ORIGINAL RESEARCH PAPER

Epidemic modelling and actuarial applications for pandemic insurance: a case study of Victoria, Australia

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Abstract

With the recent outbreak of COVID-19, evaluating the epidemic risk appears to be a pressing issue of global concern and one of the major challenges recently. In the fight against pandemics, the ability to understand, model, and forecast the transmission dynamics of infectious diseases plays a crucial role. This paper provides an overview of foundational compartment models and introduces the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model to study the dynamics of COVID-19. A meticulous data calibration procedure is employed to study the evolution trend of an actual pandemic using real-world data from Victoria, Australia. Additionally, the paper discusses innovative applications of epidemic models to the insurance industry, which are currently under investigation. Through the use of the newly developed analytically tractable model, insurance companies are able to determine fair premium levels during an outbreak. Moreover, the paper provides practical guidance for insurance companies by examining the variation in reserve levels over time.

Keywords: COVID-19; epidemiological models; pandemic insurance

1. Introduction

The recent outbreak of the Coronavirus pandemic has rendered scientific investigation of infectious diseases a globally pressing issue. From a historical perspective, infectious diseases have significant impacts on the life expectancy of humans. The records analyzed by Acemoglu *et al.* (2021) indicate that pandemic outbreaks caused substantial increases in mortality rates. For this reason, we must remain vigilant during the pandemic and develop robust statistical methods based on the latest data in conjunction with experts' insights.

Based on the research conducted by Levins *et al.* (1994), not only does the new infectious virus emerge when the disease agents adapt to the environment but also the existing contagious ailments can re-appear under such conditions.

Compartment models are the critical modeling technique in epidemiological modeling. An early version of the compartment models was proposed by Kermack and Mckendrick (1927) to evaluate the performance of contagious diseases such as the spread and duration of the diseases as well as the reproductive number of the disease agents. Afterward, Kocic and Ladas (1993) released the continuous time path constraint, and Allen (1994) extended the model into a discrete-time framework by incorporating a range of linear approximation formulas. The compartment model equations have also been reformulated in many ways. According to Kemper (1978), the compartment model could be extended by including an additional carrier state. Moreover, several

studies have been conducted using compartment models to study the vaccination effect during pandemics, for example, Cohen *et al.* (2003), Zhu *et al.* (2019), and Liu *et al.* (2021).

Compartmental models are typically developed with predefined parameters and are used to analyze the evolution of populations over time in accordance with traditional model formulations. As a result, there is a high degree of uncertainty associated with a variety of scenarios. As a way of mitigating these uncertainties, we can constrain the epidemic models based on the available data records and ensure the parameter values are selected according to the calibration results in order to reduce these uncertainties. It has been demonstrated that this process, when compared with simply using prior-determined parameters as inputs to the model, can have a significant positive impact on the fitting results and on the accuracy of the prediction. The calibration process is comprised of adjusting the parameters of the model to bring it as close as possible to the actual track, thereby enhancing the accuracy of the model. For instance, Soper (1929), Daley and Gani (2001), and Jia and Tsui (2005) explored a range of actual epidemics that have occurred throughout human history using the compartment models.

Building on a large body of literature on mathematical epidemiology, we study the evolution of populations within each compartment over time. As far as epidemiology is concerned, the basic reference model is the Susceptible-Infected-Recovered (SIR) model proposed by Kermack and Mckendrick (1927), which divides the entire population into three groups. With this basic structure in place, Bastos and Cajueiro (2020) add a Death state to disentangle the proportion of people who die from the contagious disease. The addition of the Dead compartment meets the insurance industry's requirement to work with death benefits. In light of recent vaccination campaigns, vaccination appears to play a crucial role in preventing the spread of disease. Regarding the recent insurance application, a widely accepted model is suggested by Ye *et al.* (2022), known as the Susceptible-Vaccinated-Exposed-Infected-Recovered-Dead (SVEIRD) model. Nevertheless, the fundamental formulation of this model implies that it cannot separate the number of people suffering from severe symptoms from those with mild symptoms. In this case, we use it as a baseline model and calibrate the parameters to serve as an initial reference. We develop an improved model, called Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead (SVEI3RD), that further separates infected individuals into three subgroups while keeping the apparent vaccination status. We present an analysis of the Coronavirus pandemic behavior in Victoria using both models in this paper and make a comparison of the results. The updated model produces a more comprehensive and effective approach by subdividing the infectious state, enabling us to obtain parameter values closer to those found in industry reports. Therefore, we consider this model to be more suitable for real-life applications. Taking these circumstances into account, this paper discusses the calibration of parameters and analyzes the statistical results of the first and second doses, emphasizing the effectiveness of vaccination during a pandemic. The purpose of this procedure is to take factual data and apply theoretical models to investigate the dynamics of virus spread over time.

Aside from formulating and testing the mathematical model and examining Victoria's case study, we also apply epidemiological models to the insurance modeling. As the valuation of pandemic insurance discussed by Gründl *et al.* (2021) is based on the number of infected and susceptible populations at a given time, it requires a more comprehensive calculation. This scenario utilizes the epidemiological model as a building block for actuarial measures, which provides a more robust estimation while accommodating the rapidly changing environment during the pandemic. With a more detailed description of population movements in different compartments, this paper uses the SVEI3RD model with calibrated parameter values to study the relevant insurance applications. The insurance company collects premiums from the susceptible population as part of a basic pandemic health insurance plan. In case of infection, it provides predetermined benefits based on the severity of the symptoms, as well as death benefits upon the death of the insured.

On top of the basic design, modern travel insurance also provides additional financial quarantine coverage for people exposed to the virus. As a result, the firm can calculate the zero-profit premium level by equating the premium received and the expected benefit payments. Despite this, the insurance company would like to charge a higher premium in order to ensure a high level of security throughout the coverage. This paper further explores different premium levels under various contract durations, while taking into account the importance of a prudent analysis of reserves to a company's sustainable operation.

Following the integration of several epidemiological models, our primary contribution is to implement them to innovate, value, and reserve. Given the increased uncertainties during the outbreak, it becomes imperative for the insurance industry, whose primary purpose is to provide appropriate provisions to ensure public risks, quantify the exposures, and devise pandemic-related insurance contracts. To our knowledge, the proposed work is the first attempt to establish a scientific model informing insurance companies of understanding the impacts of varying model parameters by using real state-level data sets. Referring to the first half of the paper, which provides insight into understanding the effectiveness of vaccinations during different periods of the pandemic, we highlight the importance of taking into account the external environments of specific periods for the valuation of pandemic insurance products. Since there are different sources of uncertainties during the pandemic, our reserve analysis asserts that an inadequate premium amount may lead to insufficient reserves for insurance companies during the coverage period, which merits special attention for long-term insurance contracts. In contrast, short-term contracts in areas that have reasonable medical facilities and regulations would allow insurance companies to perform an essential role in providing people with security and earn inflated profit figures.

This paper is organized as follows. Section 2 introduces the fundamental compartment models developed over time to model pandemics, including the SIR model and its extensions. One of the defining characteristics of this range of models is that they are all described by a system of Ordinary Differential Equations (ODEs). Using the actual data set, Section 3 presents a real-life case study of the spread of the Coronavirus pandemic in Victoria, Australia. As a basis, this section uses the well-accepted SVEIRD model in relation to pandemic insurance and discusses its limitations, particularly the inability to separate individuals with severe symptoms. Taking this shortcoming into consideration, we propose an improved model called the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead (SVEI3RD). It is intended to evaluate the effectiveness of Dose 1 and Dose 2 vaccinations in preventing the transmission of Coronavirus. Both models are fitted for the same time intervals during the pandemic to determine the tendency of the parameters and investigate how Coronavirus spreads. Based on the calibrated results, the extended SVEI3RD model provides a set of parameters that are considered to be more realistic and suggests that this model is more appropriate in real-life applications.

A discussion of pandemic insurance is provided in Section 4 as well as guidelines to follow when developing pandemic insurance products in reality. In this section, standard actuarial practices, namely premium pricing and reserve analysis, are discussed concerning two types of insurance arrangements. As a result of the necessity for insurance companies to provide various benefits to infected individuals depending on the level of seriousness of their symptoms, we pay specific attention to the SVEI3RD model in this section. We analyze the differences between health and travel insurance, as well as the changes in reserve levels. We do this by comparing the differences between various benefit designs while taking into account the changing needs of customers during different pandemic periods. As a further element of our analysis, we also use the calibrated parameter results and forecast the model for extended periods of time. This is a tool to evaluate how the actual and predicted reserve changes over different periods of time with multiple loadings. Additionally, we discuss the real-life implementation procedures for each insurance product

to ensure its practicality. Finally, Section 5 concludes along with some possible extensions for the future.

2. Model formulation

An overview of compartment models is provided in this section, along with an introduction to the formulation of the extended SVEI3RD model, which will be used in our subsequent case study.

We begin this section by analyzing the standard SIR model. Subsequently, we discuss the SVEIRD model, which has gained broad acceptance in the insurance industry, and point out some of its limitations, including the inability to separate patients with severe symptoms. On the basis of these foundations, we propose a more comprehensive SVEI3RD model that is considered an approach that could be more suitable for analyzing population dynamics during a pandemic. Our proposal is to use the SVEI3RD model to obtain a better understanding of the current pandemic situation and its implications for insurance risk assessment.

2.1 Formulation of the SIR model

Fundamentally, the SIR model has three compartments, denoted as S for susceptible, I for infectious, and R for recovered. Each variable $S(t)$, $I(t)$, and $R(t)$ illustrates the number of individuals within each compartment at a particular point in time t. Considering that this model is implemented over a short interval of time, it is based on the assumption that the overall population of the design remains unchanged over the computation period, which implies that $S(t) + I(t) + R(t) = N$ for every t within the time interval. Additionally, in this primary model setup, no newly born babies enter the population, nor do older people leave the population due to natural death as it pertains to the system of equations. Furthermore, the model's structure assumes that an infected individual will be granted temporary immunity within a short period after recovering from the contagious disease. This causes the Recovered state to be an absorbing state in the short-term epidemic model configuration.

