BEAM COMBINATION FOR WIDE FIELD IMAGING

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ABSTRACT. Several limitations reduce the field of view in radio-interferometry. With an optical array, two of them can be overcome to some extent according to the beam combination method. A beam combination in the pupil plane can completely overcome one of them. In the image plane, a beam combination obeying the rules of geometrical optics can overcome both limitations in principle, but is difficult to achieve in practice. We discuss particularly the real case of a Michelson Stellar Interferometer where a periscope partially re-introduces these limitations, yielding a trade-off between the extension of the field of view and the use of the periscope.

1. Introduction

In a non-heterodyne interferometer (the general case in the optical wavelengths) a beam combiner mixes the beams coming from each element of an array. This optical set-up can have many different forms [1], with different properties and performances. Regarding the field of view (FOV), a detailed review of these possibilities is not necessary. We will rather analyse the general properties concerning the FOV of each of the two families of beam combiners: pupil plane combiners where interference phenomena are observed in the superimposed exit pupils and image plane combiners where the beams are mixed in a plane conjugate to the object.

A wide FOV means that its extension is larger than some "usual" one which must be defined. We have chosen to take as a reference the FOV extension of the radio-interferometers, or what we shall call the "correlator arrays". These interferometers as described by the theory of the coherence of the waves [2] measure <u>one sample</u> of the complex degree of coherence per baseline. Heterodyne interferometers (radio and infrared) as well as optical interferometers using single mode optics fibres [3] work as correlator arrays. It is also the case for most of the present optical interferometers using separated telescopes. Several problems limit the FOV of correlator arrays [4]. Among them, we have considered the two following basic limitations, since they are handled differently in optics:

1) Each telescope has a finite diameter D which limits the FOV to λ/D: the complex amplitude of the incoming wave is averaged over the area of each telescope so that the resolution in the (u,v) plane is reduced to about

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D/λ [5]. In the visible domain, this limits the FOV to between 1 arcsec and 10 milliarcsec when D ranges from 10 cm to 10 m.

2) While the delay lines are adjusted for zeroing the optical path differences (OPD) at the centre of the FOV, an off-axis point at angular position β from this centre produces an extra external OPD equal to βb, which must be smaller than the coherence length to produce interference. For a 100 m baseline, this limits the FOV to the range 1 arcsec to 10 milliarcsec when the spectral resolution $\Delta\lambda$ ranges from 0.5 nm to 50 nm at λ=0.5 μm.

We shall say that a wide FOV is reached when at least one of these two limits is overcome.

Among the other limitations of the FOV, anisoplanatism is of particular importance in optics. In radio-interferometry this problem could be overcome with a modified form of self calibration [6]. Similarly, a data processing technique does not seem impossible in optical wavelengths. But we do not take into account such a problem since it seems not to depend on the choice of the beam combiner.

We first present the case of a beam combination in the pupil plane (Section 2). In the image plane (Section 3), we then study the ideal case of a Fizeau Stellar Interferometer and the properties of a Michelson Stellar Interferometer. We also point out how these properties are related to those of the "correlator arrays".

2. Beam combination in the pupil plane

In this beam combination scheme, interference patterns are observed in the superimposed exit pupils which are images of the entrance pupils. This concept is comparable to a correlator array rather than a usual imaging system. As stated by Mariotti [7], since the optical path between the entrance and exit pupils is independent of the angle of propagation, no internal path difference is introduced as a function of the position angle in the FOV. Thus, the FOV is still limited by the limited coherence length like any "correlator array".

On the other hand, the resolution in the (u,v) plane can be as high as possible if the pupils are combined using translations only: in this case, each baseline gives a delta function in the (u,v) plane [7]. But the (u,v) plane coverage can be increased by using the so-called "zoom effect" [8], i.e. by rotating differently the pupils before the combination. The key point in such a case is the possibility of sampling the combined pupils with an array detector. The resolution in the (u,v) plane is directly linked to the pixel size and can be chosen as small as required.

For achieving a wide FOV, this combination is particularly useful for overcoming a limitation to the resolution in the (u,v) plane when large apertures are used. For coherent large apertures (e.g. in the infrared wavelengths with an adaptive optics system), this combination allows a high resolution in the (u,v) plane using a single pixel detector per baseline or even less [1].

