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Non crystallized regions of White Dwarfs. Thermodynamics. Opacity. Turbulent convection.

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Abstract

The evolution of White Dwarf stars along their cooling sequences is governed not only by their thermal content, but also by the rate at which heat flows through the external, partially degenerate and non-isothermal layers. In particular, cooling is found to be largely influenced both by the optical atmosphere, and by the convective envelope. The first one, in fact, determines the internal density stratification, down to the point at which electron degeneracy takes over, while the second one affects the temperature stratification in the same layers. The reliability of the present generation of models of White Dwarf envelopes is discussed, on the grounds of the main physical inputs (thermodynamics, opacity, convection theory), for both H-rich and He-rich surface chemical compositions. The conclusion is that, below $Log L/L_{\odot} \leq -3$, we can build little more than *test models.*

L'évolution des naines blanches le long de leur séquence de refroidissement est gouvernée non seulement pas leur contenu thermique, mais aussi par la vitesse à laquelle la chaleur s'échappe à travers les couches externes, non-isothermes et partiellement dégénérées. En particulier, le refroidissement est largement influencé à la fois par l'atmosphère optique et par l'enveloppe convective. La première détermine la stratification interne en densité jusqu'à ce que la dégénérescence électronique prenne le dessus, alors que la seconde affecte la stratification en température dans les mêmes couches. Nous discutons la validité de la génération actuelle de modèles d'enveloppes de naines blanches, sur la base des ingrédients physiques (thermodynamique, opacité, théorie de la convection), pour à la fois des compositions chimiques de surface riches en hydrogène et en hélium, respectivement. La conclusion est que, en dessous de $LogL/L_\odot \leq -3$, il n'est guère possible de proposer autre chose que des *modèles test*.

7.1 Introduction

Observations seem to definitely show that the luminosity function of the nearby White Dwarfs (WDs) display an abrupt break-down in the luminosity range $LogL/L_\odot$= -4.0 ÷ -4.5 (Liebert et al. 1989). The obvious interpretation of this feature is the finiteness of the age of the galactic disk —in the vicinity of the Sun— such that even the first born among the WDs are not yet old enough to have cooled below this luminosity (D'Antona and Mazzitelli 1978). The alternative explanation of WDs being already in the fast Debye cooling phase (D'Antona and Mazzitelli 1989), should in fact lead to a milder decrease of the luminosity function, than apparently suggested by the observations.

In this framework, it should be legitimate to assume WDs as powerful probes for evaluating the age of the galactic disk (Winget et al. 1987). However, if this is the goal, we have first to make sure about the reliability of the present generation of stellar models, also because the external layers of the WDs —which determine the cooling rate— are at such (relatively) high densities and (relatively) low temperatures, that matter is definitely far from ideal gas conditions. Also, in old (and cold) WDs, the largest fraction of the temperature difference between surface and centre is found in the convective region. Although the convective gradient is in any case very close to the adiabatic gradient —due to the large density— the Mixing Length Theory (MLT), tuned on the sun, is hardly expected to be the more realistic model for dealing with turbulent convection in dense, partially ionized and partially degenerate conditions.

In the following, WDs models of different luminosities and surface chemical compositions will be discussed showing that, especially in the case of He-rich stars, the ρ-T region where our present physical understanding of the thermodynamics and of the heat transport properties of a partially ionized real gas is still rather poor, is met relatively soon during the evolution.

7.2 The main ingredients

There is by now wide agreement about some general features of WDs. The masses of the most of WDs, for instance, seem to lie in the range 0.5≤M≤0.6 $M_{\odot}$(Weidemann 1990). For this reason, all the following tests will be performed on a model having M = 0.55 $M_{\odot}$. Also, gravitational settling has been established as a fast and powerful mechanism to achieve, at the surface of a WD, very low metal abundances, and complete separation between hydrogen —if any— and helium (Fontaine and Michaud 1979). Then, a very low metal abundance Z=10−5 has been chosen for the models discussed below, and both pure-H and pure-He surface compositions are tested.

Finally, a more tricky point. The evolutionary models coming from the Asymptotic Giant Branch show that WDs are left with a thin hydrogen mantle, of the order of 5 10−5 $M_{\odot}$(Iben and McDonald 1986). The pulsational properties, however, seem to suggest thinner H-layers, of the order of 10−8 $M_{\odot}$or even less (Winget et al. 1982). A *compromise* between the two alternatives has been chosen here, and the H-rich models tested below have an hydrogen mantle of thickness 10−7 $M_{\odot}$.

Since the basic evolutionary sequence has been evolved starting from the Horizontal Branch and through the main Thermal Pulses phase, an artificial metal settling and He-depletion algorithm has been applied at the beginning of the WD phase, in order to be consistent with the previously discussed chemical constraints. The adopted algorithm can perhaps lead to minor numerical disturbances at the beginning of the WD evolution which, however, do not give rise to any further noise during the main cooling phase, since the duration of the high luminosity WD phases is very short, and any perturbation of the external layers is fastly reset, the WD finally setting on its cooling track. It is worth noting that, due to the presence of *breathing pulses* (Caloi and Mazzitelli 1993) at the end of the HB phase, the most of the carbon in the core is transformed into oxygen (∼ 90%, if the enhanced $12C + \alpha$ reaction rates (Kettner et al. 1982) are assumed; about 70

As for the main physical inputs:

(i) thermodynamics is from Magni and Mazzitelli (1979)
(ii) neutrinos are from Itoh et al. (1992, and references therein)
(iii) thermal conduction from Itoh et al. (1984)
(iv) atmosphere is grey, with a simple T(τ) relation

and other inputs (radiative opacities, convection theory etc.) will be discussed in the following. The numerical structure is integrated in ∼1000 mesh points, ant the evolution from the beginning of the WD phase down to final cooling ($LogL/L_{\odot}\leq$ -4.5) takes about 3000 time steps.

