GLOBALLY STABLE EQUILIBRIA OF COLLISIONLESS SELF-GRAVITATING MATTER

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ABSTRACT. We discuss the problem how to determine globally stable equilibrium states of collisionless self-gravitating matter with non-vanishing total angular momentum.

1. Introduction

We summarize the essentials of a techniques to calculate globally stable equilibrium states of collisionless matter with non-vanishing total angular momentum, that are gravitationally bound in all parts. The distribution functions of the resulting equilibria depend on Jacoby's integral. It can be seen that globally stable states cannot be rotationally symmetric for arbitrary fixed total angular momentum. With the help of a suitable relaxation model this techniques can be used to determine globally stable equilibria on macroscopic scales, which can be interpreted as final states of collisionless ("violent") relaxation (for details see Wiechen and Ziegler 1992).

2. Construction of globally stable equilibria

First we define the class of test functions. Let Φ_0 be the class of all distribution functions $f(\vec{r}, \vec{v})$ in 6-dimensional phase space meeting the following four constraints:

(i) All distribution functions $f \in \Phi_0$ correspond to the arbitrary fixed total angular momentum $\vec{L}_0 = L_0 \vec{e}_z$.

(ii) All states $f \in \Phi_0$ are accessible in collisionless, i.e. phase space density conserving dynamics. Thus all distribution functions $f \in \Phi_0$ correspond to the same arbitrary fixed memory function $g(\varphi)$ ($\Theta(x)$ denotes the unit step function)

$$\int \Theta(f(\vec{r}, \vec{v}) - g) d^3r \, d^3v = \varphi(g) \Rightarrow g(\varphi) \qquad \forall f \in \Phi_0. \qquad (1)$$

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(iii) For simplicity and clearness we restrict the space of test functions to rotationally symmetric states in configuration space.

(iv) The distribution functions $f \in \Phi_0$ have to describe states that are gravitationally bound in all parts.

To determine the globally stable equilibrium out of the class Φ_0 we have to calculate the global minimum of the energy functional $E_{f,\psi}$

$$E_{f,\psi} = \int h f(\vec{r}, \vec{v}, t) d^3r d^3v + \frac{1}{8\pi G} \int (\nabla\psi)^2 d^3r \qquad (2)$$

with $h = \frac{v^2}{2} + \psi$ (ψ: gravitational potential). We minimize the energy functional $E_{f,\psi}$ in two steps. In the first step we minimize $E_{f,\psi}$ with respect to f for arbitrary fixed ψ.

We construct the minimizing distribution functionf^*_ψ as a decreasing function of $\tilde{h} = v^2/2 + \psi - \lambda^2 r^2/2$ by an incompressible map in phase space (the fixed total angular momentum is considered by the Lagrangian multiplier λ, in equilibrium $\tilde{h}$ is equivalent to the Jacobi-integral). This yields

$$f^*_\psi(\tilde{h}) = g(\varphi_{\psi,\lambda}(\tilde{h})) \qquad (3)$$

$(\varphi_{\psi,\lambda}(\tilde{h}) = \int \Theta(R_{\psi,\lambda}(z) - r)\, \Theta(\tilde{h} - (v^2/2 + \psi - \lambda^2 r^2/2))\, d^3r d^3v)$. $R_{\psi,\lambda}(z)$ denotes the boundary of the domain in configuration space where the system is gravitationally bound.

In the second step of the minimizing procedure we minimize with respect to ψ by variation (now with $f = f^*_\psi(\tilde{h})$). The result of that variational calculus is the following Euler-Lagrange equation.

$$\Delta\psi = 4\pi G\, \Theta(R_\psi(z) - r) \int f^*_\psi(\tilde{h}(\xi)) d^3v. \qquad (4)$$

The globally stable equilibrium is given by that solution $\psi^*, f^*_{\psi^*}(\tilde{h})$ of (4) which corresponds to the global minimum of W_ψ.

It can be shown that the constraints (i)-(iv) may be so restrictive, that Φ_0 becomes the empty set. Then at least no rotationally symmetric, gravitationally bound stable equilibria corresponding to the fixed $g(\varphi)$ exist with the given total angular momentum. In that case, the globally stable equilibrium either has to be 3-dimensional (with uniform figure rotation) or it does not exist at all.

3. References

Wiechen, H. & Ziegler, H.J., 1992. *Astrophys. J.*, submitted.

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