As a complete specification of the model, we need to derive a system of ordinary differential equations to determine transition rates between separate compartments as:

$$\begin{cases} \dfrac{dS}{dt} = -\dfrac{\beta SI}{N}, \\[2mm] \dfrac{dI}{dt} = \dfrac{\beta SI}{N} - \delta I, \\[2mm] \dfrac{dR}{dt} = \delta I. \end{cases} \tag{1}$$

In this introductory SIR model, the crucial parameter β pertains to the average number of contacts per person per time. This results in $\frac{\beta I}{N}$ being regarded as the expected infected number per unit time of one susceptible, and $\frac{\beta IS}{N}$ being the expected number of new cases per unit time among S susceptible individuals. Furthermore, movements from the Infected state into the Recovered state are influenced by the parameter δ, which represents the probability of recovery over time for the individual. In addition, Hethcote *et al.* (1981) showed that under the hypothesis that waiting times are identical and independently distributed in each compartment and follow an exponential distribution, the average waiting time for an individual within the Infected state is $\frac{1}{\delta}$, and this is generally defined as the infectious period.
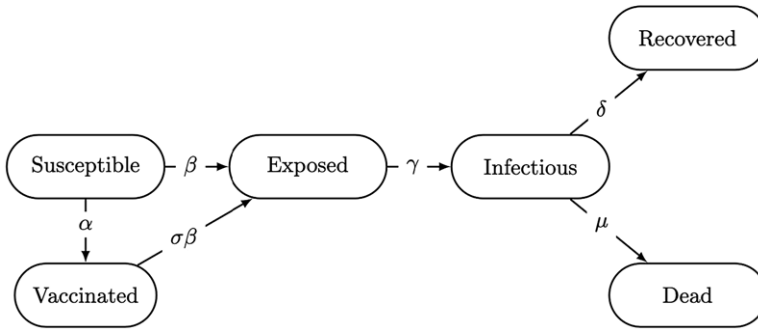
Figure 1. Transfer diagram for the Susceptible-Vaccinated-Exposed-Infected-Recovered-Dead model with the Susceptible class, the Vaccinated class, the Exposed class, the Infectious class, the Recovered class, and the Dead class.

Another key concept in compartment modeling is the basic reproductive number R_0. It is demonstrated that if the reproductive number, R_0, falls below one, the number of infected people decreases over time, and all solution paths approach steady states. Ultimately, disease-free equilibrium can be achieved with no individual infected by the disease.

2.2 Formulation of the SVEIRD model

As a consequence of the current Coronavirus pandemic, Bastos and Cajueiro (2020) extended the fundamental SIR model with an extra Death state to disentangle the proportion of people who die from the contagious disease. In this configuration, the Recovered state is divided into two sub-compartments: the Revised Recovered state and the Death state. An extra parameter μ is added to the system, which represents the death rate of the infected individuals within the system. In addition, the Death state can only be triggered once in this model because it is an absorbing state. A person who enters the state of Death is expected to remain in that state without being able to enter any other state. During this scenario, the individual remains in the Death state throughout his life, and the system's population does not change. Under this circumstance, Fernandez-Villaverde and Jones (2022) showed that there are only two ways for an infected individual to exit the Infected state, either by recovery or due to death.

There is another important model development along with the design of a Dead state, which corresponds to the inclusion of a Vaccinated state and an Exposed state, as proposed by Ye *et al.* (2022). Regarding the inclusion of the Exposed state, this configuration contributes to the circumstance in which people are infected, but do not suffer from the symptoms of the disease. In this case, they are classified as carriers or asymptomatic infected individuals based on the fact that they continue to carry the virus during this state. This means that there is a latent exposure period of E for an individual who is infected with a disease but has not yet manifested symptoms. Following this, the extra parameter γ indicates the incubation rate for individuals exposed to the disease to become infected. Moreover, there are two additional parameters related to vaccination, namely σ, which represents the vaccination inefficiency ratio, and α, which reflects the injection rate for vaccinations. In terms of the vaccination inefficiency rate, for instance, if $\sigma = 0.05$, that means that 95% of the contacts can be reduced with an effective vaccination strategy. In this model background, the contact rate β becomes $\sigma\beta$ for those persons who meet the vaccination requirements. In other words, this means that only 5% of the initial contacts are retained, and the remaining 95% vanish as a result of vaccination.

Taken all improvements into consideration, the extensive model system shown in Figure 1 accounts for both the death and vaccination effects among the whole population and the new model configuration becomes:

$$
\begin{cases}
\dfrac{dS}{dt} = -\dfrac{\beta S I}{N} - \alpha S, \\[2mm]
\dfrac{dV}{dt} = \alpha S - \dfrac{\sigma \beta V I}{N}, \\[2mm]
\dfrac{dE}{dt} = \dfrac{\beta S I}{N} - \gamma E + \dfrac{\sigma \beta V I}{N}, \\[2mm]
\dfrac{dI}{dt} = \gamma E - \delta I - \mu I, \\[2mm]
\dfrac{dR}{dt} = \delta I, \\[2mm]
\dfrac{dD}{dt} = \mu I.
\end{cases}
\tag{2}
$$

Nonetheless, a notable limitation of this model configuration lies in its treatment of all infectious individuals within the same compartment. Real-life observations suggest that additional emphasis should be placed on patients exhibiting severe symptoms, warranting the allocation of additional hospital resources to cater to their needs. Conversely, individuals displaying no or mild symptoms often can recover from the disease without extensive medical intervention. In light of this observation, it becomes essential to divide the infectious state into distinct subgroups, as outlined in the subsequent discussion.

2.3 Formulation of the SVEI3RD model

Taking the limitation of SVEIRD model into consideration, another extension of the model that can be applied is to divide the infectious people into three subgroups based on their illness severity. Since the cost of treatment varies depending on the severity of the symptoms, it is important to note that using this type of model allows insurance companies to provide adequate medical coverage to insurers based on their varying degrees of symptoms, as well as making sure the insured is satisfied with the benefits he or she receives for the premiums paid. Following this idea, the SVEI3RD model is denoted as:

$$
\begin{cases}
\dfrac{dS}{dt} = -\dfrac{(\beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3)S}{N} - \alpha S, \\[2mm]
\dfrac{dV}{dt} = \alpha S - \dfrac{\sigma(\beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3)V}{N}, \\[2mm]
\dfrac{dE}{dt} = \dfrac{(\beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3)S}{N} + \dfrac{\sigma(\beta_1 I_1 + \beta_2 I_2 + \beta_3 I_3)V}{N} - \gamma E, \\[2mm]
\dfrac{dI_1}{dt} = \gamma E - (\delta_1 + p_1)I_1, \\[2mm]
\dfrac{dI_2}{dt} = p_1 I_1 - (\delta_2 + p_2)I_2, \\[2mm]
\dfrac{dI_3}{dt} = p_2 I_2 - (\delta_3 + \mu)I_3, \\[2mm]
\dfrac{dR}{dt} = \delta_1 I_1 + \delta_2 I_2 + \delta_3 I_3, \\[2mm]
\dfrac{dD}{dt} = \mu I_3,
\end{cases}
\tag{3}
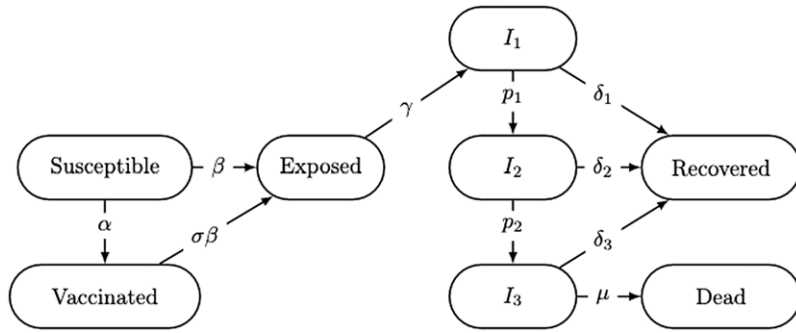$$

with an outline shown in Figure 2.

Figure 2. Transfer diagram for the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model with the Infectious class subdivided into three levels depending on the severity level of symptoms.

Under this model setting, for the infected individual, the first state of the illness occurs when he only shows mild symptoms and does not require medical treatment at the hospital. In such cases, the individuals are considered to be in state I_1, which denotes the first stage of infection. An insurance company provides only a limited number of benefit payments to patients in this state to support the basic requirements of residency. If the symptoms continue to worsen over time, he or she must be admitted to the hospital for medical treatment. If this is the case, it transfers to the second Infectious state as I_2. In addition to this, if the person is confined to the ICU due to inadequate health conditions, it enters the third phase of infection, known as I_3.

The SVEI3RD model extends the SVEIRD model by subdividing the Infectious state into three substates according to the presence and severity of symptoms. It can be seen from Equation 3 that the parameter p_1 represents the likelihood of an individual being transferred to a hospital from mild to severe symptoms. Furthermore, if the patient is already in the hospital, there is a chance that the patient will require ICU treatment represented by p_2. Furthermore, in Figure 2, people are shown that they can recover from each of the three stages of infection at varying rates of δ_1, δ_2, and δ_3 depending on the stage at which they were infected. There is evidence that individuals with mild symptoms have a higher recovery rate (δ_1) than those undergoing medical treatment at the hospital (δ_2) as discussed by Regis and Jevtie (2022). Furthermore, when comparing the recovery rate for infected individuals with ICU requirements (δ_3) to the recovery rate for infected individuals with the other two states of infection, it would be the lowest. The design of this structure assumes that only infected individuals in the ICU will die as a result of the illness, based on the idea that all infected individuals should receive medical treatment if their symptoms worsen. As a result, it is logical for insurance companies to take advantage of this model to develop appropriate insurance products during a pandemic that would be beneficial to their customers.

3. An analysis of the compartment model fitting procedure based on the case study of Victoria, Australia

In this section, the calibration of parameter values under the compartment models is conducted using the real-life Victoria dataset. Our study involves fitting both the fundamental SVEIRD model as well as the extensive SVEI3RD model. It is indicated that the SVEI3RD model fits the data more accurately and the calibrated parameter values align more closely with real-life situations. This observation suggests that the SVEI3RD model is better suited for the subsequent analysis of insurance application.

