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Numerical modelling of the paleotidal evolution of the Earth-Moon System

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Abstract. The results of a numerical simulation of the tidal evolution of the Earth-Moon system during the Phanerozoic epoch (the last 600 million years) are given. In most of the researches devoted to the solution of the problem the authors simplified and parametrized very complicated tidal phenomena to a primitive integral hump on the Earth's surface. As distinct from these the numerical model of the ocean tides in its most complete form is the core of the present study: the problem is solved for a viscous liquid in a paleoocean with variable outlines and depth allocations stimulated by the drift of the lithospheric platforms; the global interaction between the ocean and earth tides and the fluctuations of the gravitational field of the planet caused by them are taken into account. The astronomical component of the model is simplified. It is assumed that the Earth-Moon system is isolated, the Moon's orbit circular and the moment of inertia of the Earth constant during the Phanerozoic epoch. It is shown that the evolution of the Earth-Moon system during the Phanerozoic was nonuniform and that the primary role in this process belongs to the geodynamic factor.

Keywords. Paleotides, Earth-Moon system, evolution, energy dissipation, geodynamics

1. Introduction

Over a hundred years ago G. Darwin made a hypothesis about formation of an Earth-Moon system and constructed, speaking in modern terms, a mathematical model of its tidal evolution (Darwin G.H. (1978), Ball P.S. (1900)). According to Darwin, the Moon was formed from substance of the Earth which has been thrown out into an orbit by the resonant forces. The satellite appearing in close orbit induced high tides in the planet's body. Because the Moon revolved about the Earth in forward direction and the Earth's angular rate of rotation about its axis was greater than the Moon's angular orbital velocity, the tidal hump due to action of the friction forces "ran out" from under the Moon forward. Mutual attraction of hump and Moon imparted to the latter, acceleration in the orbit and slowed-down the Earth's diurnal rotation. The Moon gradually moved away from the Earth until a synchronization of the planet's rotation and satellite's revolution took place. Further on, however, the tidal influence of the Sun on the Earth slowed down its rate of rotation to a still greater extent, the tidal hump appeared to be behind the Moon and the process of evolution of the Moon's orbit continued with opposite sign. Finally, the Moon will approach the Earth so closely that the forces of attraction will disintegrate it. This, in general terms, is the picture of evolution of the Earth-Moon system calculated by Darwin.

The subsequent century did not bring fundamental changes the in theory of tidal evolution for this system, but it proved the failure of Darwin's hypothesis of the formation of our satellite. Two theories appeared: capture of the Moon by the Earth (Alfven X. & Arrhenius G. (1976), MacDonald G.J.F. (1975)) and accumulation of the Moon in the near-Earth swarm. The first theory finds it difficult to explain what happened to the energy liberated due to capture in the near-Earth orbit of a body that massive and why on the Earth no traces of this extraordinary event have been found anywhere. The theory of accumulation experiences difficulties in an attempt to correlate the length of the evolution with the Earth's age exceeding 4.5 bln years (Russkol E.A. (1975)).

The length of the evolution is defined by the magnitude of tidal friction, in other words, how high the rate of tidal energy dissipation is: the higher the dissipation, the more intensive the evolutionary process. The tidal friction can be characterized by the so-called effective lag angle, δ, indicating how far the tidal hump is carried away from under the Moon and by what angle the Moon lags compared with the tide as the meridian is passed. At the present time, the magnitude of is estimated to be $2.5 - 3^o$. The data have been obtained both in a numerical simulation of the tides in the World Ocean (Gordeev P.G., Kagan B.A. & Polyakov E.V. (1976)) and on the basis of observation data for the solar eclipses in the ancient world (Morrison L.V. (1978)). Using the present-day magnitude of δ in calculations of the tidal evolution yields its length equal to 1.5-1.75 bln years (Russkol E.A. (1975)), which is a serious argument in favor of the capture hypothesis and constitutes the main difficulty of the accumulation hypothesis. Attempts to overcome the latter are usually associated with the assumption that in the past the lag angle was smaller than the present value although justification of this assumption appears to be insufficiently convincing.

2. Construction of equations

In the present paper, in contrast with previous investigations of the tidal evolution for the Earth-Moon system, the core of the problem is a mathematical model of the tides in a most complete presentation, to which we connected a simple evolution model. Thus, in order to calculate the evolution rather than use an a prioriassigned lag angle δ, a complex tidal problem is solved and. In the tidal problem itself some quantities usually taken as constants, namely the angular rate of rotation of the Earth and the frequency and amplitude of the tide-generating force, become variable and are computed in the evolution model. A mathematical description of the problem formulation can be found in Gordeev P.G., Kagan B.A. & Polyakov E.V. (1976), Polyakov E.V. (1986).

