




RESEARCH NOTE

The role of hyperparameters in machine learning models and how to tune them

Christian Arnold¹ , Luka Biedebach², Andreas Küpfer³  and Marcel Neunhoffer^{4,5} 

¹Department of Politics and International Relations, Cardiff University, Cardiff, UK, ²Department of Computer Science, Reykjavik University, Reykjavik, Iceland, ³Institute for Political Science, Technical University of Darmstadt, Darmstadt, Germany, ⁴Rafik B. Hariri Institute for Computing and Computational Science & Engineering, Boston University, Boston, MA, USA and ⁵Department of Statistics, LMU Munich, Munich, Germany

Corresponding author: Marcel Neunhoffer; Email: marcel@marcel-neunhoffer.com

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Abstract

Hyperparameters critically influence how well machine learning models perform on unseen, out-of-sample data. Systematically comparing the performance of different hyperparameter settings will often go a long way in building confidence about a model's performance. However, analyzing 64 machine learning related manuscripts published in three leading political science journals (APSR, PA, and PSRM) between 2016 and 2021, we find that only 13 publications (20.31 percent) report the hyperparameters and also how they tuned them in either the paper or the appendix. We illustrate the dangers of cursory attention to model and tuning transparency in comparing machine learning models' capability to predict electoral violence from tweets. The tuning of hyperparameters and their documentation should become a standard component of robustness checks for machine learning models.

Keywords: Best Practice; Hyperparameter Optimization; Machine Learning

1 Why care about hyperparameters?

When political scientists work with machine learning models, they want to find a model that generalizes well from training data to new, unseen data.¹ Hyperparameters play a key role in this endeavor because they determine the models' capacity to generalize. Finding a good set of hyperparameters critically affects conclusions about a model's performance. The failure to correctly tune and report hyperparameters has recently been identified as a key impediment to the accumulation of knowledge in computer science (e.g. Henderson *et al.*, 2018; Melis *et al.*, 2018; Bouthillier *et al.*, 2019, 2021; Cooper *et al.*, 2021; Gundersen *et al.*, 2023). Is political science making the same mistake?

We examined 64 machine learning-related papers published between 1 January 2016 and 20 October 2021 in some of the top journals of our discipline—the American Political Science Review (APSR), Political Analysis (PA), and Political Science Research and Methods (PSRM). Of the 64 publications we analyzed, 36 (56.25 percent) do not report the values of their hyperparameters, neither in the paper nor the appendix. Forty-nine publications (76.56 percent) do not share information about how they used tuning to find the values of their hyperparameters.

¹ A machine learning algorithm is “a computer program [that is] said to learn from experience E with respect to some class of tasks T and performance measure P , if its performance at tasks in T , as measured by P , improves with experience E .” (Mitchell, 1997)