To begin this section, we perform an explanatory data analysis of the Victoria dataset. Subsequently, the SVEIRD and SVEI3RD models are fitted to the data by minimizing the

Table 1. Selected variables from the dataset COVID-19 data for Australia

Date	State	Confirmed	Confirmed_cum	Deaths
Deaths_cum	Tests	Tests_cum	Recovered	Recovered_cum
Hosp	Hosp_cum	ICU	ICU_cum	Vaccines
Vaccines_cum				

corresponding objective function, resulting in a set of parameter estimates. By examining these parameter values, we find that while the SVEIRD model has been extensively studied in the context of the pandemic, it still possesses certain limitations. In contrast, our newly introduced SVEI3RD model, which further disaggregates infectious individuals into subgroups, appears to be a superior approach.

3.1 An overview of the dataset

A historical record of the Coronavirus data in Victoria is obtained from the dataset,[1] which consists of a variety of variables as listed in Table 1. This enormous data set contains the daily Coronavirus data records for each state of Australia. In the analysis of this paper, records associated with Victoria are subtracted to be used in the parameter computation step of model fitting. In addition, by incorporating newly released records of everyday data, the author continues to update the data set. At the time of implementing the data analysis, data up to the end of March 2022 are used.

Before fitting the theoretical models to the real-life data, an explanatory data analysis is conducted. From the data set, Date and State are the first two variables that define the time horizon and geographical location of the Covid data, respectively. To perform the analysis, two time periods corresponding to Dose 1 and Dose 2 are selected using the Date variable, and the State variable is set to Victoria, Australia, which contains the city with the longest lockdown in the world, Melbourne. For the following variables "Confirmed" and "Confirmed_cum," they correspond to the number of newly infected individuals per day and the cumulative number of positive cases over time. However, there is a discrepancy between the cumulative number of infected people and the population of the Infectious state in the model formulation. In order to obtain the number of active cases at a given point in time, we need to manipulate the records of the data set. As a standard, we use A_t to represent the value of "Confirmed_cum" at certain date. To calculate the population of people in the Susceptible state I_t, our approach is to subtract the dead and recovered individuals from the cumulative number of confirmed cases as $A_t - R_t - D_t$.

In addition, the data set also includes categorical classification of individuals who are infected based on the severity of their symptoms. The number of infected individuals who require medical treatment is designated under the headings "Hosp" and "Hosp_cum," which indicates that these patients experience serious symptoms related to the infection. In this regard, people in the Intensive Care Unit (ICU) are reported as "ICU" and "ICU_cum." For the model fitting purpose, we use the "Hosp" and "ICU" values as I_{2t} and I_{3t} correspondingly. Thus, it is possible to determine the number of mildly infected persons, who have not yet been hospitalized, by subtracting the total number of infected persons from those in the hospital and those in the intensive care unit. Furthermore, the number of individuals who are exposed to the virus can be taken as the number of Coronavirus tests conducted under "Tests" and "Tests_cum" variables. In accordance with the announcement of the Victorian Government, people should only be tested for Coronavirus when they are experiencing moderate to severe symptoms. We assume that the individuals who undertake the tests are exposed to the virus and use the number of daily test conducted under "Tests" as E_t value in the model.

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[1]COVID-19 Data for Australia. Available online at the address https://github.com/M3IT/COVID-19_Data.
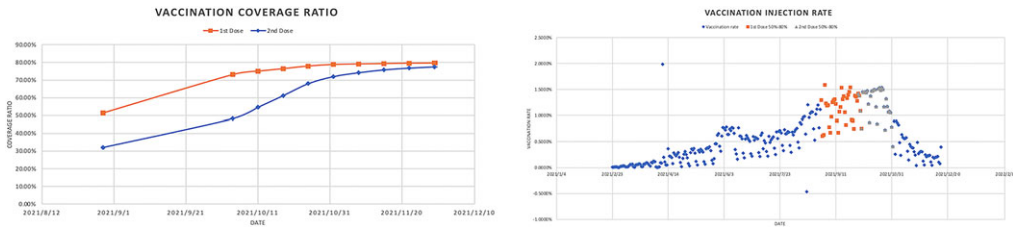
Figure 3. Coverage ratios for the first dose and the second dose in Victoria from 29/08/2021 to 29/11/2021 (left). Rates of vaccination injections in Victoria from 23/02/2021 to 14/12/2021 (right). The orange section refers to the coverage ratio of the first dose from 50% to 80% between 29/08/2021 and 04/10/2021. The gray section refers to the coverage ratio of the second dose from 50% to 80% between 01/10/2021 and 01/11/2021.

In regard to the remaining variables within the dataset, the variables labeled as "Deaths" and "Deaths_cum" represent the daily new deaths attributed to Covid and the cumulative number of deaths. The terms "Recovered" and "Recovered_cum" refer respectively to the daily number of individuals who recover from COVID-19 and the cumulative number of recovered individuals. Since the Recovered and Dead states are absorbing states, we utilize the values of "Deaths_cum" and "Recovered_cum" as the corresponding values for D_t and R_t in our model. In light of the relatively short duration of our modeling period, we assume that individuals will not transition out of these states once they enter, which corresponds with the cumulative numbers in the dataset.

There is a limitation to this data set, in that only the total vaccination digits are recorded as "Vaccines" and "Vaccines_cum," without categorizing the data by injection attributes, such as the first dose, second dose, and booster injection. For this reason, to support the data modeling, additional detailed vaccination records are obtained from the website of the Australia Government Department of Health.[2] The official weekly Coronavirus reports issued by the state government contain the vaccination rates for each dose, as well as the coverage percentages that are relevant to our modeling of vaccination.

3.2 Fitting of the SVEIRD model

Due to the rapid increase in the level of vaccination coverage, it is imperative that Vaccinated individuals be included as a separate state in the pandemic model. It is determined that two intervals are chosen to correspond to the time when the first and second dose coverage ratios increase from 50% to 80% over the entire population. There is a detailed representation in Figure 3 for the actual development of vaccination campaigns in Victoria in a given period of time.

In choosing these time ranges, it is necessary to take into account the maturity of the vaccination injection process. There were many uncertainties at the early stages of the vaccine process, and the rate of injection was very unstable. As time passed, the government medical system developed, and the attitude of the public toward vaccines changed. Since social media continued to release information on vaccination, a large proportion of the population in Victoria become receptive to dose injections in the present day. In this circumstance, the State Government is working toward achieving herd immunity against the Coronavirus by expanding vaccine coverage rapidly to a sufficient level.

In this case, the existing data in Victoria is used to fit the SVEIRD model. For estimating epidemiological parameters in the system of equations, the sum of squared errors for I_t and D_t is minimized since the number of infections and deaths are the most significant concerns during the pandemic. It is defined as the difference between the values of the model projections and the actual observations. Under this model setting, the objective function is:

[2]COVID-19 Vaccination Data. Available online at the address https://www.health.gov.au/resources/publications

$$\min_{\alpha,\beta,\sigma,\gamma,\delta,\mu} \frac{1}{2} \sum_t f[(A_t - D_t - R_t - \widehat{I}_t)^2] + f[(D_t - \widehat{D}_t)^2], \tag{4}$$

where A_t, D_t, and R_t are the cumulative numbers of infected, dead, and recovered individuals corresponding to the actual records in the data set. Additionally, the estimates $\widehat{D}_t$ and $\widehat{I}_t$ represent the estimated numbers of dead and infected individuals given the outcomes of the SVEIRD model. Accordingly, the nonlinear equation:

$$f(z) = C^2 \log \left[\left(\frac{g(z)}{C} \right)^2 \right] \text{ with } g(z) = \log(1+z),$$

suggested by Bastos and Cajueiro (2020) is used as part of the parameter estimation approach to correct for the exponential nature of the series so that the errors coming from the last values of the series do not dominate the minimization process. Furthermore, the scaling parameter is set to be $C = 2$ to soften the threshold between inliers and outliers.

In addition, we should ensure that the fitted results and the observed values are comparable. Recall that we modify the data set value "Confirmed_cum" by excluding the number of recovered and dead people to obtain the number of active cases at a certain date. This allows us to compare the fitted ($\widehat{I}_t$) values with observed $A_t - R_t - D_t$ values, where A_t represents the cumulative number of confirmed cases defined in Section 2.2. On the other hand, for the number of people in the Dead state, the comparison is taken by working out $(D_t - \hat{D}_t)^2$ where D_t and $\hat{D}_t$ refer to the number of actual and predicted death populations at time t. After the objective function has been formulated, the system of equations can be solved hierarchically by using the EpiModel package in R. In regards to solve the system of ODEs, the Runge-Kutta Method, maintained by Boyce and Diprima (1986), is the most widely used numerical method. For this insurance application, the fourth-order Runge-Kutta algorithm (RK-4) is implemented to provide an appropriate balance between implementation and accuracy.

Moreover, in this vaccine-related model, it is important to note that the number of vaccinated people varies with different modeling intervals, dependent on the formulation of the model. In the first fitting period, individuals in the Vaccinated state refer to those who have received one dose. In contrast, to attain the Vaccinated state of the model in the second fitting stage, an individual needs to complete two doses of injection following the essence of the model. Also, the vaccination rates in the two modeling stages correspond to the injection rates for the first dose and the second dose, respectively, rather than the overall daily injection ratio.

The calibrated parameter results of the fitted model are shown in Table 2. Figure 4 illustrates the corresponding fitted results. It is showcased that the vaccine inefficiency parameter σ is approximately 0.5, which means that the contact rate decreased by 50% under both modeling periods. In this circumstance, if all parameter values for vaccinated and unvaccinated people are supposed to stay the same, this difference in contact rate would result in the distinction of R_0 values as:

$$R_{0,unvacc} = \frac{\beta}{\delta + \mu}$$

and

$$R_{0,vacc} = \frac{\sigma\beta}{\delta + \mu}.$$

In practice, the factors principally affecting contact rate are the spread rate of the virus strain. Following the significant reduction in death and an increase in recovery, the reproductive number R_0 eventually decreases. Since the R_0 value is a vital indicator of the speed of the disease spread, it depicts that vaccination helps reduce the spread of Coronavirus.