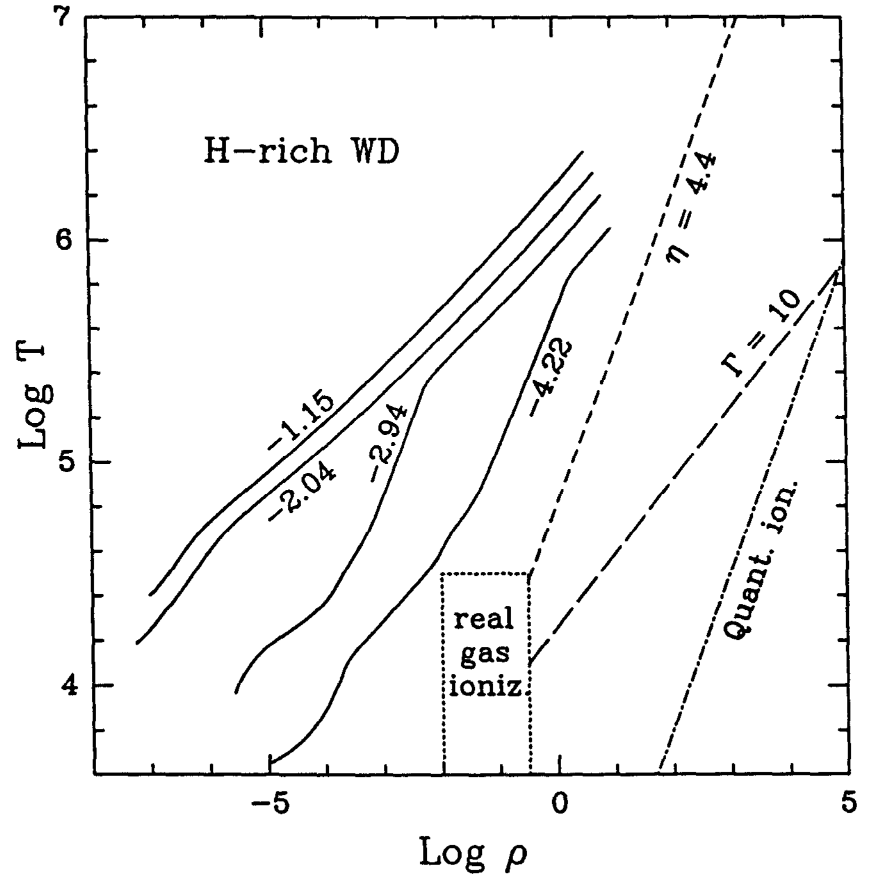

Fig. 7.1
The position on the ρ-T plane of the external, H-rich layers of a 0.55 $M_\odot$ white dwarf at several luminosities (solid lines). The dotted rectangle marks the region where pressure dissociation and ionization is approaching; the short-dashed line marks onset of electron degeneracy; the long-dashed line shows where the coulomb coupling parameter Γ reaches the value 10, and the dashed-dotted line marks the region where ions start behaving as quantum particles.

7.3 Hydrogen rich models

7.3.1 Thermodynamics

The envelope structures at different luminosities of the test WD are plotted on the ρ-T plane in Fig. 1.1.

It is clear that, at least as long as thermodynamics alone is concerned, the modeling of H-rich WD envelopes is relatively safe until down to low luminosities. In fact, only for $LogL/L_\odot \leq$ -4 a small fraction of the envelope is (marginally) close to the non-ideal gas region. Also, H-rich WDs interiors are always well far from the region where collective effect begins to be

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present, since the Coulomb coupling parameter Γ is always relatively small (its value never overcomes $\Gamma \sim 5$). Finally, at least for WDs, we have not to worry about the quantum behavior of hydrogen.

The tracks are shown down to the H/He interface. For deeper H-rich envelopes (up to the evolutionary remnant mass, which is three orders of magnitude larger than the one adopted in the present computations) the situation would not substantially change since, in the worst of the cases, the tracks would go ahead at larger temperatures and densities, eventually meeting the electron degeneracy conditions, for which a classical thermodynamical treatment is available.

Also in the case of thinner envelope masses, down to 10−14 $M_{\odot}$, as suggested by some spetroscopical features (Liebert et al. 1987), one should not expect substantial differences. In fact, as long as the *optical atmosphere* is hydrogen-dominated, the density at which the subatmospheric envelope begins is not affected by the thickness of the H-envelope and, since the slope of the tracks in the ρ-T diagram is mainly dictated by the adiabatic gradient of temperature, even after the transition to helium the tracks should not jump to ρ-T regions far different from the ones shown in Fig.1.1. One can then conclude that, from the point of view of thermodynamics alone, our present understanding of H-dissociation and ionization at high density is probably sufficient to give us reliable WD models down to very low luminosities. It is perhaps worth explicitly noting that it is not legitimate to extend the same conclusion also to Brown Dwarf stars, where the physical processes are quite different.

7.3.2 Opacity

Most unfortunately, the optimistic conclusion of the preceeding section, cannot be extended to the other relevant physical input, that is: the radiative opacities. In fact, from Fig. 1.2, one can see that, for luminosities below $Log L/L_{\odot} \sim$ -3, the optical atmosphere lies in regions of low-T and high-ρ, where no extended opacity sets are presently available.