3. Beam combination in the image plane

3.1. THE FIZEAU STELLAR INTERFEROMETER: AN IDEAL CASE

In the image plane, one can refer to the usual approach of geometrical optics and consider the interferometer as a system equivalent to a masked single aperture. Such an optical interferometer is called a Fizeau Stellar Interferometer (FSI) [9]. As with any classical "imaging system", the two basic limitations of the "correlator array" may be overcome:

1) The resolution in the (u,v) plane is related to the extension of the imaging field and can be, in principle, as high as desired. This extension is actually limited by the constraint of the hypothesis of Gaussian optics.

2) The external extra delay for any off-axis point is exactly compensated by an internal delay, as is the case with a single perfect lens.

This concept has been especially followed for interferometers with the apertures mounted on the same structure (e.g. [10-12]). It has been shown that a linear mapping between the entrance and exit pupils is the essential condition for an FSI combination, i.e. the optical layout between the entrance and exit pupil must be equivalent to an afocal system. The concept has been investigated more recently for non-monolithic arrays [13, 14]: the aperture centres are no longer in one plane, parallel to the incident wave, but are at different relative altitudes. As with an afocal system, these altitudes must be demagnified with a ratio equal to the square of the aperture demagnification ("longitudinal linear matching") [15].

The concept of the FSI is impossible to achieve perfectly and tolerances must be defined for a given FOV. Detailed studies [12, 14, 16] show that the tolerances may be very tight. The beam combination must indeed ensure lateral and longitudinal linear mapping, equality of the pupil demagnifications (each beam may have separate demagnification optics), no differential field rotation, no optical aberrations, etc. A major difficulty is to maintain all the properties while tracking the object. In any case the extension of the FOV is larger than that of a "correlator array" and depends on the amounts of the residual errors that are tolerated.

We will now study the case, important in practice, where the condition of a lateral linear mapping is not met. This condition is, indeed, generally broken intentionally by a periscope. Such a periscope first introduced by Michelson

[17] is the basis of the so-called Michelson Stellar Interferometer (MSI).

3.2. Michelson Interferometer and Field of View

From the geometrical optics point of view, a periscope introduces a violation of the Abbe sine condition [18]. On the other hand, it brings some practical advantages: the number of fringes may be reduced and kept constant (requiring fewer pixels for a suitable sampling of the image), a redundant entrance pupil may be transformed into a non-redundant exit one, and multispectral observations may be performed by re-arranging the pupils on a line and dispersing the fringe patterns.

We have already described the imaging properties of an MSI in the case of monochromatic light [5]. We extend here the derivation for a limited bandwidth $\Delta\sigma$, where $\sigma=1/\lambda$. We suppose that the brightness of a completely incoherent object, $O(\boldsymbol{\beta})$ ($\boldsymbol{\beta}$ represents the angular coordinates) does not depend on σ in the bandwidth $\Delta\sigma$, centred on σ_o. Let us consider an array of M telescopes where $P_n(\mathbf{x})$ denotes the complex transmission function of the aperture n ($P_n(\mathbf{x})=0$ outside the aperture) taking into account the corrugations of the wavefronts and the optical aberrations. $\mathbf{x}$ is a 2D-vector in the pupil plane, in metres. We assume that the delay lines are adjusted for the same point in the sky (taken as the origin of the $\boldsymbol{\beta}$ coordinates) in such a way that the average of the phase of $P_n(\mathbf{x})$ is zeroed for each aperture. The complex amplitude received in the pupil plane from an off-axis point source at the angular position $\boldsymbol{\beta}$ may be written as :

$$\psi(\mathbf{x},\sigma\boldsymbol{\beta}) = e^{i2\pi\sigma\boldsymbol{\beta}.\mathbf{x}} \sum_{n=1}^{M} P_n(\mathbf{x}-\mathbf{x}_n) \,, \tag{1}$$
$$= \sum_{n=1}^{M} \left(P_n(\mathbf{x})\, e^{i2\pi\sigma\boldsymbol{\beta}.(\mathbf{x}+\mathbf{x}_n)}\right) * \delta(\mathbf{x}-\mathbf{x}_n).$$

