FISSION AND THE ORIGIN OF BINARY STARS

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Abstract. Brief reviews of the classical 'angular momentum problem' and the statistics of upper-main-sequence binaries are presented as background for the suggestion that the close, early-type, binaries are produced by fission of rapidly rotating protostars.

Next, theoretical sequences of contracting, rotating stars are described. Recent work demonstrates that the zero-viscosity, polytropic sequences, have essentially the same properties as the McLaurin sequence. Thus, fission is possible for centrally condensed stars. Observations of close early-type binaries are compared with theoretical predictions for the minimum angular momentum in binary systems of given total mass; the agreement is excellent.

Finally, the existing theoretical objections to the fission hypothesis for the origin of binary stars are reviewed, and it is concluded that, although fission remains unproven, there are now no strong theoretical arguments against the process, and there is considerable observational support for its existence.

1. Background

A. THE ANGULAR MOMENTUM PROBLEM

It is generally believed that stars are formed from the diffuse gas and dust found in interstellar space (cf. Spitzer, 1968). Although the detailed processes of star formation are by no means well understood, a commonly accepted, over-all, model postulates conditions in the interstellar medium which would lead to large scale hydrodynamical or thermal instabilities occurring over extended regions; large masses ($M \gtrsim 10^4 M_\odot$) of gas and dust begin to collapse and then to fragment, the ultimate fragments being protostars. In a variant model suggested by McCrea (1961), fragmentation proceeds until the mass is reduced to quite small 'flocules'; these collide with one another, fuse during certain inelastic collisions, and ultimately approach proto-star proportions.

Detailed theories of star formation vary greatly in their predictions for the angular momentum content of protostars. However, all of the simpler (non-electromagnetic) theories lead to values of the angular momentum, J, much greater than observed in main-sequence stars. This point is easily made in rough quantitative fashion. Consider (a) the angular momentum of a spherical blob of gas, due simply to the fact that the local standard of rest rotates about the galaxy with a period of $\sim 10^8$ yrs; and (b) the angular momentum expected if the protostar cloud has rotational kinetic energy in equipartition with its translational kinetic energy ($|v_{tr}| \approx 10$ km/sec). In the two cases we find that a protostar with an original density of $\simeq 10^{-24}$ gm/cm^3 would have

$$\begin{aligned} J_a &\approx 10^{56}\,(M/M_\odot)^{5/3}\ \text{gm cm}^2\ \text{sec}^{-1} \quad \text{(galactic rotation)}, \\ J_b &\approx 10^{58}\,(M/M_\odot)^{4/3}\ \text{gm cm}^2\ \text{sec}^{-1} \quad \text{(equipartition)}. \end{aligned} \tag{1}$$

In comparison to this, even the most rapidly rotating main-sequence stars have

A. Slettebak (ed.), Stellar Rotation, 147–156.

$J \simeq 10^{51}$–10^{52} gm cm^2 sec^{-1}. It is interesting to note parenthetically that, in McCrea's fusion model, the estimates given by Equation (1) are considerably reduced; the angular momentum due to galactic rotation appears primarily as the rotation of the resulting star cluster and the equipartition angular momentum is reduced by the square root of the number of 'flocules' comprising a single star. The values of J given by Equation (1) are in fact much larger than equilibrium stars would be able to contain with their observed masses and radii.* This is the angular momentum problem.

In order to 'solve' the problem various authors (cf. Spitzer, 1968, for references) have suggested that magnetic forces are adequate to transfer the 'excess' angular momentum to the surrounding medium. There are two principal objections to these mechanisms. First, the time scales for angular momentum transfer are longer than the relevant free fall times of the initial condensation if the prevailing field strength is $\simeq 3 \times 10^{-6}$ gauss. Second, there is much more angular momentum in binaries than in single stars; after finding an appropriate angular momentum loss mechanism one would then be required to turn it off – prevent angular momentum loss – when binaries are to be formed. Angular momentum transfer from protostars by magnetic fields may be astrophysically important. But for the reasons mentioned above, it may be useful to consider other approaches to the angular momentum problem.

