the event of his desiring to rectify and beautify the results given by his formula.

I am, Sir,
Your obedient servant,
T. B. SPRAGUE.

Edinbro',
1 *July* 1887.

CLAIM ACCELERATION RESERVE, &c.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—For some time past I have been intending, with your permission, to correct an oversight on *J.I.A.*, xxiv, 76, for it would seem that claim acceleration reserve should be based on the theoretical instead of on the actual date for payment, at any rate as long as the fraction combined with the annuity-value in capitalizing future premiums is dependent on the date of their falling due rather than of their being received. That is, that the interval necessary for proof of death and title ought not to be taken into account, unless the grace days allowed for renewals are considered on the other side; or, in other words, if claims are payable immediately, a full half-year's (not five months') interest must be reserved, unless the $\frac{1}{2}$, or whatever it is, used with the a in valuing the premiums is not fixed by the average of their due-dates only, but regard is also had to any delay there may be in the cash reaching the office.

And, as I am writing, I would add that the formula on *J.I.A.*, xxvi, 54, looks less formidable if y be written for $\frac{x-1}{2}$; while, later on, $\frac{2x^2}{x^3}$ instead of $\frac{2}{x}$ is an ugly mishap.

I am, Sir,
Your obedient servant,
C. D. HIGHAM.

3, *Princes Street, Bank, London*,
26 *May* 1887.

FRIENDLY SOCIETY LEVIES.

To the Editor of the Journal of the Institute of Actuaries.

Sir,—In the last number of the *Journal* (p. 389) Mr. King refers to the above subject, and gives very simple demonstrations of the formula for the value of the future death levies,

$$W = \frac{m(m-1)}{2} \bar{A}_{xx},$$

where m is the number of members, x the average age, and 1 the sum paid by each member at a levy. The proof he gives of the above, by the use of contingent assurances, I may say, was suggested to me some years ago by Mr. H. J. Rothery.

It frequently happens that levies are made not only at the deaths of the members but also at the deaths of their wives, and a similar method of dealing with these leads to an equally convenient formula by which to value them. If we assume that all the members are married, and that w_1, w_2, w_3, &c., represent the ages of the wives of

the members x_1, x_2, x_3, &c., the value of the future levies on the deaths of wives, will be

$$\begin{aligned}
&\bar{A}^1_{w_1x_1x_2}+\bar{A}^1_{w_1x_1x_3}+\bar{A}^1_{w_1x_1x_4}+\ldots\ (m-1)\text{ terms}\\
&+\bar{A}^1_{w_2x_2x_1}+\bar{A}^1_{w_2x_2x_3}+\bar{A}^1_{w_2x_2x_4}+\ldots\quad\text{,,}\qquad\text{,,}\\
&\qquad\ldots\ldots\\
&+\bar{A}^1_{w_mx_mx_1}+\bar{A}^1_{w_mx_mx_2}+\bar{A}^1_{w_mx_mx_3}+\ldots\quad\text{,,}\qquad\text{,,}
\end{aligned}$$

In all there will be $m.\overline{m-1}$ contingent assurances, and if we assume as before an average age x, for members and wives alike, we have

$$m(m-1)\bar{A}^1_{xxx}=\frac{m(m-1)}{3}\bar{A}_{xxx}.$$

Mr. King considers these formulas to be theoretical, not practical, solutions, since the number of members in a society is constantly subject to change by new entrants and by secessions; but I think this objection cannot be urged with greater force in the case of these contributions than in that of the fixed annual payments of the members, and that the above formulas represent the sums at which these particular assets should be taken in the valuation. On the one hand, it is clear that no account can be taken of their possible increase by the addition of new members, nor, on the other hand, is it necessary to take any account of their possible reduction by secessions, since (provided negative values have been properly excluded from the valuation) the net liability of the society will not be increased by such means. In this respect the levies are in the same position as the fixed contributions; their estimated value may be diminished by secessions, but since, at the same time, the society's liabilities are correspondingly diminished, this is of no moment.

In order to exclude negative values from a valuation in which future levies appear as an asset, the net liability for each member, after deducting the value of contributions, but *independent of the levies*, should be taken at not less than

$$(m-1)\bar{A}_{x_1x_2}=(m-1)\bar{A}_{x_1x}\ \text{ nearly}$$

(where x_1 is the age of the member and x_2 the mean age of the remaining $\overline{m-1}$ members, which may be taken as practically $=x$, the mean age of the whole society). For the assets, as regards the value of the levies, will be diminished in case of the secession of the member x_1 by the value of the assurances on x_1 jointly with the remaining $(m-1)$ members. At the younger ages, where alone negative values will appear, the above will be somewhat less in value than

$$(m-1)\bar{A}_{xx}$$

where x is the average age of all the members, as before; this latter quantity being constant, while the former varies with the ages x_1, x_2, it may be conveniently substituted.

As an example, if we suppose a society of 150 members making a levy of 1*s.* per member at each death, and of an average age giving $\bar{A}_{xx}=\cdot600$, the constant $(m-1)\bar{A}_{xx}=4\cdot47$; and provided the net liability in respect of each member, as above (independent of the

levies) is taken at not less than this figure, negative values will be effectually excluded, and the values of the levies as given by the formula will be a perfectly good asset.

I am, Sir, your obedient servant,

5, *Whitehall, S.W.*,
24 *August* 1887.

G. F. HARDY.

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