_3}$ along stoss surfaces will generally be considerably lower than along lee surfaces. A conservative estimate is that the stoss-side concentration is less than 5% of the lee-side concentration. For $p\mathrm{CO_2} = 0.25$ bar (2.5×10^4 Pa) the total solute concentration along the center of the lee surfaces, where the concentration is greatest, exceeds that along the stoss surfaces by about 0.067 M, which corresponds to a maximum difference of about 0.10°C in melting temperature and about 8×10^{-3} M in $\mathrm{Ca^{2+}}$ concentration.

Assuming that the solute distribution is sinusoidal, as expressed in Equation (7), the product of αM_0 in Equation (13) is equal to half of this maximum difference in the melting temperature from stoss to lee side. Substituting this crude estimate into Equation (13) gives the bed-slip velocities plotted in Figure 1. As expected the effect of solutes is most pronounced at short wavelengths, because for them sliding depends mainly on the regelation process. Note that the presence of solutes reduces the transition wavelength, which is that at which the velocity is least for a fixed shear stress. For the particular solute distribution assumed in this example the transition wavelength is reduced from 60 to 20 cm.
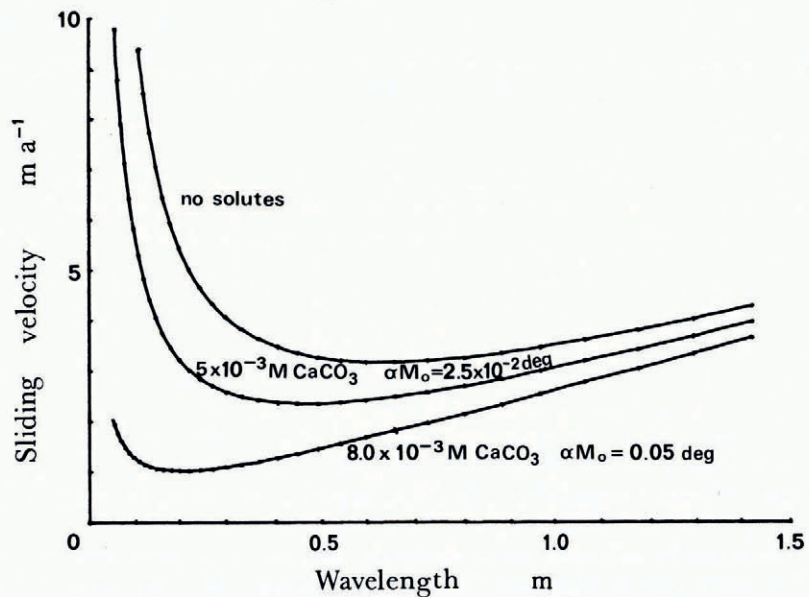
Fig. 1. Wavelength dependence of glacier sliding velocity in the presence and absence of solutes in regelation waters. Temperate ice slides over sinusoidal beds with constant roughness under a fixed shear stress of 1 bar. $CaCO_3$ dissolved in regelation waters is distributed sinusoidally. The maximum concentration difference between lee and stoss sides and the corresponding temperature deviation αM_0 from the mean are indicated. Values of 1 bar a^{-1} and 10 m^{-1} were taken respectively for the ice viscosity and the transition wavenumber. These curves, and those in Figures 2 and 3, are truncated below 0.05 m because, for shorter wavelengths. the theory is no longer valid for the highest solute variabilities considered.
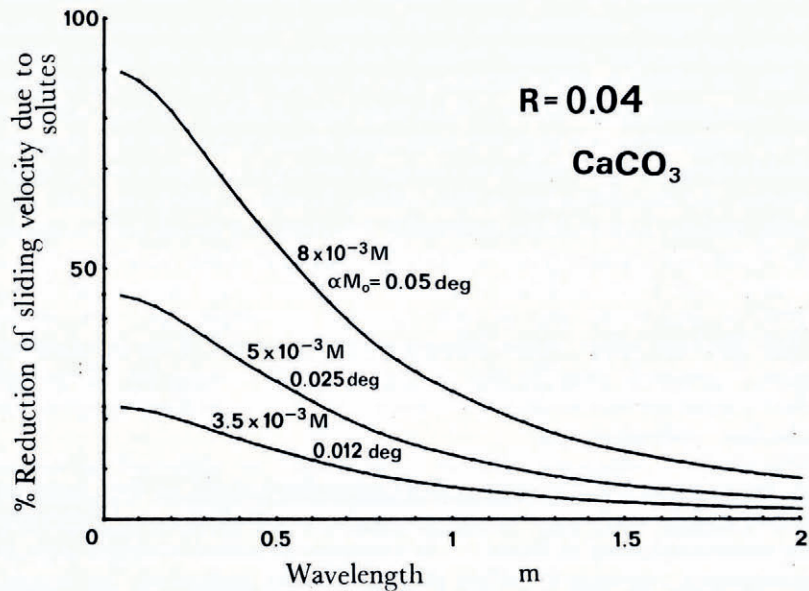
Fig. 2. Reduction in the sliding velocity due to the presence of solutes as a function of bed wavelength λ and solute concentration M_0. Basal shear stress τ is fixed at 1 bar (10^5 Pa) and the roughness R at 0.04. The percent reduction in sliding velocity approaches $100\pi\alpha M_0 R/\tau C$ for shorter wavelengths.