In any case, it is easy to see that multiple star systems can store much more angular momentum in the form of orbital motions than the several stars could readily contain as spin. Thus the angular momentum problem would be greatly alleviated (if not solved) were there a simple mechanism for transforming the spin angular momentum of a single massive protostar into the orbital angular momenta of stars in a cluster. One can imagine the process proceeding in two stages. During the initial fragmentation and, perhaps, subsequent fusion stages of star formation, much of the angular momentum is fixed in the orbital motions of massive protostars. The subsequent gravitational interactions among the massive stars can, according to the detailed numerical calculations of Van Albada (1968b), lead to the formation, of wide (visual) binaries. Returning to the massive protostars, we see that, even in McCrea's model, each object is likely to have somewhat more angular momentum than is found in single main-sequence stars. In Sections 2 and 3 we shall suggest that, so long as the kinetic energy of the contracting star is large (compared to the gravitational energy) fission will occur, perhaps repeatedly, and that the resulting stars will have the observed properties with regard to surface velocity and dualism, of the upper main sequence.

B. BINARY STARS

In order for this solution to the angular momentum problem to be plausible, it is clearly required that most stars be found in gravitationally bound double (or triple,

* Of course a one-solar mass model *could* be constructed in equilibrium with, say, $J = 10^{57}$ gm cm^2 sec^{-1}; however, it would have a radius more nearly $R \simeq 10^{11}\ R_\odot$ than $1\ R_\odot$!

etc.) systems. If we consider the upper* main-sequence stars we find our expectations confirmed (cf., for example, Blaauw, 1961). Approximately 75% of O-B stars are members of double or triple systems. An interesting, if tentative, additional piece of evidence is presented by Van Albada (1968a) and Blaauw and Van Albada (1969). Carefully examining the distribution of binary separations they find that the division of early-type binaries into close (spectroscopic) and wide (visual) pairs is probably real and not due to the obvious selection effects. For the upper main sequence, the data seem to indicate that, contrary to the earlier suggestions of Kuiper (1935), there is a real deficiency of binaries with separations in the range 10^0 AU $<a<10^2$ AU. This result, if accepted, implies that there are distinct processes for forming close and wide binary stars. Since many authors have suggested that widely separated binaries can be formed during the multiple encounters which occur in a young stellar association, it is attractive to examine the possibility that the close pairs are formed by fission.

2. Theoretical Sequences of Contracting, Rotating Stars

A. HISTORICAL BACKGROUND

The thoughts presented above are not new. They have provided part of the impetus behind centuries of elegant attacks on the problem of the equilibrium and stability of rotating, homogeneous, self-gravitating objects. Among others, McLaurin, Jacobi, Poincaré, Cartán, and Chandrasekhar have studied the subject (for a brief historical review cf. Chandrasekhar, 1967). Darwin (1916, see *Collected Works*, Vol. III) and Jeans (1929) in particular examined the question of fission. The problem, simply stated, is to enumerate the sequence of forms that an isolated object will pass through, if it is initially very large, nearly spherical, and slowly rotating, and then gradually and quasi-statically contracts. The classical problem of a *homogeneous* 'star' has been treated with considerable rigor; although real stars are not homogeneous, the classical tale bears retelling because (a) it is by now well understood, and (b) recent calculations show that similarly defined compressible sequences mimic in all essential respects the classical McLaurin sequence.

B. MCLAURIN SEQUENCE

Consider an axisymmetric star with fixed angular momentum J and mass M within which viscous and magnetic effects may be neglected. Define a model by the value of the density, ϱ, supposed uniform throughout the object. Define a limiting model, $\varrho \to 0$, to be uniformly rotating and require that the angular momentum of every

* Low mass ($M < 2\,M_\odot$) stars must be considered separately, since they have lengthy contraction phases during which the transport of angular momentum by turbulent viscosity (cf. Von Weizsäcker, 1947) or stellar winds (cf. Schatzman, 1962) is likely to be important; the upper main sequence apparently passes directly from collapse to radiative contraction (Larson, 1968), skipping the convective Hayashi phase.

annular mass element in any member of the sequence have the same value as it had in the limiting model. Define a mean radius $\bar{r} \equiv (3M/4\pi\varrho)^{1/3}$. Then one can show that each member of the sequence (i) is an oblate spheroid with axes (a, a, c), $a > c$, such that the eccentricity $e \equiv (1 - a^2/c^2)^{1/2}$ is implicitly related to $\bar{r}$ as follows:

$$\bar{r} = \left(\frac{25}{18}\right)\left(\frac{J^2}{GM^3}\right) e^3 (1 - e^2)^{1/6} \left[(1 - \tfrac{2}{3} e^2) \sin^{-1} e - e(1 - e^2)^{1/2}\right]^{-1}, \quad (2)$$