The Los Alamos opacities (Huebner 1977) do not reach temperatures below Log T=4.2, where molecules can be still present, and the OPAL ones (Rogers and Iglesias 1992), even if they reach lower temperatures (T=6000oK), are provided only for relatively low densities. The lack of radiative opacities represent a serious limit to our abilities of modeling stellar structures, since the opacity in atmosphere determines, by orders of magnitude, the density at the base of the atmosphere, at which the envelope begins. In other terms, for H-rich WDs at $Log L/L_{\odot} \leq$ -3, the location at

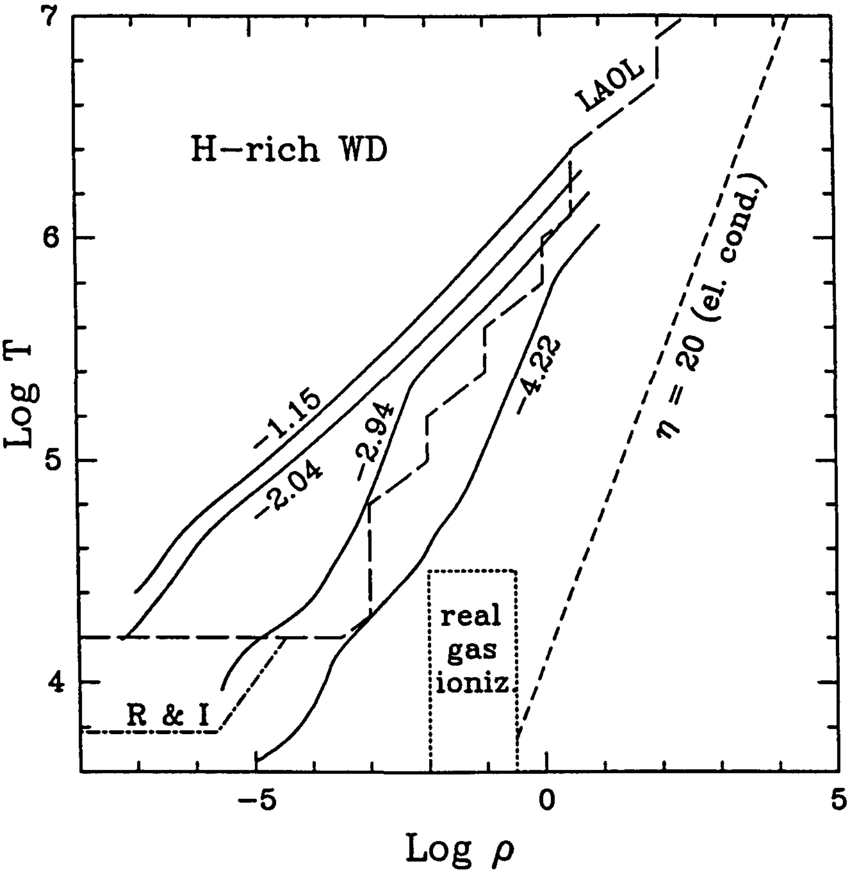

Fig. 7.2 The same as Fig. 1.1 but for the opacities. Right of the short-dashed line, electron conduction takes over. Left of the long-dashed region, the Los Alamos radiative opacities are available; left of the dotted-dashed line, the OPAL opacities are available.

which the subatmosphere begins in Fig. 1.2, can actually lie on a horizontal segment (the temperature being fixed, according to luminosity and mass) of unknown amplitude. The coolest models shown, then, are very uncertain, since they have been modeled according to a given numerical recipe for extrapolating opacity tables, and not according to physically sound opacities.

The conclusion is then that, for H-rich WDs, our present knowledge of radiative opacities confines our capability of correctly predicting the evolution of WDs to luminosities $Log L/L_{\odot} \geq$ -3. Below this limit, we can certainly build *test* models but, basing upon these last, we can scarcely draw sound quantitative conclusions about galactic evolution.

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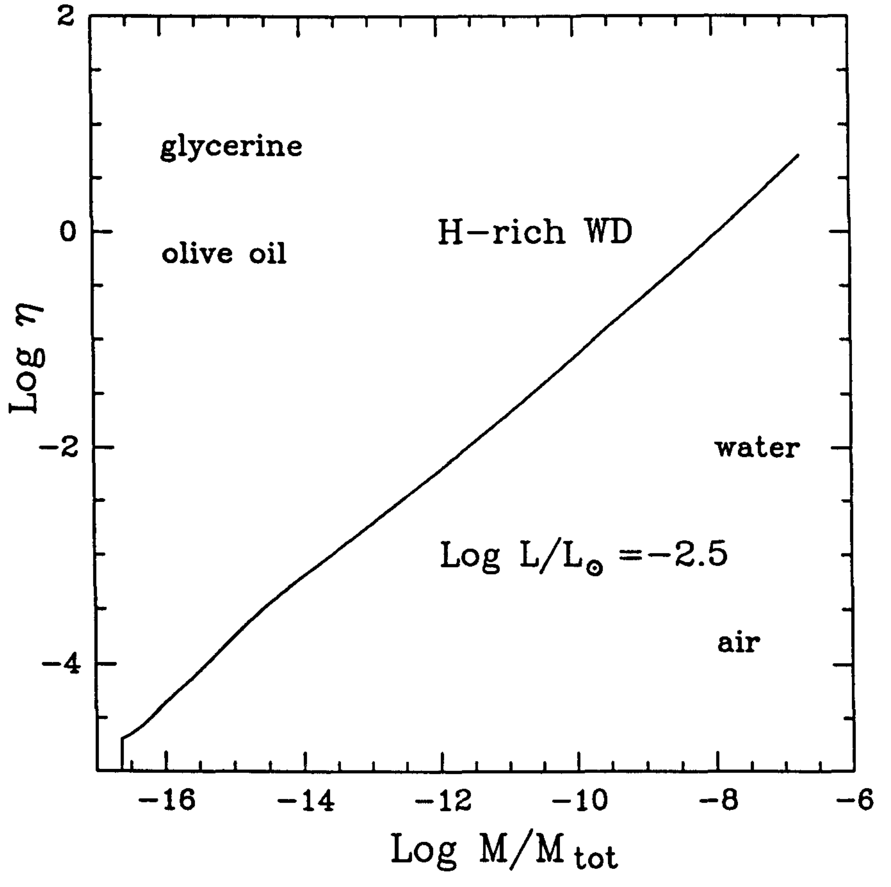