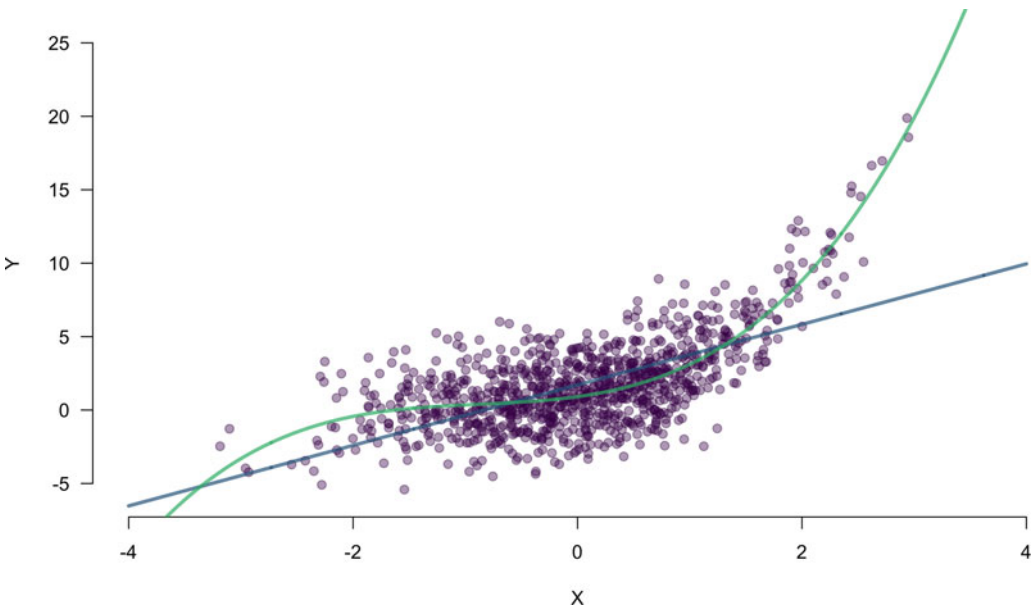


Figure 1. Example with polynomial regression. Data $X \sim N(0, 1)$. Data generating process: $Y = 1 + X + 0.8X^2 + 0.3X^3 + \epsilon$, with $\epsilon \sim N(0, 2)$. Regression Line for Bivariate OLS Model in Blue. Regression Curve for Polynomial Regression with $\lambda = 3$ in Teal.

Only 13 publications (20.31 percent) offer a complete account of the hyperparameters and their tuning. Not being transparent is a dangerous habit because readers and reviewers cannot assess the quality of a manuscript without access to the replication code.

With this paper, therefore, we raise the awareness that hyperparameters and their tuning matter. In statistical inference, the goal is to estimate the value of an unknowable population parameter. Including robustness checks in a paper and its appendix is good practice, allowing others to understand critical choices in research design and statistical modeling. The actual out-of-sample performance of a machine learning model is such an unknown quantity, too. We suggest handling estimates of population parameters and hyperparameters in machine learning models with the same loving care.

First, we explain what hyperparameters are and why they are essential. Second, we show why it is dangerous not to be transparent about hyperparameters. Third, we offer best practice advice about properly selecting hyperparameters. Finally, we illustrate our points by comparing the performance of several machine learning models to predict electoral violence from tweets (Muchlinski *et al.*, 2021).

2 What are hyperparameters and why do they need to be tuned?

Many machine learning models have parameters and also hyperparameters. Model parameters are learned during training, and hyperparameters are typically set before training. Hyperparameters determine how and what a model can learn and how well the model will perform on out-of-sample data. Hyperparameters are thus situated at a meta-level above the models themselves.

Consider the following stylized example displayed in Figure 1.² A linear regression approach could model the relationship between X and Y as $\hat{Y} = \beta_0 + \beta_1 X$. A more flexible model would include additional polynomials in X . For example, choosing $\lambda = 2$ encodes the theoretical belief that Y is best predicted by a quadratic function of X , i.e., $\hat{Y} = \beta_0 + \beta_1 X + \beta_2 X^2$. But it is also

²See also Shalev-Shwartz and Ben-David (2014) and Goodfellow *et al.* (2016).

possible to rely on data only to find the optimal value of λ . Measuring the generalization error with a metric like the mean squared error helps empirically select the most promising value of λ .

This polynomial regression comes with both parameters and hyperparameters. *Parameters* are variables that belong to the model itself, in our example, the regression equation coefficients. *Hyperparameters* are those variables that help specify the exact model. In the context of the polynomial regression, λ is the hyperparameter that determines how many parameters will be learned (Goodfellow *et al.*, 2016). Machine learning models can, of course, come with many more hyperparameters that relate not only to the exact parameterization of the machine learning model. Anything part of the function that maps the data to a performance measure and that can be set to different values can be considered a hyperparameter, e.g., the choice and settings of a kernel in a support vector machine (SVM), the number of trees in a random forest (RF), or the choice of a particular optimization algorithm.