Nevertheless, the parameter calibration result of σ under this model deviates from the industry standard. In comparison with the findings of early vaccination studies, the ratio of $1 - \sigma = 50\%$ is

Table 2. Estimated Susceptible-Vaccinated-Exposed-Infected-Recovered-Dead model parameters over the chosen first and second time periods of 29/08/2021–04/10/2021 and 11/10/2021–01/11/2021

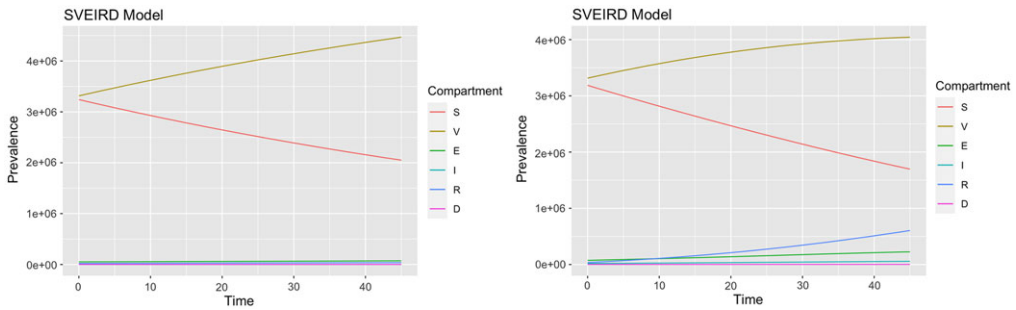| Parameter	Dose 1	Dose 2
β	1.3189	0.7258
α	0.1332	0.1026
σ	0.5429	0.5091
γ	0.0059	0.0909
δ	0.4005	0.3460
μ	0.0006	0.0005
$R_{0,\text{unvacc}}$	3.2738	1.9069
$R_{0,\text{vacc}}$	1.7774	0.9707

Figure 4. Fitted results and short-term forecasts of each compartment under the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model for the first and second vaccination periods corresponding to 29/08/2021–04/10/2021 (left) and 11/10/2021–01/11/2021 (right).

considerably lesser than the official announcement by AstraZeneca of an efficiency rate of 95%.[3] There may be a reason for this because there are only a limited number of compartments in the model formulation and it implies that the SVEIRD model does not provide a good indication of the estimated parameter values in relation to this data set.

It is important to note that, despite the fact that this model is well-known in the insurance industry, it still has a number of disadvantages. In addition, as discussed in Section 2.2, the formulation of this model limits its ability to support the insurance company's need to provide different levels of benefit payments based on patients' symptoms. Furthermore, the lack of separation among infectious individuals makes it challenging to allocate hospital resources in real-life situations, thereby limiting its usefulness in assisting the government in reducing healthcare pressure.

Additionally, it is also essential to examine the model's goodness-of-fit and we consider the fitted performance with regard to the residual standard error (RSE) values for both Dead and Infectious states. The RSE is a standard measure used to evaluate the discrepancy between actual observations and the predicted values derived from the fitted model. Using this approach, we are

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[3]The Oxford AstraZeneca COVID-19 Vaccine: what you need to know. Available online at the address https://www.who.int/news-room/feature-stories/detail/the-oxford-astrazeneca-covid-19-vaccine-what-you-need-to-know.

Table 3. Estimated Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model parameters over the chosen first and second time periods of 29/08/2021–04/10/2021 and 11/10/2021–01/11/2021

Parameter	Dose 1	Dose 2
α	0.0008	0.0009
β_1	0.2962	0.2760
β_2	0.2579	0.1532
β_3	0.0428	0.0331
σ	0.0945	0.0529
γ	0.3083	0.3176
δ_1	0.2592	0.2938
δ_2	0.2510	0.2916
δ_3	0.1013	0.1513
p_1	0.0582	0.0123
p_2	0.2933	0.1519
μ	0.0008	0.0028

able to derive the RSE values for the SVEIRD model, which could then be compared to the values obtained under the following SVEI3RD model in Section 3.3.

In spite of this, it is important to note that the performance of model fitting does not necessarily guarantee the performance of future predictions. In general, it is essential to select a model that is capable of accurately predicting future values while also being able to fit past data well. Our primary objective in this particular case is to forecast future populations in each compartment and determine the appropriate premium levels as well as analyze future reserves. Therefore, it is also necessary to assess the accuracy of the model predictions after examining the fitting performance.

3.3 Fitting of the SVEI3RD model

For Coronavirus spread within the context of a SVEI3RD model, we have used the same period for Dose 1 and Dose 2 as described in Section 3.2. In such a scenario, it would be helpful to compare the parameter values before and after the infected groups are separated.

In terms of the parameter estimation procedure, it is similar to the methodology described above. In this case, as "Confirmed_cases" includes all positive cases, including those in hospitals and ICUs, the comparable terms should be $A_t - D_t - R_t$ and $\hat{I}_{1t} + \hat{I}_{2t} + \hat{I}_{3t}$. Consequently, the objective function becomes:

$$\min_{\alpha, \beta_1, \beta_2, \beta_3, \sigma, \gamma, p_1, p_2, \delta_1, \delta_2, \delta_3, \mu} \frac{1}{2} \sum_t f[(A_t - D_t - R_t - (\hat{I}_{1t} + \hat{I}_{2t} + \hat{I}_{3t})^2] + f[(D_t - \hat{D}_t)^2], \quad (5)$$

where $f(z)$ follows the form in (4).

Based on the calibration results in Table 3, it appears that all the parameters for the contact rate β_1, β_2, and β_3 have decreased from the first modeling period to the second. There is evidence that a second dose of medication can reduce contact rates, which is consistent with the results obtained from our SVEIRD model fitting procedure described in Section 3.2. According to this model formulation, the graphical representation of the fitted results is shown in Figure 5.

By comparing the results between the first and second modeling periods, it appears that the recovery rate parameters δs increase significantly between the two stages. It can be explained by
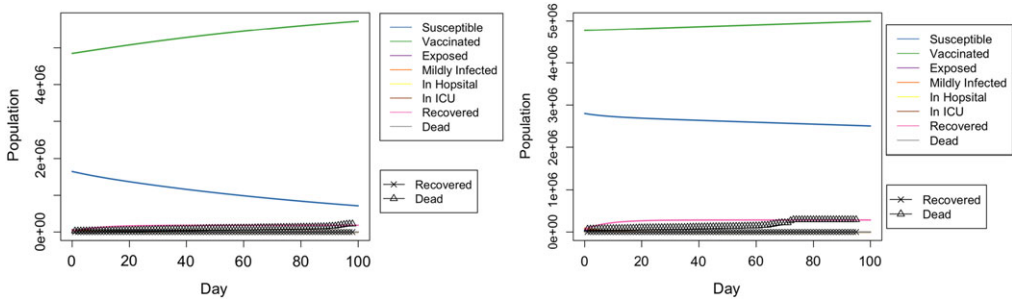
Figure 5. Fitted results and short-term forecasts of each compartment under the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model for the first and second vaccination periods corresponding to 29/08/2021–04/10/2021 (left) and 11/10/2021-01/11/2021 (right).

the fact that the prevalence of second vaccinations provides people with an increased level of protection against the disease. There is a greater likelihood that an individual who has received two vaccination injections will recover more quickly than an individual who has received only one injection. Additionally, decreases in p_1 and p_2 values indicate a reduction in transition probabilities between mild symptoms and severe symptoms, supporting the concept that two doses also provide greater protection for infected individuals.

Furthermore, it is noteworthy to point out that the death rate, as represented by μ, does not decline with Dose 2, but rather increases. The underlying reason for this is that vaccinations have reduced the number of patients transferred to hospitals and ICUs, resulting in a significant decline in the population of I_3. As μ directly correlates with the number of patients in the intensive care unit, in this case, the decline in deaths is not as significant as the reduction in the number of patients experiencing severe symptoms. This finding is in accordance with recent research obtained by Wright *et al.* (2022), which concludes that individuals with weak physical conditions are still at risk of mortality during a pandemic.

Moreover, the key parameter that illustrates the effectiveness of vaccination is the σ value. Under this model setting, the σ value, for the first period, is 0.0945, whereas for the second period, it decreases to 0.0529 as shown in Table 3. It demonstrated that the first vaccination provides $1 - \sigma = 90\%$ of protection and this protection rate can increase to 95% if a second dose of vaccination is administered as well. On the basis of the data available, the results of the SVEI3RD model are consistent with the official industry report of 95% vaccination efficiency, indicating that this model's calibration results are more practical.

Additionally, in the same way as we have done for the SVEIRD model, we also compute the RSEs for the SVEI3RD model and find that the values obtained by the SVEI3RD model are much smaller than those obtained under the SVEIRD model in Section 3.2, indicating that the newly developed model provides a better fit to the dataset. Consequently, in order to further study the forecast performance of the model, we proceed with insurance applications and analyze the prediction accuracy by comparing actual and expected reserve levels over future extended periods in Section 4.

4. Insurance applications under the SVEI3RD model

Our objective in this section is to provide a practical application for insurance companies to use as a guideline in pricing and reserving operations by using the SVEI3RD model formulation and calibrated parameter values derived in Section 3.3.

We begin this section by providing a brief overview of the settings for sample health and travel insurance products. It is done theoretically by illustrating the formulation of premiums

and reserves, and practically by discussing how to collect clinical data in real-life situations. In our study, health insurance and travel insurance were analyzed separately, but in a similar manner. In Sections 4.2.1 and 4.3.1, we use the Equivalence Principle to determine the fair premium, followed by an analysis of actual and predicted reserve levels in Sections 4.2.2 and 4.3.2. Lastly, our analysis of either insurance product concludes with a discussion of its practical significance in real-life situations.

4.1 An overview of pandemic insurance

4.1.1 A theoretical framework for pandemic insurance

The impact of contagious diseases differs from the standard concept of contingency in practice. In such instances, the epidemic model is usually employed to determine the number of policyholders in each state at various points throughout the pandemic in the insurance industry.

In this paper, we examine two types of insurance products during an epidemic. The first type of insurance is health insurance, which offers healthcare support to infected individuals under a pandemic scenario and functions similarly to traditional health insurance. A continuous premium contribution is required from healthy individuals and medical treatment assistance is provided if the insured person becomes ill.