(ii) rotates uniformly with angular velocity

$$\Omega = \left(\frac{162}{125}\right)\left(\frac{G^2 M^5}{J^3}\right) e^{-6} \left[(1 - \tfrac{2}{3} e^2) \sin^{-1} e - e(1 - e^2)^{1/2}\right]^2. \quad (3)$$

Note that, although the McLaurin sequence was defined (somewhat unconventionally) as a zero viscosity sequence and the angular velocity distribution was left to be determined by the dynamical constraints, it turned out that each member of the sequence was required to rotate as a solid body if the initial, nearly spherical member did. We may think of the McLaurin sequence as the sequence of an $n=0$ polytrope [polytrope – star in which (pressure) $\propto$ (density)$^{(n+1)/n}$]. Clearly there must exist analogous polytropic sequences having $n \neq 0$. It seems unlikely that the special property of the homogeneous McLaurin spheroids – uniform rotation – will be preserved in the more general case. Instead, even though the initial members are uniformly rotating, subsequent members of the generalized sequences are found to rotate differentially.

Returning to the $n=0$ case we find the following behavior as ϱ varies from 0 to ∞ and $\bar{r}$ from ∞ to 0. The eccentricity varies monotonically from 0 to 1, c from ∞ to 0, a from ∞ to the asymptotic value $(25\pi/108)(J^2/GM^3)$, and Ω from 0 to $(27\pi/125) \times G^2M^5/J^3$. The total energy varies monotonically from 0 to $-\infty$ and the ratio of $T/|W|$ of kinetic to absolute gravitational energy from 0 to $\frac{1}{2}$ monotonically.

The $n=0$ sequence does *not* terminate at any point; there exists an equilibrium model for every $0 \leqslant \varrho < \infty$. However, a detailed analysis of the lowest incompressible modes by Lebovitz (1961) shows that the models are neutrally stable to a nonaxisymmetric mode at $e=0.813$, $T/|W|=0.138$ and become dynamically overstable at $e=0.953$, $T/|W|=0.273$. The neutral mode occurs at the classical 'point of bifurcation' where uniform, uniformly rotating, ellipsoids become dynamically possible; however, the bifurcation point has no significance in the present discussion, because no instabilities occur there so long as viscosity is neglected. The second point ($T/|W|=$ 0.273) leads to dynamical motions examined in the nonlinear regime by Rossner (1967) and Fujimoto (1968). The motion is quite complex but might very roughly be described as the end over end tumbling of a quite prolate ($a/c \approx 25$) spheroid. An important next step would be to carry the numerical work further, relaxing the assumption, made by both Rossner and Fujimoto, that the bodies remain ellipsoidal. There are rather strong arguments (too detailed to be given here) for believing that, after the point of overstability is reached, fission will occur as the very thin, approximately spheroidal body breaks up into two or more objects orbiting about the common center of mass.

C. POLYTROPIC SEQUENCES

At this juncture the reader is no doubt wondering why the classical results have been summarized in such detail. Is it not known that these considerations are irrelevant to realistic centrally condensed stars? Has it not been shown (Jeans, 1919; James, 1964; Tassoul and Ostriker, 1970) that polytropes of index $n > 0.808$ 'fly apart' before any instabilities are reached? The answer is no. Rather, in the aforementioned papers it was shown that centrally condensed stars cannot store much kinetic energy and remain uniformly rotating. This is not surprising. We know without making any calculations that, for parts of a centrally condensed star within which the pressure forces are small compared to the inertial forces, the rotation law *must* approach Keplerian motion; that is $\Omega \propto \tilde{\omega}^{-1/2}$. If we restrict consideration to model stars forced to rotate uniformly, then we are restricting our attention to the interval $0 \leqslant T/|W| \ll 1$, within which interval we should not expect to find rotationally induced instabilities. What happens if we do not require uniform rotation?