The importance of the effect can best be visualized by plotting for fixed stress the reduction in sliding velocity as a function of the bed wavelength, as in Figure 2. Figure 3 shows similar curves for fixed solute concentration and various bed roughnesses. The curves in Figures 1 through 3 were not extended below 5 cm because of restrictions necessitated by the analysis on the solute distribution $M(x)$ in Equation (2), which are most stringent for short bed wavelengths. These figures show that an excess of dissolved $CaCO_3$ of only 8×10^{-3} M in the thin regelation water film along the lee sides of bed obstacles can drastically reduce the sliding velocity if the bed is characterized by roughness elements shorter than about 1 m.
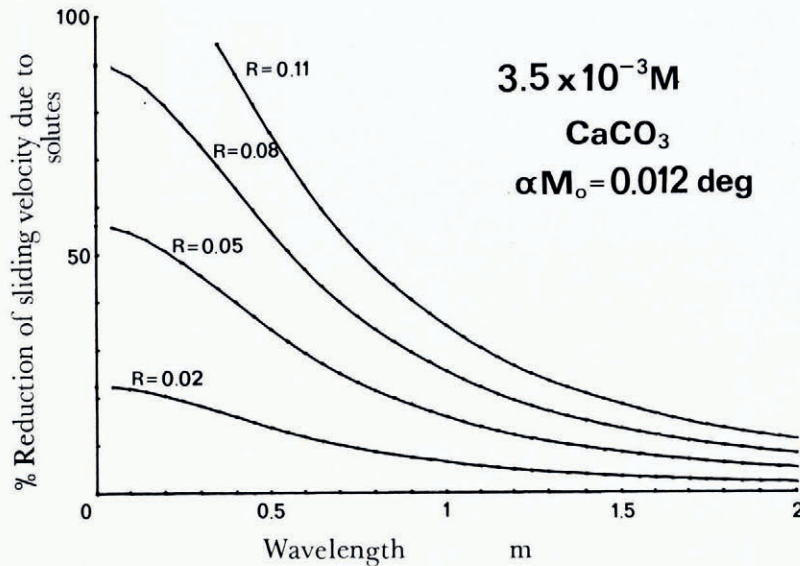
Fig. 3. *Reduction in the sliding velocity due to the presence of solutes as a function of the bed wavelength and roughness. Basal shear stress is held constant at 1 bar (10^5 Pa).*

The asymmetry in the solute distribution calculated in this analysis should be interpreted with caution, because the CO_2 content of regelation ice was assumed to be the same as that of surface ice. This assumption was necessitated by the lack of data on the gas content of regelation ice and could be considerably in error. In addition, the overburden pressure, and hence the effective pCO_2 at the lee of bed obstacles, is only a rough estimate, inasmuch as the thicknesses of temperate glaciers range from essentially zero to several hundred meters or more. Because the magnitude of the solute effect is most sensitive to the pCO_2 along lee surfaces, it may also vary over a considerable range. Finally, the undersaturation and supersaturation required for finite rates of dissolution and precipitation were ignored, as was the finite CO_2 supersaturation required for CO_2 to contribute to the formation of gas bubbles, which lead to an underestimate of the effect of solutes on the sliding velocity.