The model in question describes an Earth-Moon system isolated in space. The Moon's orbit is assumed to be circular coplanar with the plane of the Earth's equator. The latter plane is considered as an elastic-viscous body covered by a gravitational fluid shell, the ocean. The continents jutting in the ocean are in permanent movement, so that the ocean configuration changes continuously, the volume of water in the ocean remaining constant. The Moon, a point mass, induces tides in the terrestrial crust and ocean where allowance is made for bottom friction and horizontal turbulent exchange. Moving above the bottom the tidal wave causes its deformations, i.e. deflections of the crust under the crest and a rise under the wave trough. As a result of changes in the planet's form under the action of the tide-generating forces and the effects of loading and self-attraction and also redistribution of the water masses the planet's gravitational field experiences disturbances which, in turn, take part in formation of the tides in the ocean and Earth's body. In this way the effect of self-attraction of the tides is manifested.

The calculation of the Earth-Moon system evolution has been performed for the Phanerozoic epoch (to 600 mln years into the past), a short time interval compared with the system age. This limitation is connected with the fact that as of today the reconstruction of the position of the continents had been carried out only for the period specified (Zonnenshain L.P. & Gorodnitsky A.M. (1979)).

An integro-differential problem has been solved. With allowance for the assumption on the harmonic character of the oscillations, the linearized equations of tidal dynamics are written in the form (Gordeev P.G., Kagan B.A. & Polyakov E.V. (1976))

$$(r - i\sigma)\overline{\mathbf{w}} + A\overline{\mathbf{w}} - k_l\Delta\overline{\mathbf{w}} + gH\nabla\xi = gH\nabla(\gamma_L\bar{\xi}^{+} + \bar{\xi}^{\oplus}), \tag{2.1}$$

$$-i\sigma\bar{\xi} + div\overline{\mathbf{w}} = 0. \tag{2.2}$$

Here, $\overline{\mathbf{w}}$ is the integral transport vector, g is the gravity acceleration, H is the depth, $\bar{\xi}$ is the height of the ocean tide with respect to the bottom, $\bar{\xi}^{+}$ is the static tide in the ocean for a rigid Earth, γ_L is the Love factor for an elastic Earth, r and k_l are the coefficients of bottom friction and turbulent exchange, σ is the frequency of the driving force, A is the Coriolis matrix,

$$A = \begin{pmatrix} 0 & -\lambda \\ \lambda & 0 \end{pmatrix},$$

where $\lambda = 2\Omega\cos\theta$; Ω is the angular rate of rotation of the Earth. The origin of the coordinates is situated in the North Pole, θ is the colatitude, φ is the longitude. The integral term $\bar{\xi}^{\oplus}$ describes the deformations of the Earth's crust and disturbances of the gravitational potential as a response to the ocean level oscillations

$$\bar{\xi}^{\oplus} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\bar{\xi}(\theta',\varphi')\sum_n \gamma_n'\alpha_n \sum_n N_{nq}^{-1}P_n^q(\cos\theta)P_n^q(\cos\theta')\begin{pmatrix}\cos q\varphi\cos q\varphi' \\ \sin q\varphi\sin q\varphi'\end{pmatrix}\sin\theta' d\theta' d\varphi' \tag{2.3}$$

where γ_n' and α_n are coefficients for calculating induced disturbances of the static tide, $P_n^q(\cos\theta)$ is the Legendre connected function of the n order and to the q power; and N_{nq} is a normalizing factor. Set (1-3) is supplemented by the nonslip condition on the contour β of the region in question

$$\overline{\mathbf{w}}|_{\beta} = 0. \tag{2.4}$$

For epochs of the geological past, in addition, it is necessary to define the parameters ξ^{+} and σ of the driving force. For the M_2 wave the following relations are valid (Marchuk G.I. & Kagan B.A. (1983)):

$$\xi^{+} = (3Cm/4M)a^4R^{-3}\sin^2\theta\cos(\sigma t - 2\varphi), \tag{2.5}$$

$$\sigma = 2(\Omega - \omega). \tag{2.6}$$

Here, C is the amplitude coefficient † of the M_2 harmonic; m and M are the masses of the Moon and Earth, respectively, a is the mean radius of the Earth, R is the radius of the Moon's orbit, and ω is the Moon's angular orbital velocity. The values of R, Ω and

