MOTIONS OF ARTIFICIAL SATELLITES AND COORDINATE SYSTEMS

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Abstract. In order to compute satellite motions with centimeter accuracy, the reference system, to which they are referred, should be carefully chosen. In fact there are many kinds of the reference systems. In this paper advantages and disadvantages of various reference systems are discussed.

There are many different reference systems, to which satellite positions are referred. Everybody prefers one system to the others, however, there is no system which everybody prefers the best. The mean equator and equinox at a certain epoch are adopted at some institutes as their reference frame for satellite positions, whereas at others the true equator and equinox of data are adopted. Instead of the true equinox the mean equinox of a fixed date can be chosen. It is also possible to adopt the mean equator and equinox at the beginning of a year and to change the system at the beginning of every year. More precisely, there are many choices for the z-axis; namely, the figure axis or the celestial pole axis or the instantaneous spinning axis. Furthermore we may ask what kind of the figure axis should be adopted; the true or mean figure axis. I do not want to conclude here which is the best system, however, will try to discuss how we should do to compute the satellite positions with centimeter accuracy for each case.

None of the reference systems is inertial as their origins are at the geocenter which is in accelerated motion. The effects of the motion of the geocenter to satellite motions are usually included in the lunisolar gravitational perturbations. In fact the disturbing function due to the sun and the moon for the satellite motions is derived by writing their equations of motion in an inertial coordinate system with its origin at the barycenter of the solar system which is assumed to be moving with an uniform velocity.

If there is no rotational motion of the coordinate axes of the reference system with respect to the inertial system, it is not

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necessary to add any term to the disturbing function of the satellite motions, since it is a quasi-inertial system (Moritz, 1979). The coordinate system with the mean equator and equinox at a certain epoch as the reference frame is such a system. It has an advantage as it is easy to describe the equations of motion in this system. However, to write the expression of the geopotential is not so easy as the coordinates of any point are time-dependent in the system even though the solid earth is assumed. It is also possible to adopt a quasi-inertial system to formulate the equations of motion and to introduce an auxiliary reference to express the geopotential and the station coordinates. As the auxiliary reference frame usually the equator of date is adopted.

When the equator and the equinox of date are adopted as the frame of reference, the expression of the geopotential becomes simpler as the coordinates of any earth fixed point are time-independent. It has a disadvantage, however, since perturbations are produced as the coordinate axes move. To discuss more precisely on this system it is necessary to specify what axis is adopted as its z-axis. Indeed there are many axes. The system, in which the equator of date and the mean equinox at an epoch is adopted, is a kind of this system, as there is no essential difference to treat the perturbations.

Roughly speaking, there are two choices for the z-axis, the figure axis or the celestial pole axis. The figure axis is the axis of the maximum momentum of inertia. Therefore, if it is adopted C_{21} and S_{21} terms vanish in the geopotential. However, as the sun and the moon are generating tides on the earth, the figure pole is moving around its mean position by as much as 60 meters (Moritz, 1979). Therefore, the figure axis is not fixed to the earth in any sense.

However, if the mean position of the figure pole which does not move with respect to the earth is adopted as the direction of the z-axis, the coordinates of any of the observing stations are expressed as the constant values plus tidal motions and the geopotential is expressed as the sum of the averaged part and the variable part due to the tidal deformation which can be formulated or can be derived by analyzing satellite motions. The rotation of the earth around the mean figure axis is not so simple as that around the celestial pole axis. However, as its rotation rate is constant with error less than 10^{-6} at most (the distance between the mean figure pole and the celestial pole being 6 meters), the geopotential can be expressed with 10^{-10} accuracy by assuming that the rotation rate around the mean figure axis is constant. Therefore, it seems that it is easy to express both the station coordinates and the geopotential in this reference system. However, the reference system moves by the precession and the forced and free nutation in the inertial system.

When the celestial pole axis is adopted, the station coordinates are expressed by the sum of the constant part, the tidal effects and the effects of the polar motion and the geopotential is the sum of the constant part, the tidal part and the time-dependent C_{21} and S_{21} terms due to the polar displacement. The system does not move due to the free nutation with respect to the inertial system, however, of course, moves due to the precession and the forced nutation. This system has an advantage as the nutation theory is referred to this axis according to an IAU resolution in 1979. It is practically convenient to adopt CIO as the direction to the z-axis.

When any non-uniformly moving reference coordinate system is adopted the equations of motion should include additional terms due to the centrifugal and the Coriolis forces, although acceleration of the motion of the reference system is very small as it usually moves due to the precession and the nutation. However, as the velocity, the energy and the angular momentum of the satellite with respect to the system are different from those with respect to the quasi-inertial system and vary with time, the osculating semi-major axis and eccentricity change with time even for the two-body problem. Of course, the other orbital elements also include their perturbations due to the motion of the system.