Fig. 7.3 The dynamical viscosity η as a function of the depth, in the subatmospheric region of a H-rich WD at $Log L/L_{\odot}$= -2.5. The abscissa shows the Log of the fractional mass starting from the surface.

7.3.3 *Turbulent convection*

Up today, the MLT has been widely applied to the modeling of WDs convective envelopes since, in any case, the rate of overadiabaticity in the dense external layers of these objects is expected to be negligible or quite so. This seems in fact to be the case as long as the *thermal* properties of WDs are concerned, even if other properties of WDs, as the pulsational ones, are instead much more sensitive to the treatment of convection (Pelletier et al. 1986). However, the problem is perhaps not so settled, as will be shown below.

The MLT is usually tuned on the sun, and one can legitimately ask which can be its predictive power, when applied to conditions in which the density is up to six or seven orders of magnitude larger than in the solar subatmosphere. Let us examine this point more in detail.

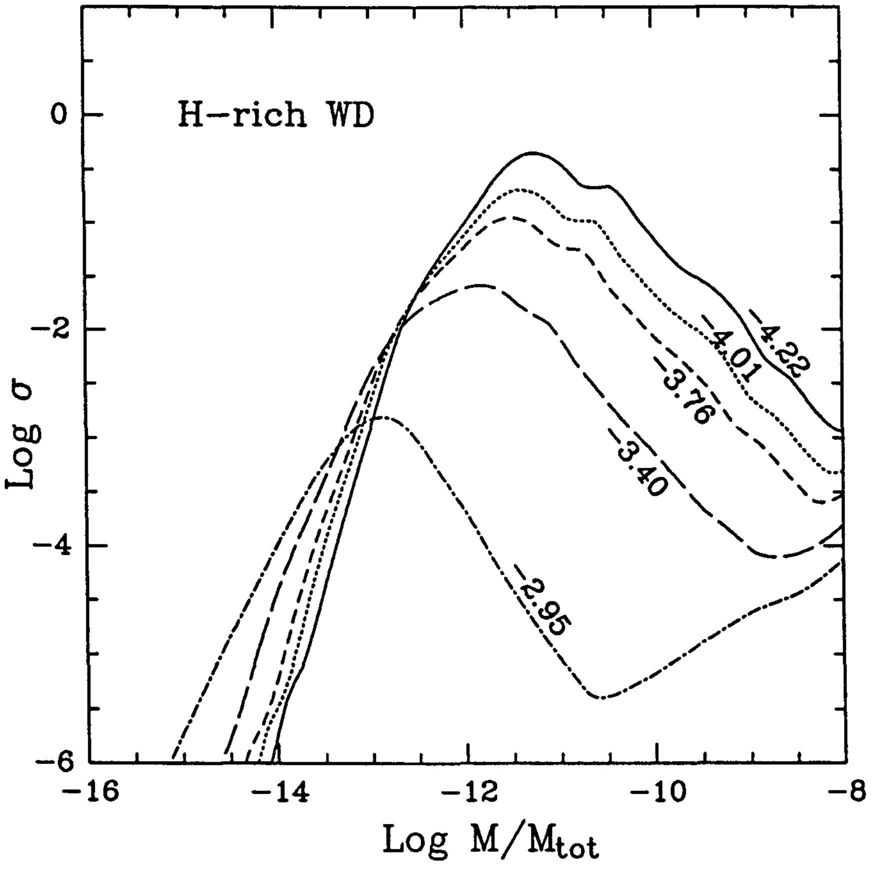

Fig. 7.4 The behavior of the Prandtl number as a function of depth, as in Fig. 1.3, at various luminosities, for a H-rich WD. For comparison, remember that the Prandtl number in the solar subatmosphere is $\sigma \sim 10^{-9}$.

Much larger densities, could imply for instance larger viscosities than in the case of the almost inviscid sun. The coefficient of dynamical ion viscosity η can be easily computed for a coulombic plasma, according to Wallenborn and Baus (1978). The results for the case of the subatmosphere of a H-rich WD, are shown in Fig. 1.3. Viscosity can be quite large, so large as to equal that of very viscous terrestrial fluids.

Actually, the knowledge of viscosity alone is not sufficient, since also for a relatively large value of η, matter in a star can still have room enough to generate a wide spectrum of viscous eddies, and full developed turbulence. The Prandtl number $\sigma = \nu/\chi$, where ν is the kinematical viscosity coefficient and χ the thermometric conductivity, is in this respect, much more significant, since it is directly connected to the amplitude of the eddies spectrum.