Another type of insurance we consider during a pandemic is travel insurance. Starting in 2021, the restrictions imposed by COVID-19 on travel have been loosened by many countries, except for some notable exceptions in East Asia, such as China and Japan. By the end of July 2022, Qin Xie[4] reports that 85 countries have largely regained normalcy. Individuals are free to travel without the requirement of a PCR test or vaccination. As a result of these facts, the travel insurance products have also been analyzed. The product provides coverage for overseas medical expenses related to treating contagious diseases during the pandemic. Additionally, we include the mandatory quarantine costs incurred during travel in our benefit design as they play a significant role in the benefit expenditures of the insurance company during a pandemic. In terms of other related terms, such as cancellation and delay costs, these could be covered based on the judgment of firms. Moreover, both insurance products include the accidental death benefit payment to support the spouses if the insured dies as a result of the pandemic.

To conduct the analysis, we use the SVEI3RD model with parameter values based on the computation results in Table 3. It is done in order to better calibrate the situation over the modeling period and also to allow the insurance companies to set different levels of coverage for their clients depending on the severity of symptoms.

As most traditional health and travel insurance policies do not include pandemic coverage in their Product Disclosure Statements (PDS), it is possible to trade our product designs as either a stand-alone product or as a component of other existing insurance contracts, which combine to provide additional coverage during the pandemic with an extra premium. Practically, this paper presents two applications for each insurance product.

In the first approach, we use the Equivalence Principle and its fundamental formula:

$$\mathbb{E}\,[\text{PV of premium received}] = \mathbb{E}\,[\text{PV of benefit paid}],$$

proposed by Feng *et al.* (2021) to determine the appropriate premium levels for relevant insurance products. By measuring the expected value of the premium received and the amount of benefit paid, the insurance company determines the minimum premium amount required for the product, also known as the fair premium level. In this scenario, the expected asset outflow equals the expected profit inflow, and the insurance company thus conducts a zero-profit operation.

The second analysis examines the reserve amount of the insurance company and calculates the accumulation of earnings over various premium levels. In reality, an insurance company would

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[4]Where can I travel without a vaccine? Available online at the address https://www.thetimes.co.uk/travel/advice/where-can-i-travel-without-a-vaccine.

charge a higher premium than the minimum required to ensure a higher level of security. At the time t, the reserve amount $V(t)$ represents the policy value of the insurance plan and is an essential factor when analyzing the solvency status. In general, a positive reserve level is achieved when a large percentage of the population remains within the Susceptible state and pays premiums to the insurance company. At the same time, fewer claims are made due to infectious diseases and death. Nevertheless, in the case of a negative reserve, the large number of infectious and death cases results in a huge amount of benefit payments to policyholders, as well as a reduction of premiums owing to the decreasing susceptible population.

Additionally, the insurance company can accumulate more reserves by charging a high premium to the susceptible population. Nevertheless, this results in a decrease in the competitiveness of the insurance product. For this type of pandemic insurance, the insurer should make a deliberate decision regarding the appropriate premium level. As such, the insurance company would find it beneficial and necessary to conduct an advanced study of the spread process of the virus.

4.1.2 A practical procedure for pandemic insurance

Regarding the insurance products' real-life implementation, a governance institute or other professional institute can issue official guidelines for pricing pandemic products in this case. By addressing the natural spread of pandemic viruses, it is possible to develop software or a set of tools that can make the pricing process more robust and easier to use every time a pandemic occurs. Furthermore, it would be sensible for insurance companies to develop a handbook or a set of standards during the preliminary stages of the development process of the pandemic insurance products.

In this case, all the data utilized for model implementation are sourced from publicly available online repositories. As the majority of these datasets originate from official government publications, they generally require minimal data cleansing procedures and can be readily employed with minor modifications. As a result, concerns regarding the availability of data resources for modeling purposes are alleviated and the reliable and readily available data further enhances the reproducibility of empirical results, which is intrinsic to scientific research.

In order to accelerate the calibration procedure more rapidly, it can be advantageous to select appropriate initial parameters during the calibration process. In this circumstance, a variety of techniques are presented that may assist insurance companies in making initial guesses for parameters and improving the efficiency of the model fitting process under real-life conditions. For example, the vaccination rate parameter (α) can usually be found in the online publication of the state government. Further, the official data publications of the World Health Organization can provide initial estimates of the recovery rate and death rate parameters as well as the value of γ, which can be calculated based on the incubation period value. It is also possible to select the parameter relating to vaccination efficiency (σ) depending on the announcements made by large pharmaceutical companies. Thus, as long as the insurance companies have appropriate starting values for each parameter, they should be able to calibrate the parameters more efficiently and without experiencing any significant difficulties.

4.2 An analysis of the design of health insurance products

4.2.1 Premiums for health insurance

In a pandemic, Krueger *et al.* (2022) uncovered that individuals are more concerned about their health and it is extraordinarily challenging for infected people to overcome their physical discomfort and medical treatment expenses. In light of the limited medical treatment resources available in the hospital, the infective period for an individual is prolonged when the pandemic first arises. There are no effective methods for doctors to heal patients within a short period. All of these factors contribute to the heightened level of pain experienced by individuals during the pandemic.

Table 4. Sample product designs for 30-day and 60-day health and travel pandemic insurance

Item	Compartment	Health insurance	Travel insurance
Benefit coverages	Exposed (q)	$0	$20
	Infectious (b_1)	$50	$50
	Hospital (b_2)	$200	$500
	ICU (b_3)	$1,000	$1,000
	Death (c)	$100,000	$10,000
Target customers		People concerned about being infected and death	People with essential needs to travel around
Premium frequency		Daily premium	One-off premium
Contract term		30 days 60 days	30 days 60 days

Consequently, there have been demands for health insurance products tailor-made to cover losses due to the pandemic. This fact may contribute to the expansion of the health insurance industry under pandemic conditions.

Our approach is similar to the analysis proposed by Lefèvre *et al.* (2017), except that we scrutinize the design of an insurance product with symptom-specific benefit support. Under this pandemic insurance model, the insurance company is assumed to collect premiums from the susceptible population at a constant rate of π per unit of time. In return, the insurer offers benefit payments at a rate of b_1 for mildly infected individuals, a rate of b_2 for patients in the hospital, and a rate of b_3 for those who require intensive care in the ICU. In the event of death, a lump-sum benefit of c will be paid. Based on this arrangement, the Equivalence Principle equation becomes:

$$\pi \int_0^T S(u) + V(u)du = b_1 \int_0^T I_1(u)du + b_2 \int_0^T I_2(u)du + b_3 \int_0^T I_3(u)du + c[D(T) - D(0)]$$

and can be rearranged to work out the premium rate:

$$\pi = \frac{b_1 \int_0^T I_1(u)du + b_2 \int_0^T I_2(u)du + b_3 \int_0^T I_3(u)du + c[D(T) - D(0)]}{\int_0^T S(u) + V(u)du}. \tag{6}$$

This Equation 6 reflects the zero-profit level for the insurance companies over a coverage period of length T.

For health insurance design, individuals continually contribute premiums while in the Susceptible state. Meanwhile, once they are infected, medical support will be provided during the entire treatment period, with various amounts depending on the seriousness of their symptoms. Afterward, the plan terminates if the individual recovers from the infectious disease. Otherwise, if the insured person dies because of the virus, his close family members will receive a one-off death benefit payment.

In the following study, a sample health insurance product is described. Details are illustrated in Table 4. This basic design includes a daily benefit payment of $b_1 = \$50$ as long as the individual is infected. Moreover, if the symptoms of the insured person develop more severely than anticipated and require further hospitalization, the benefit amount provided is increased up to $b_2 = \$200$. If the patient is transferred to the ICU, he obtains a persistent benefit amount of $b_3 = \$1,000$ from the insurance company to cover the high medical expenses. Additionally, if the insured dies as a result of the pandemic, his/her family members will be entitled to a lump-sum death benefit of $c = \$100,000$.

In regard to the duration of health insurance coverage, this paper evaluates two scenarios of 30 days and 60 days. As a general rule, a 30-day period is considered to be a standard period

Table 5. Estimated premium levels with loadings for health insurance contracts with a 30-day and 60-day validity period

Health insurance	Premium	30-day	60-day
Dose 1	Fair premium	$0.8829	$0.5173
	10% loading	$0.9712	$0.5690
	20% loading	$1.059	$0.6208
Dose 2	Fair premium	$0.2900	$0.1533
	10% loading	$0.3190	$0.1686
	20% loading	$0.3480	$0.1840

of insurance coverage for short-term health insurance products. This is due to the fact that certain insurance products, such as the one offered by Suncorp Group,[5] normally offer a one-month cooling-off period for new policyholders. Essentially, this means that insurance companies offer newly enrolled members the option of receiving a full refund of their premiums if they cancel the policy within the first 30 days of membership provided that no claims have been filed. In accordance with this term set, we determine that a 30-day period is a minimum time frame for our analysis. Additionally, policyholders may be able to prolong their contract depending on the services they receive. This leads to the insurance with a selection of an extended coverage period of 60 days. Under this setting, we are able to compare the daily premium levels for these two types of products over various time periods. For instance, the corresponding fair premium amounts for this short-term 30-day sample products are calculated as $0.8829 and $0.2900 for the first and second dose modeling period shown in Table 5. In this case, while keeping the coverage period fixed, the daily payment requirement is much lower during the second dose period. Recall that in Section 3.3, we have proved that people are better protected with two injections. As a result, the expected benefit payout amount during the second period would be smaller than in the first period. Consequently, the insurance company will charge a corresponding lower daily premium rate with respect to the lower overall payments. This pattern also occurs for the comparison between 60-day health insurance for Dose 1 and Dose 2, where the Dose 1 daily fair premium is much larger than the Dose 2 settings.

The fact that the fair premium would offer the insurance company a zero-profit position makes it unrealistic to expect an insurance company to price its products at this level. To ensure a higher level of security, insurance companies normally set actual premium amounts with extra loading factors. Based on the Australian government's Lifetime Health Cover policy,[6] we determine that a high premium level is calculated with a loading factor of 20%, and that is the Lifetime Health Cover for people who take out private patient hospital cover at the age of 40. Similarly, a low premium level is determined with a safety loading of 10%, which corresponds to the safety loadings for 35-year-old individuals who purchase their first private health insurance policy. In particular, we set the premium of each of the products at 1.1 times (or 1.2 times) the fair premium amount as shown in Table 9 and analyze how the reserve levels respond to these prices.