There is also a different kind of reference frame, such as one which has been adopted at the Smithsonian Astrophysical Observatory. It is basically a quasi-inertial system even though it has not been stated so before, since the basic equations of motion are those in the inertial system without any additional term. However, in order to make the expressions of the geopotential, the station coordinates and the perturbations much simpler an auxiliary reference for them has been introduced. It is referred to the equator of date and the mean equinox at 1950.0, with respect to which the inclination, the argument of perigee and the longitude of the ascending node of the satellite are given. However, the semi-major axis and the eccentricity are referred to the quasi-inertial system in the sense that the velocity, the energy and the angular momentum computed by formulae for the two-body problem with the osculating semimajor axis and eccentricity are those with respect to the quasi-inertial system, namely, that defined by the mean equator and equinox at 1950.0. The semimajor axis and the eccentricity, therefore, are not disturbed by the motion of the auxiliary system, that is, constant for the two-body problem, however, have some small indirect perturbations through other elements due to the motion of the equator. Therefore, one must be careful to analyze doppler data of the satellite in this system.

For the three angular elements with respect to the auxiliary system (Kozai and Kinoshita, 1973) proved that by adding $\partial i/\partial t$, $\partial\omega/\partial t$ and $\partial\Omega/\partial t$ to the right-hand sides of Lagrange's planetary equations for the inclination, the argument of perigee and the longitude of the ascending node, respectively, the equations hold for any moving auxiliary system. Similarly, any other type of equations of motion can be modified for such systems. For the other orbital elements the equations need not have any additional term as the definitions of the

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semimajor axis, the eccentricity and the mean anomaly are the same as those for the quasi-inertial system.

The partial derivatives are derived by using geometrical relations between the moving and the fixed systems. In fact as the reference system is moving the values of the three angular quantities, i, ω and Ω take values different from the original ones even if there were not perturbation at all. The partial derivatives are the time derivatives of the angular quantities without taking into account any perturbation in the orbital elements. Namely, the elements are constant in deriving the derivatives.

In the previous paper (Kozai and Kinoshita, 1973) the following expressions are derived for the partial derivatives:

$$\frac{\partial i}{\partial t} = - \frac{\partial}{\partial t} \{\sin \theta \cos (\alpha - \Omega)\} ,$$

$$\frac{\partial \omega}{\partial t} = \operatorname{cosec} i \frac{\partial}{\partial t} \{\sin \theta \sin (\alpha - \Omega)\} , \qquad (1)$$

$$\frac{\partial \Omega}{\partial t} = (1 - \cos \theta) \frac{d\alpha}{dt} - \cot i \frac{\partial}{\partial t} \{\sin \theta \sin (\alpha - \Omega)\} ,$$

where θ and α are, respectively, the inclination and the longitude of the ascending node of the moving reference plane with respect to the fixed one and are explicitly time-dependent.

When the moving references are the equator and the equinox of date and the fixed ones are the mean equator and equinox at an epoch, $\sin \theta \sin \alpha$, and $\sin \theta \cos \alpha$ are expressed as,

$$\begin{aligned} \sin \theta \sin \alpha &= \sin \varepsilon_1 \sin \psi , \\ \sin \theta \cos \alpha &= \sin (\varepsilon_0 - \varepsilon_1) + 2 \sin^2(\psi/2) \sin \varepsilon_1 \sin \varepsilon_0 , \end{aligned} \qquad (2)$$

where ψ is the arc between the ascending nodes of the ecliptic at the epoch referred to the equators and ε_0 and ε_1 are their inclinations. And ε_1 and ψ change due to the precession and the nutation and for some cases the free nutation. The expression (2) can be approximated by,

$$\begin{aligned} \sin \theta \sin \alpha &= (0.3979 + \varepsilon_1 - \varepsilon_0) \sin \psi , \\ \sin \theta \cos \alpha &= 0.3651(1 - \cos \psi) - \varepsilon_1 + \varepsilon_0 . \end{aligned} \qquad (3)$$

In order to obtain the perturbations due to the motions of the reference frame in the three angular elements it is necessary to take partial derivatives with respect to time for (1) by assuming that only θ and α are time-dependent and then to integrate them by assuming that

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Ω is also time-dependent. As ψ and ε_1 move, usually, more slowly than Ω, the perturbations introduced are smaller than the motions of the reference frame. As the lunar longitude of the ascending node which enters into the argument of the principal nutation term, 18.6 year period term, moves more slowly even than that of Lageos satellite, the amplitude of the perturbation term produced by this nutation term is smaller than that of the nutation term itself. As the 18.6 year period nutation term's amplitude is known with the accuracy of $0\overset{''}{.}001$, the perturbations in the orbital elements can be computed with centimeter accuracy. The perturbations due to other nutation terms can be computed with the same accuracy by using the existing nutation theory unless any serious resonance is introduced.

When the mean figure axis is adopted as the z-axis, the pole coordinates should be known with centimeter accuracy to compute the perturbations due to them with the same accuracy. Even when the celestial pole axis is adopted, the pole positions should be known with centimeter accuracy to compute the values of time-dependent C_{21} and S_{21}.

Even when the equations of motion are formulated in a quasi-inertial system or in a non-uniformly moving system with additional terms, the perturbation behavior for the three angular elements is not essentially different from discussed here. The semimajor axis and the eccentricity are perturbed for the moving reference system. However, they can be derived with the centimeter accuracy when the expressions of the precession, the nutation and the pole motion are known with the same accuracy. If they are not known with this accuracy, the satellite motions can improve them from time to time.

References

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