As can be seen from Fig. 1.4, during the cooling of the WD, the value of σ in the convective region steadily increases, reaching $\sim$1 below $LogL/L_{\odot}=$ -4. This leads to a consistent shrinking of the turbulent eddies spectrum with respect to an inviscid case, and the efficiency of the convective heat transfer decreases. In particular, from Canuto and Mazzitelli (1991) one can see that the integral of the energy spectrum of the turbulent eddies versus the vawenumber does not change significantly when σ decreases below $10-3$, but this is not the case for larger values of σ, since the cut-off wavelength increases proportionally to $\sigma^{3/4}$.

This effect is shown in Fig. 1.5, where the energy spectral distribution of the turbulent eddies is displayed for two different values of the Prandtl number. For comparison, remember that the corresponding spectral distribution in the MLT case would have been, in the same scale, a delta function peaked at the growth rate. Adjusting the α parameter in the MLT, corresponds then to tune the surface of the (infinitely high and infinitely thin) delta function, out of any correlation with the significant physical parameters.

The effect of viscosity on the heat transfer in WDs has not yet been satisfactorily addressed, also because the MLT does not allow viscosity to be explicitly accounted for. With the new stellar convection theory by Canuto and Mazzitelli (1991), it is instead possible to account also for viscosity. Detailed computations have not yet been performed, but in Fig. 1.6, a qualitative evolutionary scenario is shown.

In this scenario, an artificial drag was included in the models, such that, along the structure, the local overadiabatic temperature excess was forced to increase proportionally to the local value of the Prandtl number. Qualitatively, this is just the effect expected in real structures, but the simulation performed is a barely parametric one. The results have then no quantitative significance, showing only the direction towards which a consistent treatment of viscous turbulence would change the evolution.

As can be seen, when viscosity begins to increase, the corresponding overadiabaticity leads to an increase in the temperature difference between surface and centre of the star, with respect to the inviscid case. The thermal content of the star, then, remains larger for a longer time, and also the cooling times increase.

Even if the former conclusion is only qualitative, one can nevertheless look at it as another reason why we can hardly rely on our present models of low-luminosity H-rich WDs.

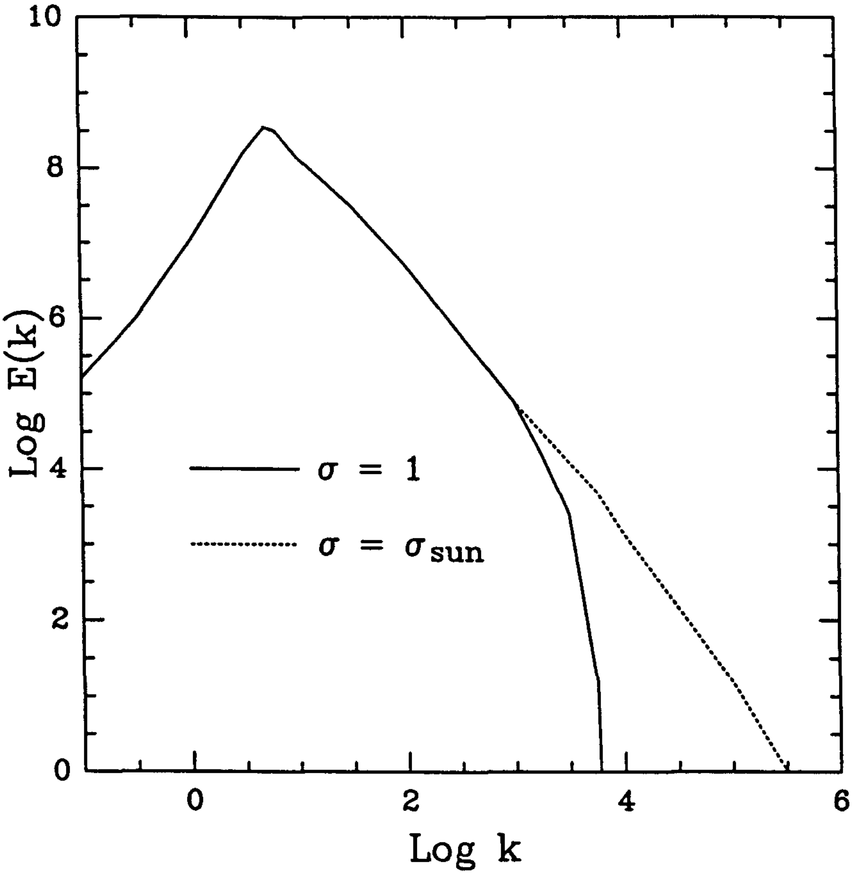

Fig. 7.5 **The true energy spectrum of turbulent eddies as a function of the vawe number, for two different values of the Prandtl number σ. The corresponding MLT spectrum would have been a delta function.**

7.4 Helium rich models

7.4.1 Thermodynamics

The case for He-rich WD models is far different from the H-rich ones. The main reason is that, at relatively low surface temperatures ($T \leq 20000oK$, helium gives an absolutely negligible contribution to opacity, which is due only to the trace metals. This implies, in turn, that the results presented in the following will have a significance only for the chosen metal abundance ($Z=10-5$). Any other metal abundance would have given completely different results, contrarily to the case for H-rich envelopes, where the above discussed results are indicative of a general *low metal* case.