3 Misselecting hyperparameters

Research on machine learning has recently identified several problems that may arise from handling hyperparameters without care. The failure to report the chosen hyperparameters impedes scientific progress (Henderson *et al.*, 2018; Bouthillier *et al.*, 2019, 2021; Gundersen *et al.*, 2023). In the face of a hyperparameter space marked by the curse of dimensionality, other researchers can only replicate published work if they know the hyperparameters used in the original study (Sculley *et al.*, 2018). In addition, it is essential to tune the hyperparameters of all models, including baseline models. Without such tuning, it is impossible to compare the performance of two different models M_a and M_b : While some may find that the performance of M_a is better than M_b , others replicating the study with different hyperparameter settings could conclude the opposite: that indeed M_a is *not* better than that of M_b . Such “hyperparameter deception” (Cooper *et al.*, 2021) has confused scientific progress in various subfields in computer science where machine learning plays a key role, including natural language processing (Melis *et al.*, 2018), computer vision (Musgrave *et al.*, 2020), and generative models (Lucic *et al.*, 2018). Reviewers and readers need to comprehend the hyperparameter tuning to assess whether a new model reliably performs better or whether a study tests new hyperparameters (Cooper *et al.*, 2021).

It is good to see political scientists also discuss and stress the relevance of hyperparameter tuning in their work (e.g., Cranmer and Desmarais, 2017; Fariss and Jones, 2018; Chang and Masterson, 2020; Miller *et al.*, 2020; Rheault and Cochrane, 2020; Torres and Francisco, 2021). But does the broader political science community fulfill the requirements suggested in the computer science literature? To understand how hyperparameters are used in the discipline, we searched for the term “machine learning” in all papers published in APSR, PA, and PSRM after 1 January 2016 and before 20 October 2021. Suppose a paper applies a machine learning model with tunable hyperparameters. In that case, we first annotate whether the authors report the final values of hyperparameters for all models in their paper or its appendix.³ We also record whether authors transparently describe how they tuned hyperparameters.⁴ Table 1 summarizes the findings from our annotations. We find that 34 (53.12 percent) publications neither report the values of the final hyperparameters nor the tuning regime in the publication or its appendix. Another 15 publications (23.44 percent) offer information about the final hyperparameter values but not how they tuned the machine learning models. In two cases (3.12 percent), we find no information about the final values of the hyperparameters but about the tuning regime. Finally, only 13 publications (20.31 percent) offer a full account of both the final choice of the hyperparameters and the way the tuning occurred in either the paper itself or its appendix.

³We call this “model transparency,” i.e., could a reader understand the final models without access to the replication code?

⁴We call this “tuning transparency,” i.e., could a reader understand the hyperparameter tuning without access to the replication code? Please see Appendix 1 for more details about our annotations.

Table 1. Can readers of a publication learn how hyperparameters were tuned and what hyperparameters were ultimately chosen? Hyperparameter explanations in papers published in APSR, PA, and PSRM between 1 January 2016 and 20 October 2021

| | | Tuning transparency | |
|--------------------|-----|---------------------|-----|
| | | No | Yes |
| Model transparency | No | 34 | 2 |
| | Yes | 15 | 13 |

Note that we annotated the literature in a way that helps understand whether reviewers and readers can assess the robustness of the analyses based on the manuscript and its appendix. Our analysis does not consider the replication code since it typically does not find consideration in the review process. In addition, we do not make any judgments about correctness. A paper without information about hyperparameter values or their tuning can still be correct. Similarly, a paper that reports hyperparameter values and a complete account of the tuning can still be wrong. It is the realm of reviewers to evaluate the quality of a manuscript. But without a complete account of hyperparameter values and tuning, readers and, in particular, reviewers cannot judge whether hyperparameter tuning is technically sound.

4 Best practice

Hyperparameters are a fundamental element of machine learning models. Documenting their careful selection helps build trust in the insights gained from machine learning models.