4.2.2 Reserves for health insurance

After the zero-profit premium level and the corresponding loadings are determined, in real-life practice, the key term that insurance companies consider is the reserve level. The purpose of this paper is to examine the relationship between the predetermined premiums set by insurance companies and the future reserves that they accumulate over time. Regarding our health insurance

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[5]Health Insurance Policy booklet. Available online at the address https://www.suncorp.com.au/content/dam/suncorp/insurance/suncorp-insurance/documents/health/suncorp-health-insurance-policy-booklet.pdf.

[6]Lifetime health cover. Available online at https://www.ato.gov.au/Individuals/Medicare-and-private-health-insurance/Private-health-insurance-rebate/Lifetime-health-cover/#Lifetimehealthcoverloading
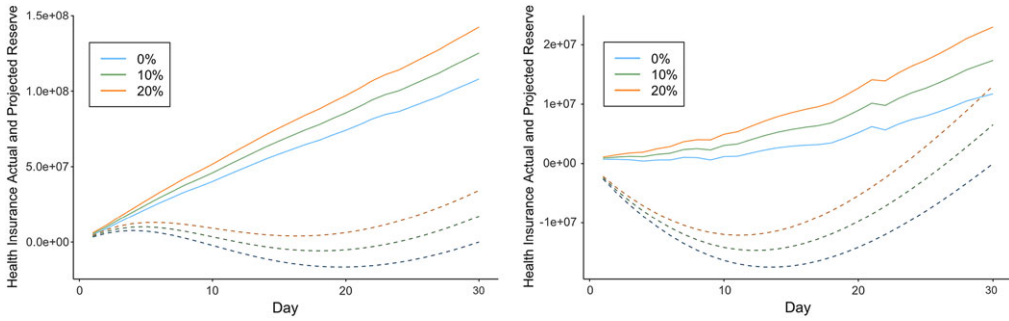
Figure 6. Comparison of actual and projected reserve levels with predefined loadings under the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model over Dose 1 (left) and Dose 2 (right) modeling period for 30-day health insurance products. The solid lines refer to the actual reserve levels. The dashed lines refer to the predicted reserve levels.

Figure 7. Comparison of actual and projected reserve levels with predefined loadings under the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model over Dose 1 (left) and Dose 2 (right) modeling period for 60-day health insurance products. The solid lines refer to the actual reserve levels. The dashed lines refer to the predicted reserve levels.
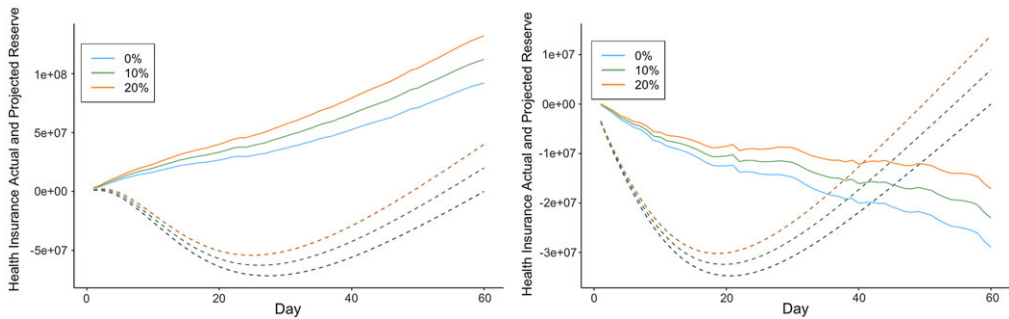
design, the reserve level can be determined as:

$$V(t) = \pi \int_0^t S(u) + V(u)du - b_1 \int_0^t I_1(u)du - b_2 \int_0^t I_2(u)du - b_3 \int_0^t I_3(u)du - c[D(t) - D(0)].$$

(7)

In relation to reserve levels, the most significant characteristic is that higher premium amounts lead to greater accumulation of reserves over time as the colored lines with high loadings always sit above the reserve line under fair premium assumption in Figures 6 and 7.

Contrary to the typical reserve progression, the profit accumulation for health insurance under a pandemic scenario appears more complicated. In the traditional actuarial field, insurance products are generally designed based on a life table. This attribute pertains to the rate of natural death of humans, which peaks at a relatively young age, shortly after birth, and as individuals get older. However, this character is incompatible with pandemic conditions. According to the beliefs of pandemic epidemiologists, the contact rate increases rapidly during the early stages of the pandemic. Afterward, when the contagious rate reaches a peak, it tends to decline until other events occur, such as the emergence of a new virus strain. This is mainly due to the fact that it takes people a substantial amount of time to discover the origin of the virus and develop the appropriate resolutions or treatments to counteract it. This results in the building of the expected reserve levels at the start of the policy period. Then, because of the increasing liabilities of infection and

Table 6. Comparison of predicted and actual premiums for health insurance contracts with a 30-day validity period over Dose 1 and Dose 2 modeling period

30-day health insurance	Premium	Predicted reserve	Actual reserve
Dose 1	Fair premium	$0	$107,961,598
	10% loading	$17,107,147	$125,152,972
	20% loading	$34,214,293	$142,344,347
Dose 2	Fair premium	$0	$11,740,755
	10% loading	$6,539,662	$17,362,531
	20% loading	$13,079,325	$22,984,306

Table 7. Comparison of predicted and actual premiums for health insurance contracts with a 60-day validity period over Dose 1 and Dose 2 modeling period

60-day health insurance	Premium	Predicted reserve	Actual reserve
Dose 1	Fair premium	$0	$92,060,005
	10% loading	$20,024,040	$112,156,366
	20% loading	$40,048,080	$132,252,726
Dose 2	Fair premium	$0	−$28,935,749
	10% loading	$6,906,539	−$23,012,804
	20% loading	$13,813,077	−$17,089,859

death benefit payouts, the predicted reserve level would decrease significantly in the middle of the insurance term. In some cases, if the virus has a relatively high contact rate, the reserve level could fall below zero, leading to the insurance company becoming financially insolvent.

Furthermore, in order to analyze the predictability of our model, we compare the predicted reserve levels with the actual reserve amounts over time. In this case, actual reserve levels are calculated based on the number of observed individuals in each compartment, using real-life data retrieved from the Victoria data set. As an alternative, predicted reserves are calculated based on the population forecasts in each compartment of the SVEI3RD model using its calibrated parameter values from the previous period listed in Table 3. The results are summarized in Tables 6 and 7. Additionally, Figures 6 and 7 provide graphical views in relate to the progress of how reserves change over time, where the projected reserve level over time is shown by the dashed lines, and the actual reserve level is represented by the solid lines. There is a tendency for the dashed lines to fall below the solid lines for both Dose 1 and Dose 2, which indicates that the actual reserves are higher than the predicted ones. The reason behind this is due to the fact that as the vaccines become more popular over time, the contact rate and death rate are expected to keep decreasing over time. Consequently, the actual number of infected individuals will be lower than our model predictions since our parameter values are based on the calibration results from the previous period.

Another conclusion that can be drawn from this analysis is that the overestimation effect is more pronounced during the Dose 1 modeling period than the Dose 2 period. According to Table 6, the final actual reserve level at the end of the 30-day period is much greater than the expected amount for the Dose 1 period, while only slightly greater for the Dose 2 modeling period. As a result of these findings, it suggests that it is more appropriate to use Dose 2 data in our modeling process because it tends to give a more stable valuation result. In case of a second dose of vaccination being administered, the prevention performance of the vaccine is more stable and this is more in accordance with the pricing procedure for insurance products. Consequently, as the second dose data becomes available to the public, insurance companies are encouraged to reevaluate their products based on the number of people who have had two doses injected. By doing this, the overestimation effect of the pricing model can be reduced.

In contrast, there is an exception that when the time period is extended to 60 days, the reserve level becomes more uncertain and the result in Figure 7 shows that over a longer period, there

Table 8. Break up of predicted and actual benefit payments for 60-day health insurance contract with 20% premium loading over the Dose 2 modeling period

Dose 2 60-day health insurance payouts	Predicted reserve	Actual reserve
Infectious payment	37,308,883 (54.02%)	40,218,000 (45.62%)
Hospital payment	4,248,823 (6.15%)	4,632,200 (5.25%)
ICU payment	21,502,927 (31.13%)	4,115,000 (4.67%)
Death payment	6,004,754 (8.69%)	39,200,000 (44.46%)

is a possibility that the actual reserve amount could largely deviate from what we predicted. In order to determine what is causing this phenomenon, we break up the total benefit payout into different pieces for our sample 60-day health insurance product with the result amount correlating to each part displayed in Table 8. In particular, we find that the main cause of this problem is the underestimation of deaths throughout the period. These findings are in line with the results obtained during the fitting procedure of the SVEI3RD model in Section 3.3. Keeping in mind that the death rate computed by μ in the second modeling period increases compared to the Dose 1 period as shown in Table 3, we conclude that people with fundamentally weak physical conditions are still at risk of death during the pandemic. Consequently, although vaccinations provide better protection for most people, the virus still causes significant death rates among the individuals with weak bodies. Therefore, it is necessary for insurance companies to apply an adjustment for their long-term pandemic-related products and ensure that their reserve levels are above the safe line.

4.2.3 Discussion of real-life health insurance practices

An essential aspect of the insurance process is the variable benefit amount paid by the insurance company to each infected patient based on the severity of their symptoms. Suppose an insurance company gives the same subsidy to all individuals who are infected without regard to their circumstances. Consequently, patients with severe symptoms and high medical costs may not obtain adequate benefits to cover their medical expenses. Thus, they may choose to leave their current contracts and be insured by other insurers in the market. Additionally, if an insurance company imposes a fixed benefit level across all infected persons, it is expected that the average premium rate will be more than when support is provided based on the fitness status. Mildly symptomatic patients are unwilling to accept the high premium price for the insurance product. As a result, people with better health conditions are disadvantaged, and thereby, the company loses this group of customers and is left with a clientele made up primarily of the old and the infirm. This is similar to the insurance term of adverse selection, referring to the situation where healthy individuals gradually leave the insurance contract, leading to a pool of high-risk policies.