$\mathbf{x}_n$ is the coordinate of an aperture centre in the pupil plane, $*$ denotes the convolution operator and δ the Dirac delta function. When the apertures are shifted from $\mathbf{x}_n$ to $\mathbf{x}'_n$ by a periscope, the wave on each aperture is unchanged and the complex amplitude becomes simply :

$$\psi'(\mathbf{x},\sigma\boldsymbol{\beta}) = \sum_{n=1}^{M} \left(P_n(\mathbf{x})\, e^{i2\pi\sigma\boldsymbol{\beta}.(\mathbf{x}+\mathbf{x}_n)}\right) * \delta(\mathbf{x}-\mathbf{x}'_n). \tag{2}$$

Following a calculation similar to the one made in [5], a point spread function can be derived for the bandwidth $\Delta\sigma$ if we assume that the optical combiner provides a perfect Fourier transform:

$$I_o(\boldsymbol{\beta}',\boldsymbol{\beta}) = \frac{1}{A\Delta\sigma}\int d\sigma\, \sigma^2\, F\!\left(\frac{\sigma-\sigma_o}{\Delta\sigma}\right) |\tilde{\psi}'(\sigma\boldsymbol{\beta}',\sigma\boldsymbol{\beta})|^2 \,, \tag{3}$$

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where $\tilde{\psi}'$ denotes the Fourier transform of ψ', A is a normalisation factor and F(t) is a normalised function describing the "bandwidth pattern" centred on σ_o. As in [5], this point spread function is not shift invariant. The Fourier transform of an image thus keeps the following general form:

$$\tilde{I}(\mathbf{u}) = \int d^2\mathbf{\beta}\; O(\mathbf{\beta})\; \tilde{I}_0(\mathbf{u},\mathbf{\beta}). \tag{4}$$

Putting the Fourier transform of Eq. (3) with respect to the image coordinates $\mathbf{\beta}'$ into Eq. (4) and using the Fourier transform of Eq (2) yields:

$$\tilde{I}(\mathbf{u}) = \tilde{O}(\mathbf{u}) \left(\sum_{n=1}^{M} \frac{1}{A\Delta\sigma} \int d\sigma\, F(\frac{\sigma-\sigma_o}{\Delta\sigma})\; \tilde{S}_{nn}(\frac{\mathbf{u}}{\sigma}) \right)$$
$$+ \sum_{n=1}^{M} \sum_{m\neq n}^{M} \frac{1}{A\Delta\sigma} \int d\sigma\, F(\frac{\sigma-\sigma_o}{\Delta\sigma})\; \tilde{O}(\mathbf{u}+\sigma(\mathbf{x}'_{nm}-\mathbf{x}_{nm}))\; \tilde{S}_{nm}(\frac{\mathbf{u}}{\sigma}+\mathbf{x}'_{nm}) \tag{5}$$

where $\tilde{O}$ is the Fourier spectrum of the object, $\mathbf{x}_{nm}=\mathbf{x}_n-\mathbf{x}_m$ is the baseline vector for the pair (n,m) of telescopes, $\mathbf{x}'_{nm}$ is the corresponding exit baseline, and $\tilde{S}_{nm}(\mathbf{x})= P_n(-\mathbf{x}) * P^*_m(\mathbf{x})$ is the crosscorrelation function of the apertures n and m (autocorrelation when m=n). The first summation is the low spatial frequency part of the image spectrum, whereas the second one gives the M(M-1) "fringe peaks". When $\Delta\sigma=0$, as already stated in [5], at the spatial frequency $\mathbf{u}=-\sigma_o\mathbf{x}'_{nm}$ corresponding to the central frequency of one "fringe peak" $\tilde{S}_{nm}$, we observe the object spectrum at the frequency $\mathbf{u}=-\sigma_o\mathbf{x}_{nm}$, i.e. the central frequency of the fringe peak <u>before</u> the periscope. This is a sort of conversion of the spatial frequency. Problems arise from the wavelength dependence of this frequency conversion when $\Delta\sigma\neq 0$. So, even if the shift applied to the fringe peak is reversed as proposed in [5] for the monochromatic case, the object-image relationship of the FSI can no longer be recovered. What is the effect on the object shape ?

