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SOLVABILITY OF NON SELF-ADJOINT AND HIGHER ORDER DIFFERENTIAL EQUATIONS WITH JUMPING NONLINEARITIES

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This thesis is concerned with problems of the type

$$Lu - g(u) = f(x) \tag{1}$$

We assume that the nonlinearity in (1) can be written as $g(u) = \mu u^{+} + \nu u^{-} + \psi(u)$ where $\mu, \nu \in \mathbb{R}$, $\mu = \lim_{u\to\infty} u^{-1} g(u)$, $\nu = \lim_{u\to-\infty} u^{-1} g(u)$. In the Ambrosetti-Prodi result the nonlinearity 'jumps' over the first eigenvalue of the linear problem, that is, $\nu < \lambda_1 < \mu$ (see [1]).

Knowledge of the set $A_0 = \{(\mu, \nu) \in \mathbb{R}^2 :$ there exists a non-trivial solution of $Lu = \mu u^{+} + \nu u^{-}\}$ is important if one wishes to apply homotopy invariant index methods to establish the existence of solutions. We obtain results for the set A_0 for a wide class of operators L.

A_0 is in general difficult to calculate but one problem for which results are known is the second-order self-adjoint problem:

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$$-u''(x) - \mu u^+(x) - \nu u^-(x) = 0, \quad x \in (0,\pi)$$
$$u(0) = u(\pi) = 0$$

Knowledge of A_0 for this problem has been suggestive of a number of more general results. (See [2].)

We calculate the set A_0 explicitly for two problems:

(i) The second-order non self-adjoint problem,

$$-u''(x) - \mu u^+(x) - \nu u^-(x) = 0, \quad x \in (0,\pi)$$
$$u(0) + u(\pi) = 0, \quad u'(\pi) = 0$$

(ii) The fourth-order self-adjoint problem suggested by Fučik in [4],

$$-u^{iv}(x) - \mu u^+(x) - \nu u^-(x) = 0, \quad x \in (0,\pi)$$
$$u(0) = u(\pi) = u''(0) + u''(\pi) = 0$$

These results compare interestingly with the known case and in the second-order case we obtain a counter-example to a potential generalisation of a classic theorem of Dolph.

Dancer's results in [2] for the complementary set, $\mathbb{R}^2 \backslash A_0$, are extended for higher-order and non self-adjoint problems. Here the use of the Maximum Principle is foregone and the theory of Positive Operators used instead.

More general theorems are obtained concerning the local behaviour of the set A_0 near (λ_k, λ_k) both when λ_k is a simple eigenvalue and when λ_k is an eigenvalue of higher multiplicity for the corresponding linear problem. The difficulty here is the non-smoothness of the nonlinearity in (1). (It is not even C^1). A modification of the Implicit Function Theorem is used.

Finally, in the case when the linearisation has one-dimensional kernel, we provide a negative answer to a question posed by Dancer in [3]: Does A_0 contain an open set?

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