4.1 Selecting hyperparameters for performance tuning

Without automated procedures for finding hyperparameters, researchers need to rely on heuristics (Probst *et al.*, 2019). The classic approach to hyperparameter optimization is to systematically try different hyperparameter settings and compare the models using a performance measure. Machine learning splits the data into training, validation, and test data (Friedman *et al.*, 2001; Goodfellow *et al.*, 2016). The model parameters are optimized using the training data. The validation data is used to optimize the hyperparameters by estimating and then comparing an estimate of the performance of all the different models. Finally, the test data helps approximate the performance of the best model for out-of-sample data. Researchers should train a final machine learning model for a realistic estimate of the model’s performance. This model relies upon the identified best set of hyperparameters, uses a combined set of the training and validation data, and is evaluated on the so far withheld test set. Note that this last evaluation can be done only once to avoid information leakage. Tuning hyperparameters is therefore not a form of “p-hacking” (Wasserstein and Lazar, 2016; Gigerenzer, 2018) where researchers try different models until they find the one that generates the desired statistics. On the contrary, transparently testing different hyperparameter values is necessary to find a model that generalizes well.

In hyperparameter grid search, researchers manually define a grid of hyperparameter values, then try each possible permutation and record the validation performance for each set of hyperparameters. More recently, some instead suggest randomly sampling a large number of hyperparameter candidate values from a pre-defined search space (Bergstra and Bengio, 2012) and recording the validation performance of each set of sampled hyperparameter values.⁵ This random search can help explore the space of hyperparameters more efficiently if some

⁵How many permutations from the search space should be tried depends on the search space size and the available computational resources.

hyperparameters are more important than others. Both approaches typically yield reliable and good results for practitioners and build trust regarding the out-of-sample performance.

But the tuning of hyperparameters might be too involved for grid or random search in light of resource constraints. It is then useful to not try all combinations of hyperparameters but rather focus on the most promising ones.⁶ Sequential model-based Bayesian optimization formalizes such a search for a new candidate set of hyperparameters (Snoek *et al.*, 2012; Shahriari *et al.*, 2016). The core idea is to formulate a surrogate model—think non-linear regression model—that predicts the machine learning model's performance for a set of hyperparameters. At iteration t , the underlying machine learning model is trained with the surrogate model's suggestion for the next best candidate set of hyperparameters. The results from this training at t are fed back into the surrogate model and used to refine the predictions for the candidate set of hyperparameters in the next iteration $t + 1$.⁷

Without a formal solution, the selection of hyperparameters requires human judgment. We suggest relying on the following short heuristics when tuning and communicating hyperparameters.⁸

1. **Understanding the model.** What are the available hyperparameters? How do they affect the model?
2. **Choosing a performance measure.** What is a good performance for the machine learning model? Depending on the respective task, appropriate measures help assess the model's success. For example, a regression model is trained to minimize the mean squared error. Classification models can be trained to maximize the F1 score. With an appropriate performance measure, it is also possible to systematically tune the hyperparameters of unsupervised models (Fan *et al.*, 2020).
3. **Defining a sensible search space.** Useful starting points for the hyperparameters can be the default values in software libraries, recommendations from the literature, or own previous experience (Probst *et al.*, 2019). Any choice may also be informed by considerations about the data-generating process. If the hyperparameters are numerical, there may be a difference between mathematically possible and reasonable values.
4. **Finding the best combination in the search space.** In grid search, researchers should try every possible combination of the hyperparameters of the search space to find the optimal combination. In random search, each run picks a different random set of hyperparameters from the search space.
5. **Tuning under strong resource constraints.** If the model training is too involved, adaptive approaches such as sequential model-based Bayesian optimization allow for efficiently identifying and testing promising hyperparameter candidates.

Researchers should describe in either the main body or the appendix of their publication how they tuned their hyperparameters and also what final values they chose. Only then can reviewers and readers assess the robustness of machine learning models.

4.2 Illustration: Comparing machine learning models to predict electoral violence from tweets

To illustrate our point, we compare machine learning models trained to predict electoral violence from tweets. Muchlinski *et al.* (2021) collected Tweets around elections in three countries

⁶For other promising strategies, see the thorough overviews in, e.g., Hutter *et al.* (2015), Luo (2016), Probst *et al.* (2019) and Bischl *et al.* (2023).