3.3. FIRST ORDER EFFECT

We now derive an approximate effect on the object by neglecting the wavelength dependency of the functions $\tilde{S}_{nm}$. This means that the spectral bandwidth meets the condition $\Delta\sigma/\sigma < r_o/D$ [19] where r_o is the Fried parameter and D the diameter of the telescopes. Introducing the approximation into Eq. (5) yields :

$$\tilde{I}(\mathbf{u}) \approx \tilde{O}(\mathbf{u})\; \frac{1}{A} \sum_{n=1}^{M} \tilde{S}_{nn}(\frac{\mathbf{u}}{\sigma_o})$$
$$+ \sum_{n=1}^{M} \sum_{m\neq n}^{M} \frac{1}{A}\; \tilde{S}_{nm}(\frac{\mathbf{u}}{\sigma_o}+\mathbf{x}'_{nm})\; \tilde{O}_{Fnm}(\mathbf{u}+\sigma_o(\mathbf{x}'_{nm}-\mathbf{x}_{nm})), \tag{6}$$

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$$\text{with} \quad \tilde{O}_{Fnm}(\mathbf{u}) = \mathcal{F}_{2D}\left\{ O(\beta)\ \tilde{F}(\Delta\sigma\beta.(\mathbf{x}'_{nm}-\mathbf{x}_{nm})) \right\}, \tag{7}$$

where $\tilde{F}$ is the 1D-Fourier transform of the "bandwidth pattern". Equation (6) shows a similar relationship as the one obtained for monochromatic light. As in Eq. (5), a spatial frequency conversion appears. Nevertheless, even if this conversion is corrected, no convolution relationship can be recovered: indeed, Eq. (6) shows that the object brightness is modified differently by the function $\tilde{F}$ for each baseline, so that the interferometer observes a different object through each baseline.

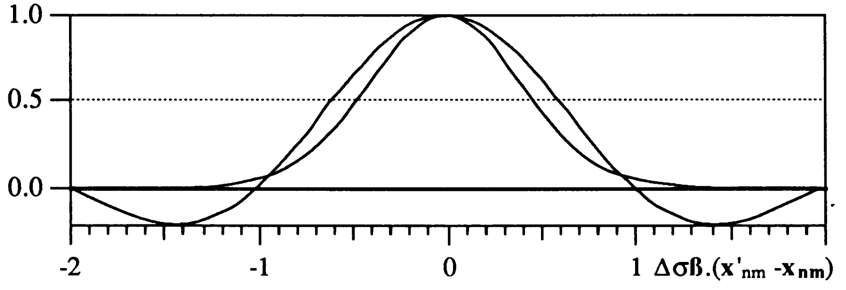

Fig. 1. The FWHM of the function $\tilde{F}(\Delta\sigma\beta.(\mathbf{x}'_{nm}-\mathbf{x}_{nm}))$ does not depend greatly on the shape of the bandwidth F. Its value is 0.94 and 1.2 when F is a gaussian or a rectangular function respectively.

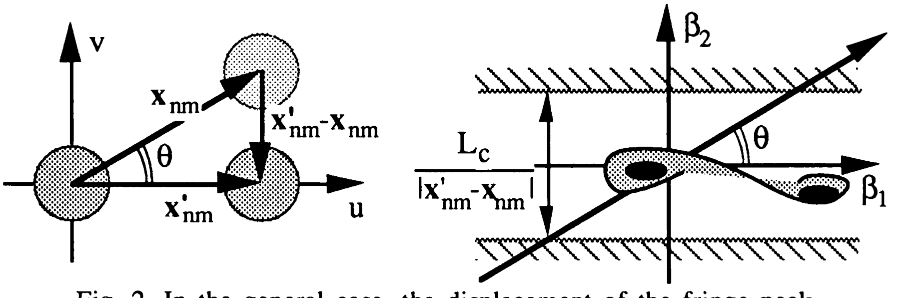

Fig. 2. In the general case, the displacement of the fringe peak $(\mathbf{x}'_{nm}-\mathbf{x}_{nm})$ is not collinear with the entrance baseline vector $\mathbf{x}_{nm}$ (on the left), so the narrowest width of the FOV will not necessarily be perpendicular to the direction of the baseline.