Due to the very low radiative opacities at low temperature, the He-rich optical atmospheres turn out to be very thick (in mass) and dense, so that, in the ρ-T diagram , the tracks of the subatmospheric envelopes are shifted

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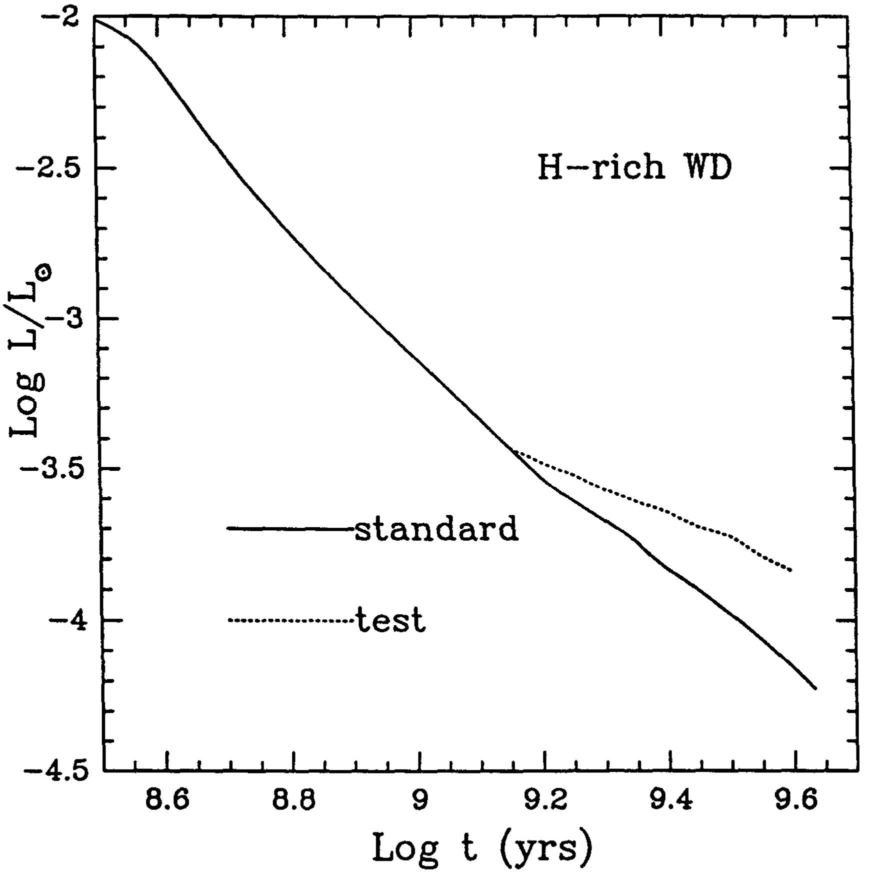

Fig. 7.6 The temporal evolution of a H-rich WD, both ignoring the effect of viscosity on turbulent convection (solid line) and testing a possible scenario for viscous convection (dotted line).

towards much larger densities with respect to the H-rich case, and larger densities mean complicances due to real-gas effects, collective effects and quantum effects. It is worth explicitly noting here that, since the chances are that the metal abundances in the atmospheres of cold He-rich WDs could be even lower than Z=10−5, the following discussion elucidates perhaps the *more favourable case*, the atmospheres of real He-rich WDs being probably even thicker and denser.

Figure 1.7 elucidates the thermodynamic problems met in the case of He-rich envelopes. As soon as the luminosity of the WD drops below $LogL/L_{\odot}$= -2, part of the convective region begins to lie in a non-ideal gas zone. Since the thermal structure of the envelope is largely sensitive to the adiabatic temperature gradient, it is also clear that a very good equation of state for partly-ionized and partly-degenerate real gas is required.

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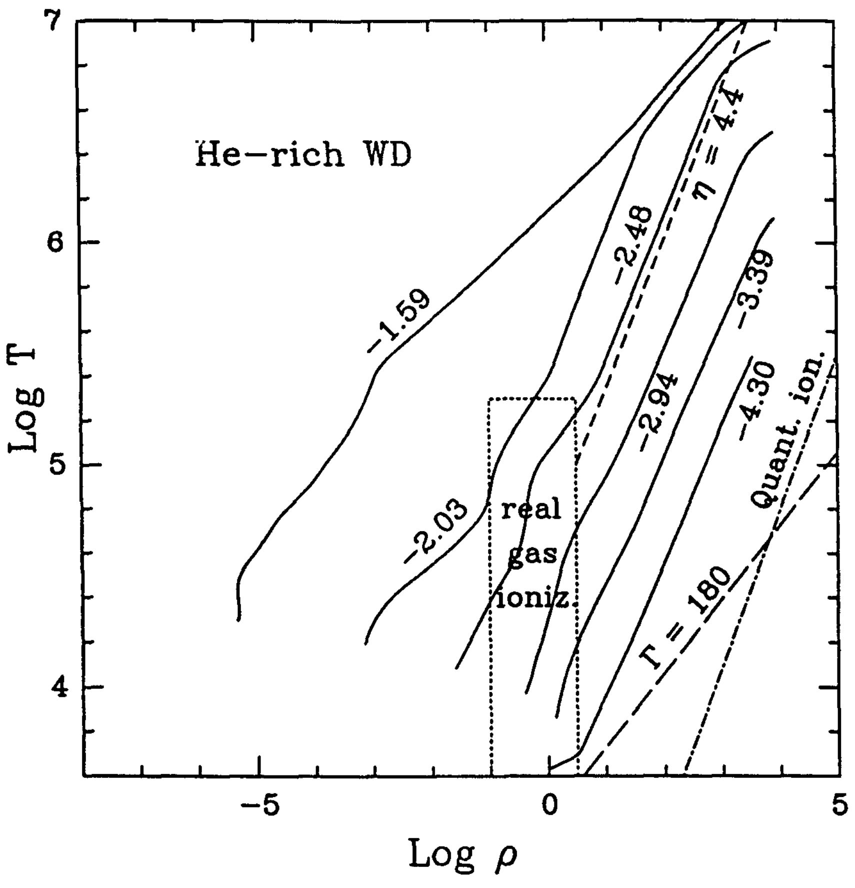

Fig. 7.7 The analogous of Fig. 1.1, but for He-rich atmospheres. In this case, the non-ideal gas conditions are met very soon during the evolution.