† According to Marchuk G.I. & Kagan B.A. (1983) $C = (1 - 5e^2/2)\cos^4(i/2)$, where e is the eccentricity and i is the inclination of the Moon's orbit. Although the set of evolution equations has been written for the case of the satellite's circular orbit ($e = 0$) lying in the plane of the planet's equator ($i = 0$), the magnitude of C is assumed to be equal to the present magnitude $C_0 = 0,9081$ in order to obtain results corresponding precisely to the M_2 wave and subsequently compare them with those of further investigations.

ω are defined using the equation for impulse moment conservation in the Earth-Moon system

$$I\frac{d\Omega}{dT} + \frac{Mm}{M+m}\frac{d}{dT}(R^2\omega) = 0, \tag{2.7}$$

Kepler's third law is

$$R^3\omega^2 = G(M+m) \tag{2.8}$$

and the expression for moment L of the tidal force zonal component [8]

$$L = I\frac{d\Omega}{dT} = 6.4\pi\rho_0\gamma_L\frac{3}{4}CGma^4R^{-3}D_{22}^{-}\sin\varepsilon_{22}^{-}, \tag{2.9}$$

where T is the time in the geological scale directed into the past, I is the Earth's moment of inertia equal to $0,3315Ma^2$ (Marchuk G.I. & Kagan B.A. (1983)), G is the gravitational constant, ρ_0 is the mean density of water, D_{22}^{-} and ε_{22}^{-} are parameters obtained from the coefficients for expansion of the amplitude of level $\bar{\xi}$ into a series using the spherical function in a system of coordinates with westerly direction of the longitudinal axis φ, therefore here, in contrast to Marchuk G.I. & Kagan B.A. (1983) in expression (11), use is made of parameters D_{22}^{-} and ε_{22}^{-}, in place of D_{22}^{+} and ε_{22}^{+}.

Using (8) and (9), Eq. (7) takes the form

$$\frac{d}{dT}R^{1/2} = -\frac{L}{Mm}\left(\frac{M+m}{G}\right)^{1/2}. \tag{2.10}$$

Set (1)–(6), (8)–(10) is complemented with initial conditions corresponding to the present epoch: $\Omega_0 = 7.2921 \cdot 10^{-5}s^{-1}$, $\omega_0 = 2.6617 \cdot 10^{-6}s^{-1}$. The proposed formulation of the tide problem makes it possible to find the solution based only on the masses of the Moon and the Earth, the mean radius of the latter, and the depth of the ocean bottom relief: $m = 7.348 \cdot 10^{22}$ kg, $M = 5.976 \cdot 10^{24}$ kg, $a = 6.371 \cdot 10^6$ m, $H = H(\theta, \varphi)$.

3. Results

It should be noted that the tidal model in question has been worked out for solving an independent problem on interaction between the ocean and Earth tides, precalculation of the tides in an ocean with an elastic bottom and investigation of the tidal variations in the force of gravity. Having been used for its direct purpose it has recommended itself quite well. The results of calculations of the tidal harmonics M_2, S_2, K_1 and O_1 for the present–day epoch (Fig. 1) are in satisfactory agreement with data by other authors (Accad Y. & Pekeris C.L. (1978), Zahel W. (1978)) and observations (Schwiderski E.W. (1978)). This makes it possible to acheive some credibility for the solutions of the tidal problem also for past epochs (Fig. 2–4).

A note on the angle δ: clearly, the tides in the ocean have nothing in common with the hump and are distinguished by a complex spatial structure. Adding together the moments of the attraction forces which act between the Moon and each of the particles in the ocean we obtain integral moment L of the tide-generating forces. One can imagine a tidal hump producing a moment equal to L in magnitude. The crest of the hump producing moment L must be spaced from the point under the Moon by the angle δ.