The zero viscosity polytropic ($n \neq 0$) sequences defined earlier have been studied over the past few years by a group at Princeton University Observatory. The equilibria are numerically found using the SCF method (Ostriker and Mark, 1968) and the normal modes of oscillation studied using the virial techniques described by Tassoul and Ostriker (1968). Detailed results will be published elsewhere, but the important points are easily summarized by saying that *in all essential respects the $n \neq 0$ sequences resemble the classical $n = 0$ McLaurin sequence*. In particular, the sequences do not terminate. That is, for given (J, M) a model can apparently be constructed with central density ϱ_c, $0 \leqslant \varrho_c < \infty$. There is no 'rotational breakup'. For $n \neq 0$, neither the density nor the rotation are uniform nor, in general, are the equipotential surfaces spheroids. However, the dependence of a, c, Ω, E, $T/|W|$, and the normal modes on ϱ_c are topologically similar to the $n = 0$ case. In all cases examined, a point of bifurcation occurs at $T/|W| \doteqdot 0.14$ and overstability is reached at $T/|W| \doteqdot 0.26$. The numerical values of $T/|W|$ at the critical points are remarkably independent of n; to the level of accuracy presently available in the numerical work these critical values are constants. Thus it appears that real stars may, after all, undergo fission. The over-all picture is schematically reviewed in Figure 1. From a somewhat different point of view Roxburgh (1966) anticipated these conclusions without a detailed study of stability, and revived the fission hypotheses for the origin of close binaries among the low mass stars (W Ursae Majoris stars).

3. Applications of Theory

A. WHITE DWARFS

White dwarf stars obeying the Chandrasekhar equation of state are similar to polytropes, but the central densities are not arbitrarily adjustable. One can, however,

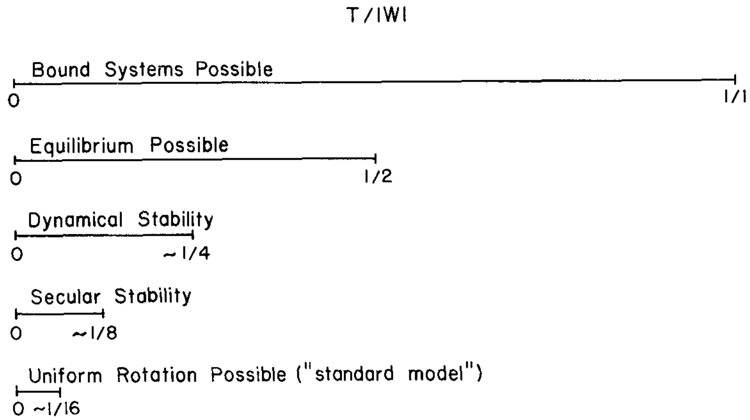

Fig. 1. Schematic diagram showing the limits placed by equilibrium and stability requirements on the ratio $T/|W|$ between kinetic energy of rotation and gravitational energy. See text in Section 2.C.

consider a star of given mass and angular momentum distribution and construct the sequence of stars having larger and larger total angular momenta. The computations (Ostriker and Bodenheimer, 1968) again indicate that the sequences do not terminate. A maximum in Ω is found along the sequence but neither 'rotational mass loss' nor 'rotational breakup' ever occur. However, a stability analysis (Ostriker and Tassoul, 1969) indicates that for $T/|W| \doteqdot 0.26$ the models are overstable and probably subject to fission. Nevertheless, for a variety of reasons it is very unlikely that close pairs of white dwarfs can be formed by fission, and in fact no such pairs have yet been observed.

B. PRE-MAIN SEQUENCE EVOLUTION

A more promising line of research (with respect to the fission problem) has been followed by Bodenheimer (1969) and Bodenheimer and Ostriker (1970), who have examined the pre-main sequence evolution of massive stars. The calculations (for 3–12 $M_\odot$ stars) begin at the start of the radiative contraction phase. For each star the mass, M, and total angular momentum, J, are assumed fixed. Since all the suggested processes for redistribution of angular momentum require longer than a Kelvin time, we further assume that the angular momentum of each annular element is preserved. Thus, the angular momentum distribution (chosen to be that of a uniformly rotating, uniform sphere) is an invariant. The study reached the following principal conclusions:

(1) For given M, the evolutionary track of a star with small J is like the non-rotating track, nearly horizontal in the ($\log L/L_\odot$, $\log T_{\rm eff}$) plane. With increasing J the luminosity decreases and the tracks become more nearly vertical. Aside from the dependence of track shape and orientation on J, the high angular momentum models resemble slowly rotating models of lower mass with respect to contraction times and position (near the main sequence) after nuclear burning commences.