Unfortunately, due to the presence of two electrons, the case for helium is far more difficult to be dealt with than the case for hydrogen and, up today, no extensive numerical results from a realistic thermodynamic treatment of this situation are available.

The conclusion is then that thermodynamics puts a severe constraint on our understanding of He-rich WDs already at relatively large luminosities. Another constraint, at much lower luminosities, comes from the fact that the convective region begins approaching the conditions where the quantum corrections take over, but this occurs only at $Log L/L_{\odot} \leq -4 \div -4.5$.

7.4.2 Radiative opacity

Even worse is the situation for He-rich WDs when we consider the radiative opacities. In fact, as can be seen in Fig. 1.8, we do not have reliable opacities

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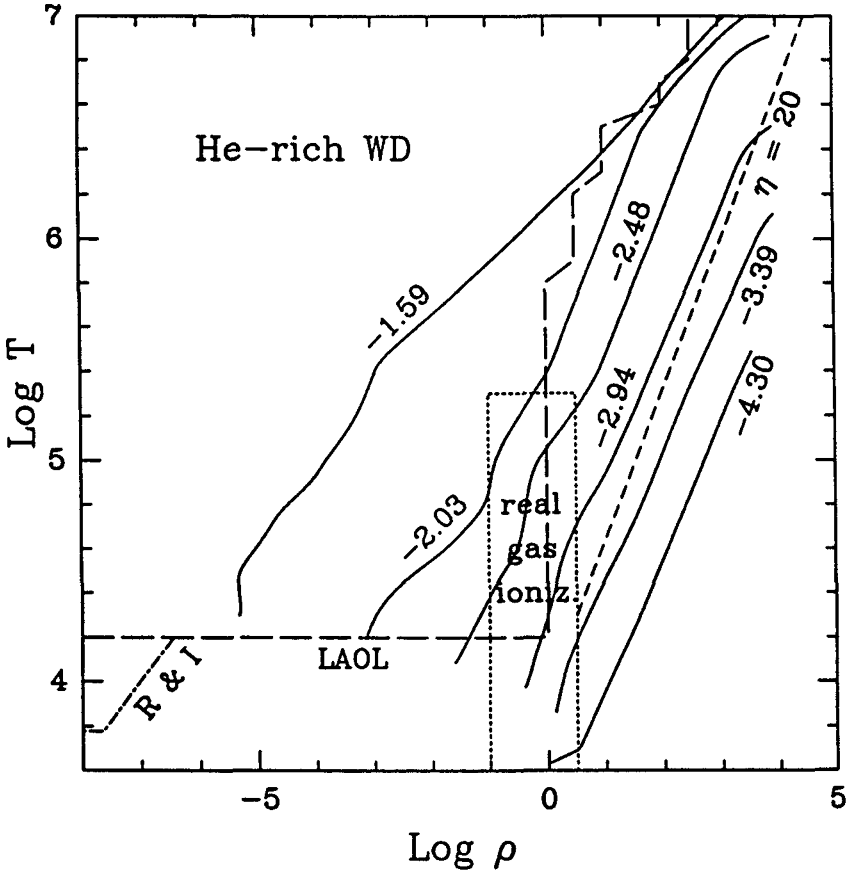

Fig. 7.8 **The same as Fig. 1.2, but for He-rich WDs.**

for He-rich atmospheres below $LogL/L_{\odot}$= -2. Also the LAOL opacities at relatively high densities are probably not very realistic, since they have been computed in the ideal-gas approximation. However, in convective layers, this is perhaps a minor problem, since the thermal structure of WDs in these ρ-T regions is mainly dictated by thermodynamics. As soon as convection sets in, opacities become relevant only in the optical atmosphere.

The situation is furtherly complicated by the fact that, even before trying to evaluate radiative opacities, a good knowledge of the real-gas thermodynamics is required. At present, it seems perhaps possible to make progresses on thermodynamics by means of Free Energy minimization schemes, or by means of the brute force of a Monte Carlo treatment but, unfortunately, none of these two alternatives is able to provide the detailed knowledge of the bound levels required for evaluating opacities. This is even more true since it is not only the structure of the bound levels of helium to be required, but mainly the structure of the (pressure perturbed) bound levels of the trace metals, radiative opacity being due almost only to these last elements.

To add further complicances to an already messy situation, let us focus on the already quoted dependence of opacity on metal abundance. The possible presence of small amounts of hydrogen or metals due to interstellar accretion, or to levitation from inside due to radiation pressure on the lines, makes models of He-rich WDs little more than a computer exercise, as soon as the surface temperature drops below the level at which the He-contribution to opacity becomes negligible.

In practice, from the point of view of radiative opacities, the same conclusions can be drawn as for the case of thermodynamics, even if somehow more severe: for luminosities below $LogL/L_{\odot}\sim$ -2, the models can be only *test* models, and they are likely to become absolutely meaningless below $LogL/L_{\odot}\sim$ -3.

7.4.3 Turbulent convection

In the case of He-rich WDs, viscosity in the convective envelopes turns out to be always negligible,the Prandtl number being about 3-4 orders of magnitude lower than for hydrogen. However, we are in the presence of another tricky feature.

