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CORRECTION

Britton, T. and Lindholm, M. (2009). The early stage behaviour of a stochastic SIR epidemic with term-time forcing. *J. Appl. Prob.* **46,** 975–992.

1. Description of the errors made

In the paper by Bacaër and Ed-Darraz [1] some results were pointed out that are in conflict with an example in the above paper. After some consideration we have come to the conclusion that the theorem upon which the example is based contains an error. The error stems from the fact that the probability generating function $\chi(s)$, as defined in (1) of the above paper, does not describe the dynamics of the branching process under study. Instead it defines the behaviour of the *averaged* offspring distribution of the environmentally driven branching process under study, given a single completed environmental cycle.

As a consequence, part of Theorem 1 of the above paper is wrong. The convergence results of the epidemic to the branching process in random environment remain true, it is the analysis of the approximating branching process in random environment that is incorrect.

Below we present a corrected version of Theorem 1 of the above paper.

1.1. Corrected formulation of Theorem 1.

Let $\tau(t) = \max\{n\colon \sum_{i=1}^{n}(T_{i1} + \cdots + T_{ik}) \leq t\}$, where k denotes the number of environmental states, and define $\chi_{\tau(t)}(s)$ as follows:

$$\chi_{\tau(t)}(s) := \xi_1(\xi_2(\dots\xi_{k-1}(\xi_k(s, T_{\tau(t)k}), T_{(\tau(t)-1)(k-1)})\dots, T_{21}), T_{11}), \qquad |s| \leq 1. \tag{1}$$

Let ρ, the probability of extinction, be defined as

$$\rho = \mathbb{E}[\lim_{t\to\infty} \chi_{\tau(t)}(0)]. \tag{2}$$

Theorem 1. *Let $\{(S^{(n)}(t;\Lambda(t),\gamma), I^{(n)}(t;\Lambda(t),\gamma))\}_{t\geq 0}$ denote an epidemic process in a term-time forced environment defined by the rates in Table 1 of the above paper, and let everything else bed defined as above. Moreover, Let $\mathbb{E}^{(n)} := n - S^{(n)}(\infty;\Lambda(\infty),\gamma)$. Then $R_\star$ defined by*

$$R_\star = \frac{\sum_{i=1}^{k}\mathbb{E}[T_i]\lambda_i}{\gamma\sum_{j=1}^{k}\mathbb{E}[T_j]}$$

works as a threshold, such that $\mathbb{E}^{(n)} \xrightarrow{\mathrm{D}} \mathbb{E}$ as $n\to\infty$, and $\pi := \mathbb{P}(\mathbb{E} = +\infty)$ satisfies the relation $\pi = 1-\rho$. In addition, $0 < \pi < 1$ if and only if $R_\star > 1$.

1.2. Comments on Theorem 1.

As pointed out above, what went wrong in the above paper is the analysis of the approximating branching process. A derivation of $R_\star$ is readily found in [1]. A brief motivation of the intuition behind $R_\star$ is as follows: the underlying model is a linear birth-and-death process, which, conditional on the states of the environmental process, has a stochastic growth after n

completed generations given by

$$\begin{aligned}\mathbb{E}[\#\text{infected individuals at generation } n \mid \mathcal{F}_n] &= \exp\left[\sum_{i=1}^{n}\sum_{j=1}^{k}(\lambda_j - \gamma)T_{ij}\right] \\ &= \exp\left[n\sum_{j=1}^{k}(\lambda_j - \gamma)\sum_{i=1}^{n}\frac{1}{n}T_{ij}\right] \\ &\sim \exp\left[n\left(\sum_{j=1}^{k}\mathbb{E}[T_j]\right)\gamma(R_\star - 1)\right],\end{aligned}$$

which indicates how $R_\star$ behaves as a threshold. One can of course directly use the definition of the (random) probability generating function $\chi(s)$ given in (1) together with standard limit theorems to come to the above conclusion.

Regarding the probability of extinction (explosion), (1) together with (2) provide one way of calculating these probabilities numerically, by restraining calculations to some large value of $\tau(t) = n$.

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References

[1] Bacaër, N. and Ed-Darraz, A. (2014). On linear birth-and-death processes in a random environment. *J. Math. Biol.*, **69,** 73–90.