(2) For sufficiently high values of J (with given M) the stars become unstable to

nonaxisymmetric modes before they have contracted to the main sequence. It is interesting to note that if J is slightly less than the critical value $J_c(M)$, the surface velocity of the $12\,M_\odot$ model at the main sequence is ≈ 400 km/sec, approximately the maximum velocity observed* for that mass star. In all probability it is instability to fission that limits the angular momentum and hence the surface velocity of main sequence stars.

(3) Stars with angular momentum just greater than the critical value, $J_c(M)$, will presumably form close binary systems. We can compare the relation $J = J_c(M)$ with the observed relation between angular momentum and mass in the closest (lowest angular momentum) early-type binaries. The agreement is excellent, with the theoretical curve lying between the curves for binaries with mass ratio one $J = J_b(M, 1)$ and mass ratio two $J = J_b(M, 2)$. As is seen in Figure 2,

$$J_b(M, 1) > J_c(M) > J_b(M, 2). \tag{4}$$

A similar comparison for the low mass stars was made by Roxburgh (1966).

4. Summary

In conclusion, let us review the arguments that have been put forward *against* the fission theory for the origin of binary stars.

Early in this century mathematicians thought it possible for stars to proceed quasi-statically through a series of uniformly rotating configurations to a detached binary system; the evolution envisaged was first along the McLaurin sequence of spheroids, then, at the point of bifurcation, to the Jacobi ellipsoids, and then, at a second point of bifurcation, to the 'pear-shaped' configurations studied in detail by Darwin. At this stage James felt there was 'little doubt' that fission would occur and lead to two

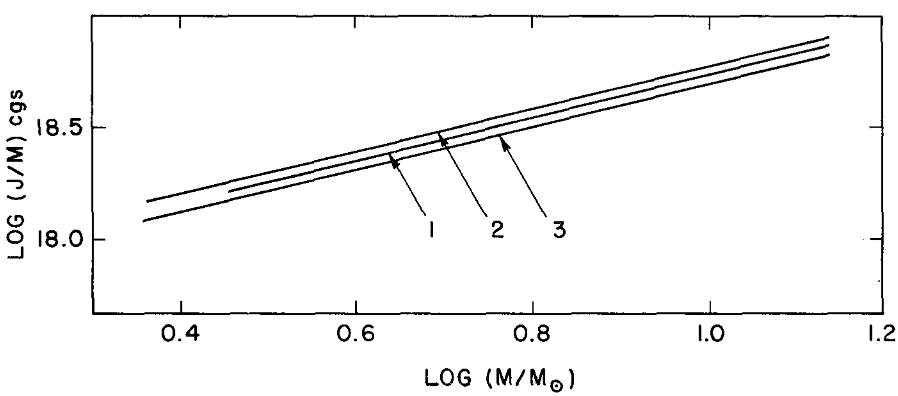

Fig. 2. Estimated relations between mass and angular momentum per unit mass adapted from Bodenheimer and Ostriker (1970): (1) $J_c(M)$ – differentially rotating models which are suspected to undergo fission just prior to the onset of nuclear reactions; $J_b(M, 1)$, $J_b(M, 2)$ – main sequence contact binaries adapted from Kraft (1969), with the given total mass and mass ratios of 1 and 2 respectively.

* The 'observed' velocities for rapidly rotating stars perhaps underestimate $v_e \sin i$ by as much as a factor 1.4 according to Hardorp and Strittmatter's (1968) calculations of gravity darkened models.

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detached, unequal, masses. However, Cartán (1922) proved unequivocally that the Jacobi ellipsoid is unstable at the point where the pear-shaped configurations branch off so that quasi-static evolution to a binary system was recognized as impossible. The possibility nevertheless remained that binary stars could form as a result of a dynamical instability. The argument for continuous quasi-static contraction fails on other grounds as well. It now seems unlikely that viscosity plays an important role during the early evolution of massive stars; in the absence of any dissipation a homogeneous star will not enter the Jacobi sequence at all but will follow the McLaurin sequence until $T/|W|=0.27$, at which point it is dynamically unstable. However, in the course of the subsequent dynamical evolution (described by Rossner, 1967) it may break into two or more pieces. Dynamically induced fission is probable here, but remains unproven.