With the recent Coronavirus pandemic background, the treatment cost varies based on the severity of the symptoms experienced by infected individuals. According to a recent medical study by Ohsfeldt *et al.* (2021), the medical costs for treating patients in the ICU are almost five times the amount for treating other patients in the public hospital. Thus, we believe that our previous assumption of a subsidy of $200 for patients receiving regular hospital care and $1,000 for those undergoing intensive care is reasonable in real-life circumstances.

In addition, the availability of public medical resources is another critical factor insurance companies should take into account when developing the pandemic insurance product in practice. Concerning Coronavirus, in most countries, the local government is responsible for providing treatment costs for patients. It means that residents who are identified as infectious will be admitted to a public hospital and treated as public patients, with all treatment expenses being fully covered by the government. In this case, since most people receive adequate health care services
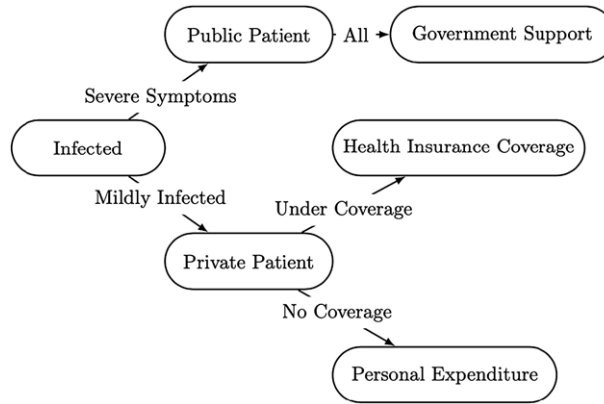
Figure 8. Treatment approaches for patients with various symptoms during the COVID-19 pandemic in Australia.

under the mature national medical system without utilizing private health insurance, insurance companies are less likely to payout large amount of private treatment fees during the pandemic. If the resident wishes to be treated as a private patient and to use their private health insurance coverage to obtain more comprehensive care at the clinic, most health insurance plans[7,8] provide their customers with coverage of private hospital admissions for lung and chest conditions, including most Coronavirus-related treatments. Additionally, due to the popularity of vaccination, a number of health insurance companies[9] also cover treatment for rare adverse reactions to Coronavirus vaccinations. On the contrary, there are no benefits payable on a health insurance policy for food or accommodations, as its primary objective is to assist customers with medical expenditures only. This approach is depicted in Figure 8.

Unlike in developed countries, insurance companies operate differently in some developing countries. When the fast-spreading virus strain Omicron reached the public, the number of confirmed cases increased dramatically each day, causing immense pressure on the local healthcare systems. Under this situation, many developing countries did not have adequate resources to handle the rapid increase in patient numbers and were unable to provide treatment to each infected person. This resulted in some countries starting to categorize infectious persons based on their symptoms. The Thai Government,[10] for example, classified infected people into three categories, green, yellow, and red, depending on the severity of their symptoms. People in the red level were provided with priority medical attention under the national healthcare system. However, due to the lack of medical resources available, mildly infected individuals were unlikely to be able to receive free public treatment.

There was, as a consequence, an increase in the use of private healthcare options and the need to obtain private health insurance coverage. From the perspective of an insurance firm, a significant amount of medical and death benefit payments were made during this period, which resulted in a dramatic drop in the reserve balance. Specifically, two major Thai insurance companies

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[7]COVID-19 (Coronavirus) and your cover. Available online at the address https://www.nib.com.au/health-information/member-services/coronavirus-and-your-cover

[8]COVID-19 What you need to know about Coronavirus. Available online at the address https://www.bupa.com.au/health-insurance/covid-19.

[9]Our Response to COVID-19. Available online at the address https://www.aia.com.au/en/individual/help-support/covid-19.html.

[10]Thailand Situation Update Coronavirus (COVID-19). Available online at the address https://www.businesseventsthailand.com/en/situation-update-coronavirus-COVID-19.

announced liquidation decisions in January 2022.[11] Thus, in order to ensure a sustainable day-to-day operation, insurance companies need to take into consideration external medical conditions when implementing pandemic product configurations as well.

4.3 An analysis of the design of travel insurance products

4.3.1 Premiums for travel insurance

Travel insurance is another form of insurance associated with the pandemic. The Australian Government re-opened its internal border for business and travel at the end of 2021. Despite the existence of Coronavirus, individuals are permitted to travel within the nation and internationally. This waiver of travel restrictions is primarily motivated by the availability of vaccinations in all Australian states. The investigation indicates that vaccination injection aids in decreasing the death rate and lowering the intensive medical requirements for infected individuals as well. In the present state of affairs, most people with the contagious disease can recover after several days of self-quarantine without having to seek medical attention. Nevertheless, the requirement for close contact to undertake isolation is critical to preventing the spread of the virus. The person exposed to the virus is not permitted to go out to work for a short period and must stay in a hotel or at home for two weeks to ensure that he is not infected.

The nature of certain careers means that some job positions do not support work-from-home functionality, indicating that individuals required to perform self-quarantine may not be able to work during this time. In consequence, some insurance companies have developed a new form of insurance to provide financial support to people during their quarantine period. This type of insurance policy is designed to benefit employees whose work involves travel and who are currently facing the possibility of coming into close contact with infected people during their day.

As the timeframe for our study, we chose 30 days and 60 days as the coverage periods for travel insurance, which is the same as the case for health insurance discussed in Section 4.2.1. It remains close to the current market condition. As an example, we find that most travel insurance policies, such as the one offered by Allianz,[12] charge the same amount for travel periods of less than 30 days. Alternatively, a small discount is only offered when the client requests a longer period. As a result, we will continue to model travel insurance products based on the original 30-day and 60-day term settings.

In this subsection, a typical example of such a travel insurance product is provided with a summary in Table 4. In the first instance, the insured individual with weak symptoms will receive a benefit amount of $b_1 = \$50$. According to real-life conditions, overseas hospital treatment expenses are expected to be much higher than domestic hospital treatment expenses, which has led insurance companies to provide greater benefit payments as $b_2 = \$500$ and $b_3 = \$1,000$ for infected patients in hospitals and intensive care units, respectively, compared to the local health insurance product design in Section 4.2.1. Additionally, as part of the product design, a one-time death benefit payment of $c = \$10,000$ is also involved. There is a fundamental difference between this travel insurance and the formerly discussed health insurance in that it includes a quarantine subsidy of $q = \$20$, which is available to those who are exposed to the virus and forced to undertake self-isolation. Furthermore, travel insurance normally requires a one-off premium at the beginning of the contract ($t = 0$), as opposed to a series of payments in health insurance. We use a similar manner as Health Insurance and the same set of parameter values as in Table 3 for this emerging travel insurance product and evaluate the fair premium amount as:

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[11] Thailand: 2 insurers petition for liquidation under the weight of COVID-19 claims. Available online at the address https://www.asiainsurancereview.com/News/View-NewsLetter-Article/id/79438/type/eDaily.

[12] Allianz Travel Get a Quote. Available online at the address https://www.allianztravelinsurance.com/compare-plans.

Table 9. Estimated premium levels with loadings for health insurance contracts with a 30-day and 60-day validity period

Travel insurance	Premium	30-day	60-day
Dose 1	Fair premium	$28.60	$32.89
	10% loading	$31.46	$36.18
	20% loading	$34.32	$39.47
Dose 2	Fair premium	$9.88	$10.34
	10% loading	$10.87	$11.37
	20% loading	$11.86	$12.41

Table 10. Comparison of predicted and actual premiums for travel insurance contracts with a 30-day validity period over Dose 1 and Dose 2 modeling period

30-day travel insurance	Premium	Predicted reserve	Actual reserve
Dose 1	Fair premium	$0	$132,690,964
	10% loading	$18,573,304	$151,349,745
	20% loading	$37,146,608	$170,008,526
Dose 2	Fair premium	$0	$26,073,386
	10% loading	$7,485,049	$32,489,246
	20% loading	$14,970,098	$38,905,105

$$\pi = \frac{b_1 \int_0^T I_1(u)du + b_2 \int_0^T I_2(u)du + b_3 \int_0^T I_3(u)du + q \int_0^T E(u)du + c[D(T) - D(0)]}{S(0) + V(0)}. \tag{8}$$

The premium values for this travel insurance sample design are summarized in Table 9. In this case, due to the fact that the 60-day insurance plan covers a long period, it is evident that a higher premium is required at the commerce of the contract when compared to the shorter one-month coverage. For travel insurance, the one-off premium required at the beginning of the coverage period is expected to cover the entire insured period benefit payouts. Consequently, the premium amount should increase when the contract period is extended. In order to create suitable travel insurance product, the same loadings of 0%, 10%, and 20% have been placed on top of the desired premium levels similar to what we have done for health insurance product design in Section 4.2.2 with the premium values shown in Table 9. As we discussed previously, these differences in price levels are microcosms of the judgment of the firm regarding the insurance policy.

4.3.2 Reserves for travel insurance
Following a similar technique as discussed in Section 4.2.2, the accumulated reserve level for travel insurance can be worked out as:

$$V(t) = \pi[S(0) + V(0)] - b_1 \int_0^t I_1(u)du - b_2 \int_0^t I_2(u)du - b_3 \int_0^t I_3(u)du$$
$$- q \int_0^t E(u)du - c[D(t) - D(0)].$$

with results shown in Tables 10 and 11.

For travel insurance, we also examine the reserve development path under three one-off premium settings with the final reserve amounts at the end of coverage period shown in Table 10.

In contrast to health insurance, whose reserve curves fluctuate over time, the accumulation of profits for travel insurance tends to decrease over time under all scenarios as shown in Figure 9.

