

## THE MATHEMATICAL ASSOCIATION

The fundamental aim of the Mathematical Association is to promote good methods of mathematical teaching. A member receives each issue of one or more of *The Mathematical Gazette*, *Mathematics in School*, *Primary Mathematics* (according to the class of membership chosen), together with Newsletters. Reports are published from time to time and these are normally available to members at a reduced rate. Those interested in becoming members should contact MA Headquarters for information and application forms. The address of the Association Headquarters is **Charnwood Building, Holywell Park, Loughborough University Science and Enterprise Park, Leicestershire LE11 3AQ, UK (telephone 0116 221 0013)**. The Association should be notified of any change of address. If copies of the Association periodicals fail to reach a member through lack of such notification, duplicate copies can only be supplied at the published price. If change of address is due to a change of appointment, the Association will be glad to be informed. Subscriptions should be submitted to the Treasurer via Headquarters. Correspondence relating to **Teaching Committee** should be addressed to TC Chair. The **Association's Library** is housed in the University Library, Leicester.

Views expressed in the *Mathematical Gazette* by authors or advertisers are not necessarily those of the Association.

## THE MATHEMATICAL GAZETTE

### *Editor:*

Dr Gerry Leversha,  
15 Maunder Road,  
Hanwell,  
London W7 3PN  
*g.leversha@btinternet.com*

### *Problem Corner:*

Mr Nick Lord,  
Tonbridge School,  
Tonbridge,  
Kent TN9 1JP  
*njl@tonbridge-school.org*

### *Production Editor:*

Mr Bill Richardson,  
Kintail,  
Longmorn,  
Elgin IV30 8RJ  
*wpr3145@gmail.com*

### *Reviews Editor:*

Mr Owen Toller,  
4 Caldwell House,  
48 Trinity Church Road,  
London SW13 8EJ  
*owen.toller@btinternet.com*

Potential advertisers e-mail Charlotte Dyason at: [charlotted@media-shed.co.uk](mailto:charlotted@media-shed.co.uk)

Material for publication should be sent to the Editor.

Books for review should be sent to the Reviews Editor.

### *Advice to authors of notes and articles.*

Study the format of articles in the *Gazette*. Please note the format for references, which should be listed in their order of appearance in an article. MSS may be submitted electronically, preferably in pdf format, or, if sent by post, should be typed and two copies included. (Mathematical expressions may be hand written.) Please send electronic files by e-mail. This edition of the *Gazette* was produced on an Acorn machine using TechWriter and Draw.

10.1017/mag.2023.88

## CONTENTS (continued)

### Notes 107.14 to 107.27

Does a trapezium exist whose side lengths form a geometric progression?	<i>Victor Oxman, Moshe Stupel</i>	301
Fruit diophantine equation	<i>Dipramit Majumdar, B. Sury</i>	302
An expression for the prime-composite characteristic function	<i>Matthieu Contensou</i>	306
Two curios related to lattice polygons	<i>Nick Lord</i>	307
A two-variable approach to some standard optimisation problems	<i>Nick Lord</i>	312
A quadratic harmonic approximation	<i>Mark B. Villarino</i>	316
Euler's constant and the speed of convergence	<i>Peter R. Mercer</i>	320
Proof Without Words: An inverse tangent inequality	<i>Mehdi Hassani, Gerry Leversha</i>	323
Quick proofs of two inequalities related to the digamma function	<i>Rasul Khan, Allan Silberger</i>	324
Location of the inarc circle and its point of contact with the circumcircle	<i>Martin Lukarevski</i>	327
How to cut cubes into dodecahedra and icosahedra	<i>Hidefumi Katsuura</i>	332
A refinement of Griffiths' formula for the sums of the powers of an arithmetic progression	<i>José Luis Cereceda</i>	340
A difference theorem involving $k$ -gonal and centred $k$ -gonal numbers	<i>Günhan Caglayan</i>	342
The discrete renewal theorem with bounded inter-event times	<i>Rohan Manojkumar Shenoy</i>	343

### Teaching Note

Trigonometric identities from the mystic rose	<i>Paul Stephenson</i>	349
---	------------------------	-----

<b>Feedback</b>		356
-----------------	--	-----

<b>Problem Corner</b>	<i>Nick Lord</i>	359
-----------------------	------------------	-----

<b>Student Problems</b>	<i>Tuya Sa</i>	365
-------------------------	----------------	-----

<b>Reviews</b>		368
----------------	--	-----

## CONTENTS

### Articles

Groups, conics and recurrence relations	<i>A. F. Beardon</i>	193
The repayment of financial debt: some mathematical considerations	<i>John Stubbs, Jacob Adetunji</i>	204
How effective is the efficiency gap?	<i>Thomas Q. Sibley</i>	218
Yet more characterisations of parallelograms	<i>Mowaffaq Hajja, Panagiotis T. Krasopoulos</i>	225
The lost boarding pass problem: converse results	<i>Shohei Kubo, Toshio Nakata, Naoki Shiraishi</i>	234
Integer triangles with integer circumradii	<i>Emrys Read</i>	241
Developing new picture proofs that the sums of the first $n$ odd integers are squares	<i>Chris Sangwin, Fenner Stanley Tanswell</i>	249
A graph related to the Euler $\phi$ function	<i>Nima Ghanbari, Saeid Alikhani</i>	263
Approximating Lipschitz and continuous functions by polynomials; Jackson's theorem	<i>G. J. O. Jameson</i>	273

### Matter for Debate

What makes a good maths teacher?	<i>Tony Gardiner, Chris Tew</i>	286
----------------------------------	---------------------------------	-----

© The Mathematical Association 2023

Typeset by Bill Richardson

**CAMBRIDGE**  
**UNIVERSITY PRESS**



Printed in Great Britain by Henry Ling Limited, The Dorest Press, Dorchester, DT1 1HD  
ISSN 0025-5572