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N-body codes

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Abstract. We review some advances relating to direct N-body codes. In particular, there has been significant progress in dealing with large-N systems containing a few dominant members. The simulation of massive black holes also requires treatment of relativistic effects for strongly bound two-body orbits. Although somewhat costly, the addition of post-Newtonian terms is still straightforward when used in connection with regularization methods. Several versions of multiple regularization are especially well suited to studying black hole problems. We also report on a new stability criterion for the general three-body problem which will provide a robust test in systems where hierarchies are a troublesome feature, as in the case of star cluster simulations with primordial binaries.

Keywords. stellar dynamics, methods: n-body simulations, celestial mechanics

1. Introduction

Many technical challenges arise in the quest for realistic simulations of globular clusters and galactic nuclei. Already special-purpose GRAPE and parallel supercomputers enable the study of systems with up to 10^6 members over significant times. This development has focused attention on problems involving massive objects in the form of black holes, which are of great astrophysical interest. However, until recently, a careful treatment of dominant two-body motions was mostly missing and the effect of relativistic terms rarely included. In this short review we concentrate mainly on these aspects which are vital for further progress. Since compact subsystems also play an important role in general star cluster models, we summarize a new development which gives rise to more reliable stability criteria for hierarchical systems.

2. Large-N simulations

Several large N-body simulations have included massive bodies representing black holes without the introduction of relativistic terms. Agreement with theory was found for the growth of a density cusp around a massive central object and up to 2.5×10^5 particles using GRAPE hardware (Preto *et al.* 2004). This simulation employed chain regularization for the central subsystem. Simulations of three massive objects in systems with $N \leqslant 128\,\mathrm{K}$ have also been carried out with gravitational radiation added (Iwasawa *et al.* 2005). This work employed the standard Hermite scheme with different softening for the Newtonian interactions and produced conditions leading to coalescence for relatively large binary masses and short times, partly aided by large eccentricities. The long-term evolution of rotating systems containing two massive bodies was studied on two GRAPE clusters for models based on direct summation with $N \leqslant 4 \times 10^5$ and relatively small softening (Berczik *et al.* 2006). Here the hardening rate was independent of N, albeit for a modest range in shrinkage of the semi-major axis.

3. Multiple regularizations

Given the large range in length scale required for achieving GR coalescence, it is desirable to explore regularization methods which are able to deal with small separations. This still leaves the serious problem of small time-scales even if only a few particles are involved. Chain regularization has been tried with some success for modest binary shrinkage in combination with the GRAPE-6 (Szell *et al.* 2005). However, the time-transformed leapfrog method (Mikkola & Aarseth 2002) is more accurate for studying significant orbital shrinkage in the massive binary problem. An attractive alternative method has also been developed recently based entirely on time transformations combined with the leapfrog integrator (Mikkola & Merritt 2006). In the so-called algorithmic regularization, a time-symmetric leapfrog scheme can be constructed even when the forces depend on velocities, as in the post-Newtonian case. Provisional tests of the stand-alone code are very promising and indicate that a full-scale simulation code would be effective for studying extreme configurations, albeit at some cost.

The search for powerful methods has lead to an implementation of the so-called wheel-spoke regularization which was developed to deal with one massive object surrounded by strongly bound particles (Zare 1974). Here the basic idea is to regularize all the members in a compact subsystem with respect to the dominant body and include a small softening for the other internal interactions. Recent tests show that substantial binary shrinkage can be treated, including coalescence due to post-Newtonian terms (Aarseth 2006).

4. Hierarchical stability

Following Wisdom (1980), a new stability criterion for the general three-body problem has been constructed using the concept of resonance overlap (Mardling 2006). Resonances between the inner and outer orbits of a hierarchical triple can be identified via a Fourier expansion of the disturbing function. A given configuration is defined to be resonant if at least one Fourier argument, say ϕ, *librates*, that is, $\phi_{\rm min} < \phi < \phi_{\rm max}$, where $|\phi_{\rm max} - \phi_{\rm min}| < 2\pi$. If two or more Fourier arguments librate with different libration frequencies, the system is said to be in a state of resonance overlap and hence chaotic. Almost all chaotic triples are unstable to the escape of one of the bodies: the resonance overlap stability criterion provides a powerful way to identify unstable configurations.

The new stability criterion employs a simple analytic formula expressed in terms of the orbital parameters of the system to determine its resonant (or otherwise) state. A single number is calculated for several 'neighbouring' Fourier components; if two or more of these numbers are negative the system is identified as unstable to the escape of one of the bodies.

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