Table 11. Comparison of predicted and actual premiums for travel insurance contracts with a 60-day validity period over Dose 1 and Dose 2 modeling period

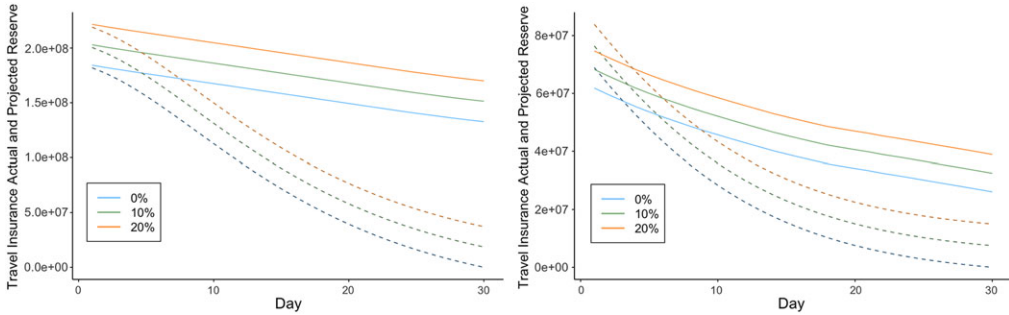| 60-day travel insurance	Premium	Predicted reserve	Actual reserve
Dose 1	Fair premium	$0	$131,487,165
	10% loading	$21,358,628	$152,944,088
	20% loading	$42,717,255	$174,401,011
Dose 2	Fair premium	$0	−$987,591
	10% loading	$7,832,661	$5,726,227
	20% loading	$15,665,322	$12,440,044

Figure 9. Comparison of actual and projected reserve levels with predefined loadings under the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model over Dose 1 (left) and Dose 2 (right) modeling period for 30-day travel insurance products. The solid lines refer to the actual reserve levels. The dashed lines refer to the predicted reserve levels.

This is because the travel insurance provider collects the total premium payments before the policy is issued. Thus, the insurance company has its highest reserve level at the beginning of the contract. As time passes, when people become infected or exposed to the virus, appropriate benefits are expected to be provided, resulting in a reduction in the reserve level of the insurer for the product. As a result, for an insurance corporation to avoid possible bankruptcy, it is vital that they receive enough premium payments at the commencement of the contractual period and ensure sufficient accumulations in their account to cover future liabilities of the policyholders.

When it comes to comparing actual and predicted reserve levels under travel insurance, the results are similar to those discussed for health insurance in Section 4.2.2. In particular, the actual reserve level also fell below zero during the extended 60-day period under the second modeling period, indicating that the insurance company suffered a loss of profit. In order to prevent the situation in Figure 10 from occurring in real-life cases, the company can adjust the price according to the rapidly changing external environment for travel insurance products with a coverage period exceeding 30 days. Since the one-time premium for travel insurance is fixed at the beginning of the contract period, a possible way is to include extra compensation terms in relate to the policy's extended coverage term. It is expected that the loading on the one-off premium level will be higher than for short-term travel insurance. The results of our sample case indicate that if the loading of the premium increases to 30%, the insurance company will be able to accomplish a positive profit on the 60-day travel insurance sample product. Meanwhile, higher prices are unfavorable for customers, and insurance companies may lose their competitive advantage in the market. Consequently, finding the right balance for pricing such products is highly dependent on the accuracy of their pricing models.
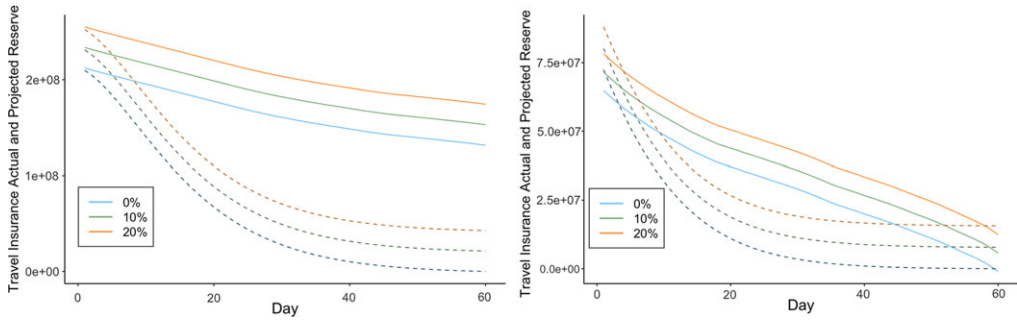
Figure 10. Comparison of actual and projected reserve levels with predefined loadings under the Susceptible-Exposed-Infected-Containing-3-Substates-Recovered-Dead model over Dose 1 (left) and Dose 2 (right) modeling period for 60-day travel insurance products. The solid lines refer to the actual reserve levels. The dashed lines refer to the predicted reserve levels.

4.3.3 Discussion of real-life travel insurance practices

For travel insurance, the most critical aspect of its real-life implementation is that the insurance provider must set the adequate premium levels in a prudent manner. In the event that a new virus strain spreads rapidly with a high contact rate among the public, the likelihood of a large number of people becoming infected can be expected within a short period of time. Considering the concept of superimposed effects, the contagious virus spreads more rapidly when there is more infection. As a consequence, a wide range of people would be in close contact and require mandatory quarantines. Due to the design of travel insurance products, insurance companies will be under great pressure from the upcoming claims. Consequently, insurance companies need to collect adequate premiums at the beginning of the contract period in order to cover upcoming risks.

Concerning the recent pandemic, there is a variety of insurance policies[13,14] that offer coverage for traveling under Coronavirus. These refer to the policies that cover overseas medical expenses and provide coverage for infected individuals who require hospitalization while abroad. Furthermore, some insurance products also provide compulsory quarantine subsidies when the insured person or a member of the traveling party is diagnosed with Coronavirus or confirmed as having close contact during the trip. Generally, the cost of quarantine support is primarily set for accommodation and food supply. Meanwhile, travel insurance does not provide coverage for quarantine expenses if the necessity of isolation is acknowledged before departure. For instance, at the time this paper was written, the Australian Government enforced mandatory isolation for all returning residents. This requirement is known by travelers before they plan their journey, so the insurance company will not cover this expense. It is evident that the insurance company provides coverage for uncertain events, but not for preexisting conditions. Therefore, it is critical to carefully review the PDS of the relevant insurance policy.

Along with the features we discussed previously, there are a number of other factors that may also contribute to the various pricing levels in the implementation of travel insurance products, including the traveler's current location, the destination, the product's excess policy, the age of the policyholder, and so on. Accordingly, the underwriting process should be carefully followed by the insurance company prior to issuing an insurance policy. For instance, RACV[15] offers different

[13]Travel insurance for Coronavirus (COVID-19). Available online at the address https://travel.insurance.qantas.com/coronavirus-cover.

[14]COVID-19 Travel Insurance Benefits. Available online at the address https://www.medibank.com.au/travel-insurance/covid-benefits/.

[15]Compare Travel Insurance. Available online at the address https://racv.tmmatravel.com.au/quotes/steps/2.

travel insurance plans as Basics, Essentials, and Comprehensive, in increasing order of coverage, which can be tailored to the preferences of customers.

5. Discussion and conclusion

In this paper, we examine the concept of epidemiological modeling and investigate the spread of Coronavirus over time in Victoria, Australia. Based on the comparison of the estimated parameters of epidemic models in different periods, this paper points out the significance of vaccination with the use of the actual data in Victoria. It appears from the estimated parameter results that individuals who have undergone a complete vaccination injection are better protected against the Coronavirus. According to the words of Avery *et al.* (2020), modeling results play a critical role in informing policy decisions in a variety of ways. Our findings suggest that vaccination is an effective method for reducing the death rate and that the government should increase the dose injection campaign among the general public.

Regarding the pandemic insurance application, the insurance company receives premium payments from individuals who are susceptible to the disease. In return, it provides benefits to the insured person when he becomes infected, as well as one-time death benefits to his family members in the event he dies. With regard to subsidies for infectious individuals, the amount of the benefit depends on the level of symptoms experienced by the infected individual. Furthermore, the travel insurance offers additional quarantine support for people exposed to the virus. In both product configurations, all benefit payments are considered as continuous payouts except for the death benefit, which is a one-off payment. By computing the integral over time and adding the extra death benefit amount at the end, the total expenditure for an insurance company can be estimated over time. Moreover, the zero-profit premium level is determined by equating the premium received and the expected benefit payments. In practice, the insurance firm typically determines the premiums with loadings to ensure higher level of security. In general, the higher the premium pricing, the greater the expected reserve amount of the insurance corporation over time. In terms of real-world implementation, insurance companies must determine premium levels that are sufficient to earn profit as well as maintain their competitiveness in the highly competitive market.

In this paper, the following contributions are made. This paper proposes a more comprehensive SVEI3RD that overcomes the limitations of the standard SVEIRD model by separating out patients who needed healthcare during the pandemic. Furthermore, due to the widespread popularity of vaccination, we intend to calibrate parameter values using real-life Coronavirus data in order to model the spread of the virus and examine the effectiveness of vaccination. In this case, this paper examines the effects of the first and second doses, both of which contribute to reducing contact rates among the general population. In addition, the newly proposed SVEI3RD model provides more practical results and fits better with the changes in observed population dynamics. Consequently, this updated model is considered to be more effective when it comes to insurance-related applications.

In addition, two types of insurance products are developed on the basis of pandemic models in order to assess the possible impact of an outbreak on the insurance industry. The insurance setup is essentially a generalized version of the primary design proposed by Feng *et al.* (2021). Aside from the base settings, the more comprehensive SVEI3RD model offers insurance companies the ability to provide different benefit subsidies based on the severity of symptoms, thus enhancing the pricing procedure for pandemic-related insurance products. Additionally, we introduce a new form of travel insurance that extends the design to include additional benefits related to quarantine during times of pandemic outbreaks. Currently, both types of insurance are in their infancy and have a great deal of untapped potential for growth in the foreseeable future. Regarding real-life practices, we also form discussions on the actual implementation of products, which can serve as a guideline for insurance companies while developing their products during the pandemic.

As a measure of the realistic feasibility of the model, we examine the profit reserve level of an insurance company under different premium charges as well as the difference in the reserve curve over time for both health and travel insurance. Furthermore, we perform a comparison between the actual reserve levels and those predicted by our model in order to assess the practicality of our model. In relation to practical insights, we uncover scenarios in which insurance companies fail to develop products correctly, as well as discuss the real-life procedures involved in implementing this type of insurance. Our research analyzes the potential risks associated with pandemic-related insurance products, and how the insurance industry could assist the government health care systems by developing pandemic insurance products. By doing so, our results are linked closely to actual industry procedures, thus forming a bridge between the theoretical framework and real-life insurance practices.

In conclusion, considering the recent occurrence of infectious disease outbreaks, scientists and researchers should prioritize the study of vaccines in the field of healthcare. For actuaries who are also skilled at quantifying risk, we envision more future research aiming to integrate epidemiological models and risk quantification under a dynamic framework.

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