⁷Adaptive hyperparameter optimization is conveniently implemented in many software frameworks: for R see, e.g., `mlr3` package on CRAN (Lang *et al.*, 2019), for Python, e.g., `scikit-optimize` (Pedregosa *et al.*, 2011) or `keras` (Chollet *et al.*, 2015).

⁸See also Sculley *et al.* (2018), Bouthillier *et al.* (2021) and Cooper *et al.* (2021).

Table 2. Performance benchmarking of Muchlinski *et al.* (2021) on different classifiers using our scraped data.

| Classifier | Default F1 | Tuned F1 | Default F1 | Tuned F1 | Default F1 | Tuned F1 |
|------------|---------------|-------------|-----------------|-------------|---------------|-------------|
| | Ghana | | The Philippines | | Venezuela | |
| NB | 0.000 | 0.538 | 0.000 | 0.390 | 0.000 | 0.308 |
| RF | 0.341 | 0.603 | 0.400 | 0.160 | 0.237 | 0.479 |
| SVM | 0.381 | 0.727 | 0.357 | 0.561 | 0.080 | 0.465 |
| CNN | 0.632 | 0.604 | 0.356 | 0.500 | 0.319 | 0.304 |

On the left: results with default values for the hyperparameters. On the right: results from tuned hyperparameters

(Ghana, the Philippines, and Venezuela) and annotated whether these messages described occurrences of electoral violence. We re-scraped the data based on the shared Tweet IDs. To predict these occurrences from the content of these Tweets, we use four different machine learning models—a naive Bayes classifier (NB), random forest (RF), a support vector machine (SVM), and a convolutional neural network (CNN).

Table 2 summarizes our results. In the left column of each country, we report the results from training the models with default hyperparameters. On the right, we show the results after hyperparameter tuning.⁹ Hyperparameter tuning improves the out-of-sample performance for most machine learning models in our experiment.¹⁰ Table 2 also shows how easy it is to be deceived about the relative performance of different models—if hyperparameters are not properly tuned. The performance gains from tuning are so substantial that most tuned models outperform any other model with default hyperparameters. In the case of Venezuela, for example, comparing a tuned model with all other baseline models at their default hyperparameter settings could lead to different conclusions. Researchers could mistakenly conclude that (a tuned) NB classifier (F1 = 0.308) is at eye-level with a CNN model (F1 = 0.319) and better than any other method; or also that the RF is the better model (F1 = 0.479), or the SVM (F1 = 0.465), or the CNN (F1 = 0.304). In short, model comparisons and model choices are only meaningful if all hyperparameters of all models are systematically tuned and if this tuning is transparently documented.

5 Tuning hyperparameters matters

Hyperparameters critically influence how well machine learning models perform on unseen, out-of-sample data. Despite the relevance of tuned hyperparameters, we found that only 20.31 percent of the papers using machine learning models published in APSR, PA, and PSRM between 2016 and 2021 include information about the ultimate hyperparameter choice and how they were found in the manuscript or the appendix. Furthermore, 34 papers (53.12 percent) neither report the hyperparameters nor their tuning. This is a dangerous habit since handling hyperparameters without care can lead to wrong conclusions about model performance and model choice.

The search for an optimal set of hyperparameters is a vibrant research area in computer science and statistics. For most of the applications in our discipline, acknowledging and discussing how the choice of hyperparameters could influence results in combination with a proper and systematic search for appropriate hyperparameters would go a long way. It would allow others to understand original work, assess its validity, and thus ultimately help build trust in political science that uses machine learning.

⁹In line with (Muchlinski *et al.*, 2021), we chose the F1 score as the performance metric. We include details on the tuned hyperparameters, the default values we chose, the search method, the search space for each model, and any random seeds in the Online Appendix.

¹⁰In cases where hyperparameter tuning does not improve the performance over default hyperparameter values, the default values are closer to the optimal solution than the best-performing hyperparameters from a cross-validation procedure. However, the only way to find this out is through systematic hyperparameter tuning.

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