As shown in Fig. 1, the FWHM of $\tilde{F}$ is close to 1 and does not depend greatly on the shape of the bandwidth pattern. Taking this value as a limit, the FOV β_o must meet the following condition :

$$\Delta\sigma\ \beta_o.(\mathbf{x}'_{nm}-\mathbf{x}_{nm}) < 1 \tag{8}$$

The extension of the FOV does not depend on an entrance baseline $|\mathbf{x}_{nm}|$ as in radio-interferometry but on the re-arrangement vector $|\mathbf{x}'_{nm}-\mathbf{x}_{nm}|$. In the case of an FSI $(\mathbf{x}'_{nm}=\mathbf{x}_{nm})$, the FOV is not limited. The extension of the

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FOV also depends on the orientation of this vector, as is shown in Fig.2. The change of the width and the orientation of the FOV window on each baseline shows how complex the effect on the object can be. There is a similar (though simpler) problem in radio-interferometry, however the spectral bandwidths are narrow enough to make the effect negligible. Until such time as a method is proposed for taking into account the effect described here, we must use similar conditions in the optical domain.

3.4. RELATION WITH RADIO-INTERFEROMETRY

Let us point out the differences between this presentation for the optical domain and the usual relationships derived in radio-interferometry. Starting from the general Eq.(6), we isolate a single fringe peak in the (u,v) plane by considering a single term of the second summation, and integrate it:

$$r_{nm} = \int d^2\mathbf{u}\ \frac{1}{A}\ \tilde{S}_{nm}\left(\frac{\mathbf{u}}{\sigma_o} + \mathbf{x}'_{nm}\right)\ \tilde{O}_{Fnm}(\mathbf{u} + \sigma_o(\mathbf{x}'_{nm} - \mathbf{x}_{nm}))$$

$$= \int d^2\mathbf{ß}\ O(\mathbf{ß})\ \tilde{F}(\Delta\sigma \mathbf{ß}.\mathbf{x}_{nm})\ S_{nm}(\mathbf{ß})\ e^{i2\pi\sigma_o \mathbf{ß}.\mathbf{x}_{nm}}\ . \qquad (9)$$

r_{nm} corresponds to the expression for the output signal of a correlator in a radio interferometer [20]. It is the integral of the product of 4 terms: the object brightness, the "bandwidth pattern", the "antenna pattern" and the "fringe pattern". We can see that r_{nm} is independent of $\mathbf{x}'_{nm}$, i.e. of the re-arrangement of the baselines, and that the two basic limitations of the FOV associated with "correlator arrays" are exactly recovered: it is limited by the finite coherence length ("bandwidth pattern") and the finite resolution in the (u,v) plane ("antenna pattern"). It can be seen that, for the FSI, the benefit of the exact OPD compensation of a geometrical optics system is lost.

4. Conclusion

A high resolution in the (u,v) plane is a necessary condition for a wide FOV, whatever the choice of the beam combination method. Furthermore, in the image plane, the limitation from the finite coherence length can be overcome completely with an FSI, which stands as the ultimate solution to reach a wide FOV but implies difficult instrumental constraints. When the condition of "lateral linear mapping" is not met between the entrance and exit pupils (e.g. effect of a periscope), the FSI becomes an MSI and is no more a perfect optical system (no convolution relationship). We have shown that its capabilities as an imaging system are however not destroyed but that some limitations reappear on the FOV: for each baseline, the object brightness is multiplied by a "windowing function" whose width depends on a compromise between the re-arrangement of the baselines and the spectral bandwidth (Eq. 8). The FOV extension may range between the one of a "correlator array" and the unlimited one of a perfect geometrical optics system: according to our definition, the FOV may be considered as wide.

The effects of the "windowing functions" on an object are never taken into account in data processing, so they must be negligible like in radio-interferometry, so condition (8) could be somehow optimistic. To what extent these effects could be corrected must be determined.The effects on the object of the other departures from the ideal case of an FSI also deserves more work.

Studying wide FOV interferometry (as we have defined it) in the optical domain is somehow a projection into the future, but yields a better understanding of the existing instruments.

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