Application of the model for calculating the evolution of the Earth–Moon system has led to unexpected results which may be used as a reason for revision of long-standing concepts about the character of secular changes in the system parameters and eliminate

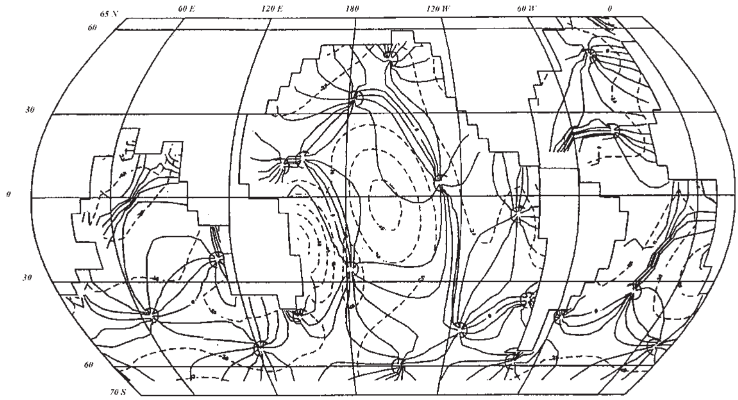

Figure 1. Tidal map. M_2 wave. Present epoch.

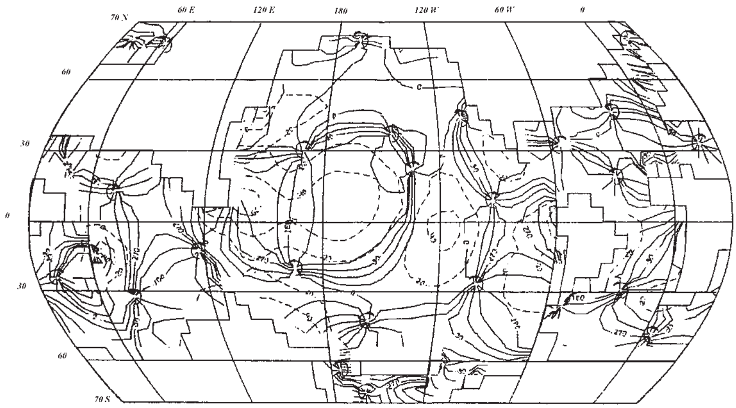

Figure 2. Tidal map. M_2 wave. 70 Myr ago.

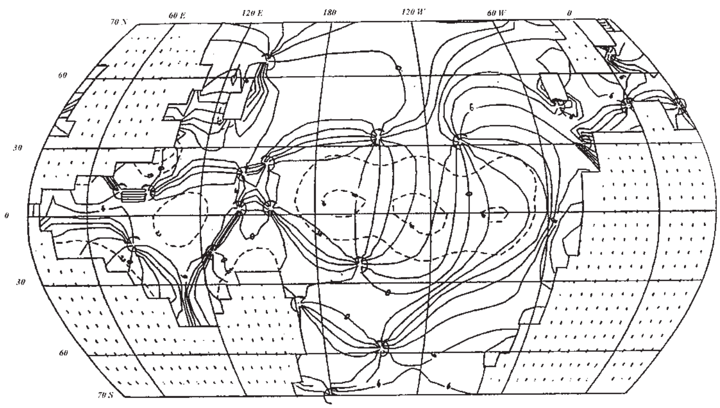

Figure 3. Tidal map. M_2 wave. 240 Myr ago.

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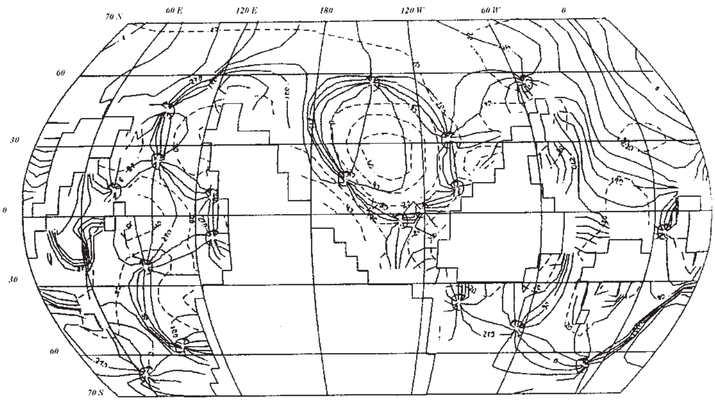

Figure 4. Tidal map. M_2 wave. 570 Myr ago.