From an inspection of Fig. 1.7 it can be seen that, at low luminosities, the star seems to approaches the region where Γ = 180 —that is: crystallization— not only in the central layers, but also in the convective envelope, just below the optical atmosphere.

This is more evident from Fig. 1.9, where the behavior of Γ is shown as a function of the depth, starting from the bottom of the optical atmosphere. The values of Γ in the more external region, below $10-10\ M_{\odot}$, are meaningless, since matter very close to the surface is not completely ionized, but the peak around Γ=170 is indeed significant. The reason why Γ increases outwards up to a maximum is due to its power dependence $\Gamma \sim \rho^{1/3}/T$, and to the fast drop of the temperature when approaching the surface, whereas density does not decrease so fast as in normal stars.

Within the (admittedly narrow) limits of reliability of the above models, we then reach the conclusion that, at very low luminosities ($LogL/L_{\odot}\leq$ -4.3), He-rich WDs can start forming a *solid crust* close to the top of the convective region! The implications of this curious physical feature are far from being clear. Does this mean that convection is abruply stopped and, in the solid region, the temperature gradient suddenly jumps to the radiative

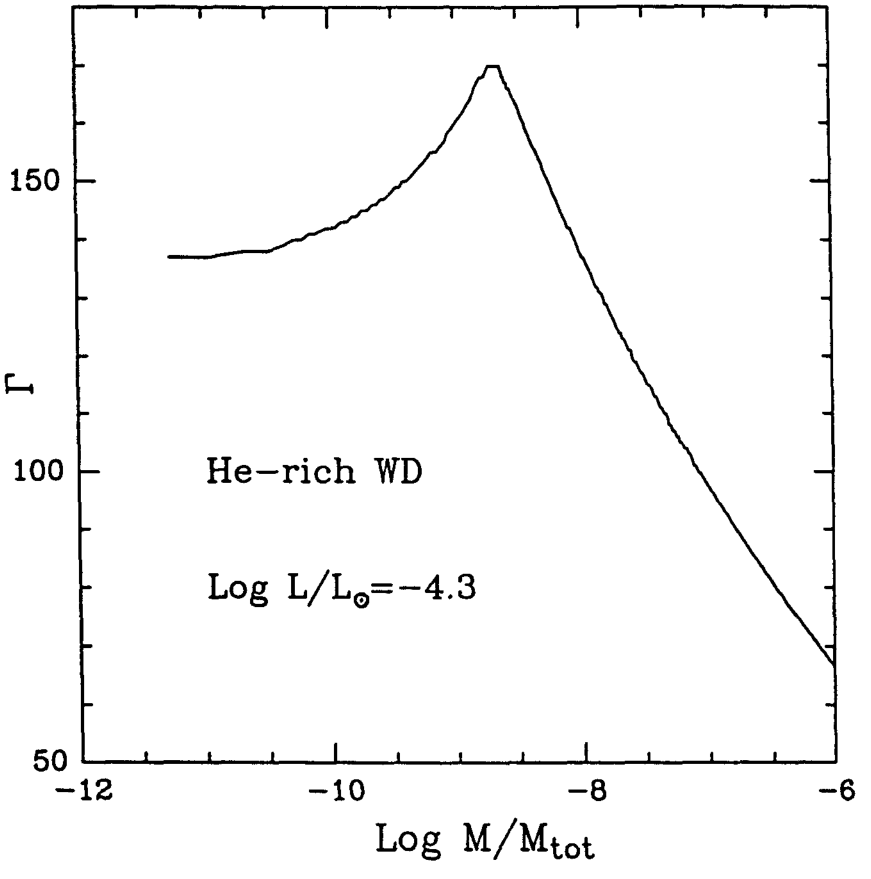

Fig. 7.9 The value of the Coulomb coupling parameter Γ inside the subatmosphere, as a function of the fractional mass (starting from the surface).

one? Or does it mean that convection mixes downward matter, so that a dynamical equilibrium between crystallization and melting is reached?

The only qualitative conclusion which can be drawn from these considerations is that we can in any case expect some sort of drag on convection, for very cold and faint He-rich WDs. However, since crust crystallization occurs well ahead of the onset of other physical processes which make our understanding of He-rich WDs uncertain, this feature is at present little more than a curiosity.

7.4.4 Mixing

Just for the sake of completeness, it is worth recalling the reader's attention upon one final complicance, which arises if the hydrogen-rich envelope on H-rich WDs is thinner than $2 \div 3 \cdot 10{-}8\ M_{\odot}$—a value which cannot be

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dismissed on the basis of our present interpretation of astroseismological data.

In that case, in fact, the sinking of the H-convection at low luminosity is such that, below $LogL/L_{\odot}\sim$ -3.5, the H-convective and the He-convective layers join, and full mixing is achieved. Hydrogen at the surface is then diluted by orders of magnitude, and the problems with He-rich WDs are met again, adding one further degree of freedom to an already unmanageable problem.

7.5 Conclusions

From the above discussion, it should now be clear that our understanding of the main physical inputs entering the modeling of very cool and old WDs, no matter if H-rich or He-rich at the surface, is still rather poor. Perhaps, radiative opacities in the optical atmospheres are by far the missing ingredient which makes so unsure our knowledge of the final cooling phases of these objects, even if the problems with thermodynamics and turbulent convection are still far from being settled.

For these reasons, one derives the feeling that webetter try to be cautious in rising stringent constraints on the age of the galactic disk, from fits between observed luminosity functions and WDs evolutionary models; the technique is promising, but the error bar weighing on this kind of comparisons is still quite large and —even worse— its amplitude is presently unknown (maybe even in the range of billions of years?).

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