Despite the grossly oversimplified model used here for the glacier bed topography and the solute distribution, it is apparent that dissolved $CaCO_3$ in the subglacial water film can inhibit basal sliding markedly if the wavelength characteristic of the bed does not exceed about one meter.
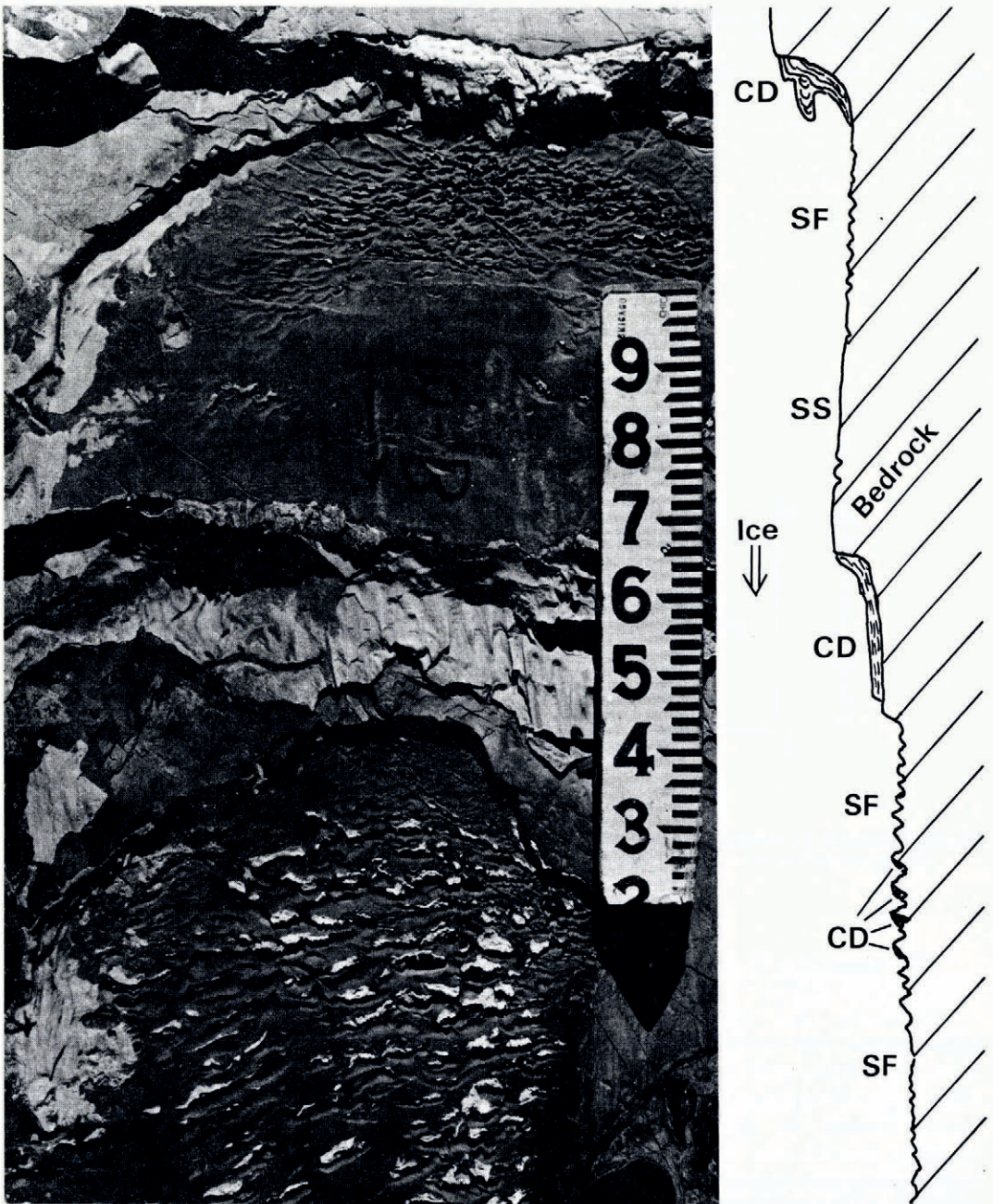
Fig. 4. *Dissolutional furrows (darker areas) and chemical deposits (lighter areas) formed under a sliding temperate glacier. The schematic cross-section, taken through the center of the photograph, shows where the calcite deposits (CD), solutional furrows (SF) and striated surfaces (SS) are located with respect to the micro-topography. The ice was formerly sliding in contact with the bed, without cavitation, in the direction indicated by the foot-long (30 cm) ruler.*

Discussion

It appears unlikely that the mechanics of sliding over a bed of arbitrary form would be markedly different from that of sliding over a sinusoidal bed of equal roughness. If so, the importance of the solute effects depends on the surface characteristics of actual glacier beds. Kamb (1970, p. 707–08) has suggested that roughness elements shorter than a few meters parallel to the ice flow direction are generally absent from glaciated bedrock because they are presumably removed by selective glacial abrasion. Spectral analyses of three detailed topographic profiles of recently deglaciated bedrock surfaces, measured by the author, indicate clearly, however, that roughness elements of shorter wavelengths are in fact present. Moreover, there are indications that, instead of tending to erode the glacier bed flat, subglacial processes at times accentuate and even create certain roughness features. Of particular interest are closely spaced solutional furrows on stoss surfaces that clearly result from subglacial bedrock dissolution. These solutional furrows, which are roughness elements of very short wavelength, are illustrated in Figure 4. They tend to be oriented perpendicular to the direction of former ice flow because, with this orientation, they produce more melt water and hence greater dissolution rates, and thus tend to grow faster than furrows in other orientations. Inasmuch as glaciated bedrock surfaces, which are presumably similar to subglacial ones, seem characteristically to include small roughness elements, it appears reasonable to expect solutes to have a generally significant effect on glacier sliding.