Main characteristics of tidal evolution of the Earth–Moon system. M_2 wave

T, 10^6 yr	Ω, 10^{-5} s^{-1}	ω, 10^{-6} $s{-}1$	σ, 10^{-4} s^{-1}	$A+$, cm	L, 10^{16} $n \cdot m$	δ, deg	Q	$\dot{R}$, cm/yr	$\dot{\omega}$, "/cent^2	$\dot{\tau}$, s/cent
0	7.292	2.662	1.405	24.23	3.42	5.18	11.0	2.91	19.7	1.59
10	7.306	2.665	1.408	24.28	3.14	4.75	12.0	2.68	18.4	1.46
50	7.349	2.675	1.416	24.46	2.17	3.27	17.5	1.83	12.4	0.98
100	7.382	2.682	1.423	24.60	0.94	1.43	40.0	0.81	5.6	0.43
200	7.407	2.688	1.428	24.71	0.40	0.61	93.9	0.34	2.3	0.18
300	7.423	2.691	1.431	24.77	0.53	0.81	70.7	0.46	3.2	0.24
350	7.439	2.695	1.434	24.85	0.93	1.41	40.6	0.80	5.5	0.42
400	7.452	2.698	1.436	24.89	0.66	1.01	56.7	0.57	3.9	0.30
450	7.476	2.703	1.441	24.99	1.80	2.70	21.2	1.54	10.7	0.80
500	7.527	2.715	1.451	25.21	2.95	4.48	12.8	2.53	17.7	1.31
570	7.617	2.736	1.469	25.60	2.54	3.82	15.0	2.19	15.5	1.11

the contradiction, cited above, between the age of the system and the length of its tidal evolution.

It appears that moment L of the tide-generating forces and the lag angle δ (see table) changed in the Phanerozoic in a complicated manner. Thus, 500 mln years ago they had maximum magnitudes ($2.95 \cdot 10^{16}$ nm and 2.24^o, respectively) close to present magnitude ($3.42 \cdot 10^{16}$ nm and 2.59^o) and during periods from the Devonian to the Cretaceous (400 – 100 mln year) their magnitudes were a factor of 3 – 8 below the present values. The table, apart from L and δ, shows the mechanical quality Q of the Earth's ocean-crust oscillatory, the system rate of separation $\dot{R}$ and secular acceleration of the Moon's $\dot{\omega}$, and the lengthening of the terrestrial day $\dot{\tau}$. The magnitudes of the above parameters obtained from an analysis of movement of the satellites and calculated on a model for the present epoch are in satisfactory agreement.

Figure 5 shows graphs for the number of days in one year N and for the length of the terrestrial day τ in the past. It is not difficult to see that these parameters changed unevenly in the Phanerozoic. Their smallest changes correspond to the period with low rate of dissipation $-\dot{E}$. The curve of relative dissipation $-\dot{E}/\dot{E}_o$, where $-\dot{E}_o$ is the

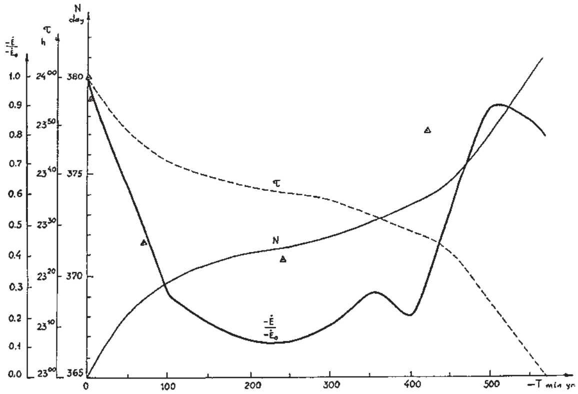

Figure 5. Changes during the Phanerozoic in the dissipation $-\dot{E}$ of the tidal energy normalized by its present-day (calculated) magnitude $-\dot{E}_o = 2.41 \cdot 10^{12} W$, length τ of the mean solar days and number N of days in the year

present-day rate of dissipation, is nearly coincident in shape with curves for L and δ (not given in Fig. 5).

4. Analysis

This complex behavior of the dissipation (or L or δ) in the past cannot be explained only by a change in the Moon's orbit. It remains to assume that the character of the evolution of the Earth–Moon system is also dependent on the ocean configuration varying as a result of the movement of the lithospheric plates.

Figure 6 shows the results of checking this assumption. In order to construct the figure, 81 numerical experiments have been performed, each representing a solution of the problem about tides in the paleo-ocean (and subsequent calculation of tidal energy dissipation) for all possible combinations of nine ocean configurations (one, present-day, plus eight paleoreconstructions borrowed from Zonnenshain L.P. & Gorodnitsky A.M. (1979)) and nine different radii of the Moon's orbit. It can be clearly seen that the movement of the continents has a stronger influence on the dissipation than the increase of the Moon's orbital radius.