A second objection is due to Lyttleton (1953). Since the dynamical evolution does not require friction, it must be time reversable. But, if we reverse the direction of time in an existing binary system it clearly does not revert to a single star but simply remains a binary system with the orbital directions reversed. Therefore a single system cannot evolve without dissipation into a binary. This argument is irreproachable as stated. However, other stages of stellar evolution proceed in a notably nonadiabatic way so it seems unreasonable to assume that the fission process alone should be energy conservative. Without the restriction to energy conservative systems, the argument of time reversability fails and Lyttleton's objection loses force.

As Roxburgh pointed out, a far more serious objection arose from the discovery of Jeans' (1929) and James' (1964) that centrally condensed ($n>0.808$) uniformly rotating objects could never reach the points of bifurcation or overstability. However, it is now clear that this result was an artifact of the constraint to uniform rotation. The self-consistent zero-viscosity, polytropic sequences ($n>1$ and $n<1$) described in Section 2.C do not terminate and they do become overstable at $T/|W|\doteqdot 0.27$. Thus, it seems compressible stars do have the possibility of becoming binaries by undergoing dynamically induced fission.

Finally, the observational evidence presented by Roxburgh (1966) and Bodenheimer and Ostriker (1970) concerning the angular momentum of close binaries provides experimental support for the fission hypothesis. While there is still a considerable amount of work to be done (cf. comments in Section 2.B) before one can say, with certainty, that the process is theoretically possible, there are now no strong theoretical arguments against it.

In the light of present observational and theoretical work, the fission hypothesis seems attractive, once again, as a partial solution to the angular momentum problem, and as a natural way of producing close binaries.

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Discussion

Jaschek: I would just like to comment that the investigation by Blaauw and Van Albada should be extended to later-type stars in which the statistics seem to behave very differently from what they found. In particular no break between close and wide binaries was found. This break is furthermore located at the very point where the incidence of selection is largest, so that its reality should not be accepted without further investigation.

Ostriker: I am not qualified to comment on the reliability of Blaauw and Van Albada's statistical conclusions. Van Albada's mechanism for producing wide (visual) binaries is in any case interesting and, I believe, plausible.

Fricke: I don't feel very satisfied with the neglect of angular momentum transport by meridional circulations during the pre-main-sequence evolution. The ratio of centrifugal force to gravity becomes of order 1 in your models and angular momentum transport within the Kelvin-Helmholtz time scale over appreciable distances of the star should be expected. I suggest that constant angular velocity might be the more reasonable constraint one should impose during the contraction phase.

Ostriker: For most of the proposed contraction phase the ratio of centrifugal to gravitational force is small. Even for the most rapidly rotating part of the star with the largest angular momentum considered and taking that star at its most rapidly rotating (most condensed) part of the contraction phase – even in this most extreme case, the ratio of centrifugal force to gravity is only 0.4. That is, I believe that circulation currents should be included and intend to do so in future work, but I doubt the conclusions will be very much altered.

Furthermore, it is not obvious to me that circulation would lead the star towards constant angular velocity; in the absence of any detailed calculations constant angular momentum density seems, a priori, equally probable.

Collins: You have established that a sequence of models is in equilibrium. Have you established that any two neighboring models can be reached, i.e., can you move from one model to another without violating any physical laws?

Ostriker: The models are connected to one another in the same sense as models along the classical McClaurin Sequence. I have not investigated for either sequence what form (if any) the quasi-static contraction must take in order to transform one equilibrium model into the next one along the sequence.

Roxburgh: As I pointed out in 1966 it is to be expected that non-uniform rotation is developed during pre-main sequence contraction, and I used this in my papers on the fission theory. However, as I pointed out, during the adjustment period from fully convective phase to the radiative phase a high degree of differential rotation develops with an approximately radial variation.

The question I wish to put is whether your set of models, which with a misuse of the term are homologous, are in fact a sequence? That is whether you can in fact go from one model to the next in a sequential sense. You assume that in each model the angular velocity is constant on cylinders and that the angular momentum per unit mass is conserved. This may not be possible without at the same time destroying the angular momentum conservations for each individual fluid element that you must have in an inviscid fluid.

Ostriker: Your question was answered in part when I replied to Collins' question; the sequences are constructed so that every element conserves vorticity – and of course angular momentum as well. On your first point, we treated the contraction phase of high mass stars (5–12 $M_{\odot}$) specifically because these, according to recent calculations of Larson and Bodenheimer, may not have any Hayashi phase. The non-dynamical contraction phase is expected to be purely radiative for these stars and our calculations begin at the beginning of this quasi-static radiative contraction phase.

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