1) The members of P which displace the integer n are even in number because the algebraic sum of the displacements of n by members of P must be 0.

2) The members of P which displace the integer n are precisely those which do not change the relative order of the other integers $1, 2, \ldots, n-1$. Hence if the integer n is ignored the other members of P are adjacent-transpositions on $(1, \ldots, n-1)$ with a product which is the identity permutation. By the inductive assumption, their number must be even.

Thus N is the sum of two even integers and is therefore even.

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A LEMMA ON CONTINUOUS FUNCTIONS

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The point of this note is to get a lemma which is useful in treating homotopy between paths in a topological space [1].

As explained in the reference, two paths joining a given pair of points in a space E are homotopic if there exists a mapping $F: I \times I \longrightarrow E$ (I being the closed interval $[0,1]$) which deforms one path continuously into the other. In practice, when two paths are homotopic and the mapping F is constructed, then the verification of all its required properties, with the possible exception of continuity, is trivial. The snag occurs when F is a combination of two or three functions on different subsets of $I \times I$. Then the boundary lines between these subsets have to be given special consideration, and although the problems resulting are routine their disposal can involve some tedious calculation and repetition. In the development [1] of the fundamental group of a space, for example, this sort of situation comes up four or five times.

The calculation can be by-passed by means of the

LEMMA. Let S, T, be topological spaces; $A=\bar{A}$ (the closure of A), $B=\bar{B}$, two closed subsets of S; and $f: A \to T$, $g: B \to T$, continuous mappings on A and B (considered as subspaces of S) such that $f(x) = g(x)$ whenever $x \in A \cap B$. Define $h: A \cup B \to T$ by

$$h(x) = f(x), \quad \text{if } x \in A,$$

$$h(x) = g(x), \quad \text{if } x \in B.$$

Then h is continuous on $A \cup B$.

Proof. h is a well-defined mapping. For $p \in A \cup B$, let N be an arbitrary neighbourhood of $h(p)$. If $p \in \bar{A} \cap \bar{B}$, i.e. $A \cap B$, then by the continuity of f and g there are neighbourhoods (in the topology of S) $J(p)$ and $K(p)$ for which

$$(1) \qquad f(J \cap A) \subset N, \quad g(K \cap B) \subset N.$$

If $p \in (A \cup B)-\bar{A}$, there is a neighbourhood $J(p)$ such that $J \cap A$ is void; furthermore $p \in B$ and g is continuous at p; so that (1) still holds. In the same way (1) remains true if $p \in (A \cup B)-\bar{B}$. (1) then is valid for all $p \in A \cup B$, and it follows with $L = J \cap K$ that

$$f(L \cap A) \cup g(L \cap B) = h(L \cap (A \cup B)) \subset N.$$

This establishes the continuity of h.

REFERENCE

1. A.H. Wallace, An Introduction to Algebraic Topology (New York, 1957).

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