In Fig. 6 are compared two curves characterizing the separation of the Moon from Earth. Curve 1 has been calculated in a traditional manner on the assumption of the constant lag angle; curve 2 has been obtained as the problem in question has been solved. Calculation of the evolution based on the tidal model in which the continental drift has been allowed for leads to a more than two-fold reduction of the Moon's separation. This strong influence of the ocean configuration on the tidal energy dissipation should be sought for in resonant nature of the excitation of the ocean tides. If we assume that the dissipation is proportional to the tidal energy, then based on Fig. 6 one may come to some conclusion about evolution of the ocean resonant properties. Namely: at the present time and 500 mln years ago the ocean configuration contributed to "adjustment" of the tides to frequency of the disturbing force and the amplitude of the tidal waves increased, resulting in growth of their gravitational interaction with the Moon and intensification of the evolutionary process. Vice versa, mismatching of the frequencies in a range of 400-150 mln years ago led to slowing down of the evolution.

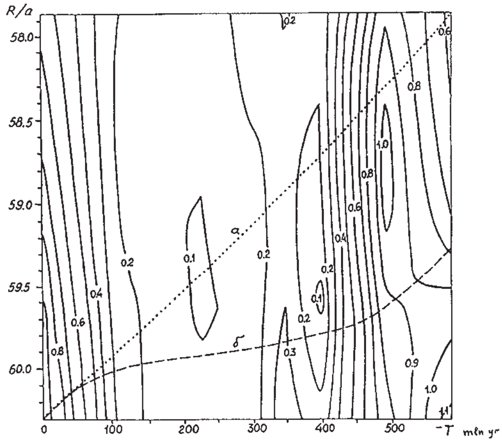

Figure 6. Dependence of tidal energy dissipation $(-\dot{E}/\dot{E}_o)$ on the evolution of the Earth-Moon system (along the ordinate axis) and the drift of the lithospheric plates (along the abscissa axis). Superimposed two plots of the separation of Moon from Earth as a function of the time ine past: 1 - with constant lag angle δ; 2 - based on a calculation of the paleotides

5. Conclusions

The results presented indicate that reaching a consistent and noncontradictory solution to the problem of Earth-Moon system evolution is only possible in the case of joint consideration of the astronomical, geophysical and geodynamic influencing factors, the latter two, as it has become clear, having an important role.

References

Accad, Y. & Pekeris, C.L. 1978, *Phil. Trans. Roy. Soc.* A290, 235

Alfven, X. & Arrhenius, G. 1976, *NASA Tech.*, p. 512

Ball, P.S. 1900, *Mathesis*, Odessa, p. 104, (in Russian)

Darwin, G.H. 1978, *Phil. Roy. Soc.* A170, 447

Goldreich, P. 1975, *Mir Publ.*, Moscow, 97, (in Russian)

Gordeev, P.G., Kagan, B.A. & Polyakov 1976, *Dokl. Akad. Nauk SSSR.* 228(4), 817, (in Russian)

Hansen & Kirk S. 1982, *Rev. Geophys. Space Phys.* 20(3), 457

Kagan, B.A. & Maslova, N.B. 1994, *Earth, Moon, Planets* 66,173

MacDonald, G.J.F. 1975, *Mir Publ.* Moscow, 9, (in Russian)

Marchuk, G.I. & Kagan, B.A. 1983, *Gidrometeoizdat Publ.* Leningrad, p. 360,(in Russian)

Morrison, L.V. 1978, in: P. Brosche & J. Sundermann (eds.) *Tidal Friction and the Earth Rotation*, Springer-Verlag, p. 35

Polyakov, E.V. 1986, *Izv. Akad. Nauk SSSR. Oceanic and Atmospheric Physics.* 22(5), 505, (in Russian)

Ray, R.D., Bills, B.G. & Chao, B.F. 1999, *J. Geophys. Res. - Solid Earth* 104(B8), 17653

Russkol, E.A. 1975, *Nauka Publ*, Moscow, p. 189, (in Russian)

Schwiderski, E.W. 1978, Naval Surface Weapons Center. - Dahlgren Lab. - Dahlgren. p. 88

Zahel, W. 1978, in: P.Brosche & J.Sunderman (eds.) *Tidal Friction and the Earth's Rotation*, Springer-Verlag, p. 132.

Zonnenshain, L.P. & Gorodnitsky, A.M. 1979, *Nauka Publ.*, Moscow, 327, (in Russian)