It should be emphasized, however, that the subglacial solute distribution associated with glacier sliding over a bed whose surface is characterized by an entire spectrum of roughness elements may be considerably different from that considered here. The principal divergence would probably arise from the interference, within the subglacial film, of water and ionic fluxes due to different bed harmonics.

Solutes will affect basal sliding as long as the concentration is appreciably higher along the lee sides than along the stoss sides of bed obstacles. Where glaciers flow over calcareous bedrock, the non-uniform calcite solubility resulting from the accumulation of CO_2 along lee surfaces assures that even if the concentration along stoss surfaces reaches saturation, it is still much lower than along lee surfaces. For other systems the pCO_2 has no direct effect on the mineral solubility; therefore the saturation concentration is likely to be essentially uniform along the water film. If the rate of dissolution is sufficiently slow, the water flowing along stoss surfaces would not have sufficient time to approach saturation. The limited amount of solute contained in the water is continuously concentrated by freezing along lee surfaces. Eventually the concentration of the solution will reach saturation and will probably be considerably greater than along stoss surfaces. Hence solutes other than $CaCO_3$ should also influence sliding, as long as they do not dissolve too rapidly.

Unfortunately, there does not appear to be any clear field evidence at present concerning the possible effect of solutes on basal sliding. An ideal situation would be one where a temperate glacier flows across a contact between essentially insoluble rocks and calcareous ones. The rate of basal sliding ought to be reduced over the calcareous terrain, if the bed roughness and the basal shear stress are uniform.

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APPENDIX

THE THICKNESS OF THE REGELATION WATER FILM IN THE PRESENCE OF SOLUTES

This calculation is identical to that of Nye (1973[b], p. 192) with the modifications necessary to include the effect of solutes in the subglacial regelation water film. The modified general expression for the Fourier transform of the ice velocity normal to the bed is to first order the following:

$$\overline{W}(k) = \frac{-ik^3 \bar{f}(k) U}{k_\star^2 + k^2} - \frac{\alpha k \overline{M}(k)}{2\eta C(k_\star^2 + k^2)} \,. \tag{A-1}$$

This equation and Equation (4) are analogous to Equation (26) of Nye (p. 452). Now consider the bed of the glacier and the solute distribution to be sinusoidal as respectively described by Equations (6) and (7). Expressions for the basal pressure distribution $P(x)$ and the velocity of ice normal to the bed $W_h(x)$ can then easily be obtained by substituting Equations (6) and (7) in Equations (4) and (A-1), which have been expressed in terms of untransformed variables. They are as follows

$$P(x) = \left(\frac{k^2 \cos kx}{k_\star^2 + k^2}\right)\left(2\eta UAk^2 + \frac{\alpha M_0}{C}\right), \tag{A-2}$$

$$W_h(x) = \left(\frac{k \cos kx}{k_\star^2 + k^2}\right)\left(-AUk^2 + \frac{\alpha M_0}{2\eta C}\right). \tag{A-3}$$

The thickness of the water film is that which allows for the discharge of water through the film deduced from Equation (A-3) under the pressure distribution described by Equation (A-2). Treating the subglacial water flow as a viscous flow between parallel plates and carrying through to the solution, the film thickness is found to be

$$t = \left[\frac{6\rho_i \eta_w}{\rho_w \eta k^3}\left(\frac{2\eta CAUk^2 - \alpha M_0}{2\eta CAUk_\star^2 + \alpha M_0}\right)\right]^{\frac{1}{4}} \tag{A-4}$$

where ρ_i and ρ_w are the densities of ice and water, and η_w is the viscosity of water. Without solutes ($M_0 = 0$), Equation (A-4) is of course identical to the corresponding expression of Nye (1973[b], p. 193). For a fixed sliding velocity, solutes tend to increase the pressure gradients and reduce the ice velocity normal to the bed because sliding by regelation is impeded by the presence of solutes in subglacial waters. A thinner film suffices for the passage of regelation waters when solutes are present because the pressure gradient along the film is higher and, in addition, the water flow is lower due to the reduced rate of ice melt. The reduction of the film thickness due to solutes is not, however, considerable because of the small exponent on the right side of Equation (A-4). For example, the film thickness is reduced only by 30% for sliding at 10 m a^{-1} over a sinusoidal bed with a wavelength of 0.05 m, an amplitude of 2×10^{-3} m and αM_0 equal to 0.05 deg, and the effect of solutes decreases with increasing wavelengths.