Quantities to be *deducted* on account of *pence* occurring in the price of the annuity.

1*d*.	·000160256	·000179426	·000198413	·000217220
2	·000320513	·000358852	·000396825	·000434439
3	·000480769	·000538278	·000595238	·000651659
4	·000641026	·000717703	·000793651	·000868878
5	·000801282	·000897129	·000992063	·001086098
6	·000961538	·001076555	·001190476	·001303318
7	·001121795	·001255981	·001388889	·001520537
8	·001282051	·001435407	·001587302	·001737757
9	·001442308	·001614833	·001785714	·001954976
10	·001602564	·001794258	·001984127	·002172196
11	·001762821	·001973684	·002182540	·002389415

*** We print this modification of Orchard's tables in the form adopted by our correspondent; but we think no good purpose is served by giving more than five decimal places. In practice, it would probably be better to make the corrections *additive:* thus, taking the above example,

£8. 17*s*. 10*d*.=£9−2*s*. 2*d*.

Value for the £9	=	523·809524
Add for the 2*s*.		4·761905
,, ,, 2*d*.		·396825
Value, as above,		528·968254

Ed. *J. I. A.*

ON "TEN YEAR NON-FORFEITURE POLICIES."

To the Editor of the Journal of the Institute of Actuaries.

Sir,—If leisure had permitted I intended to have given in the last Number of the *Journal* a development of the suggestion contained in your foot note to my letter in the January Number, and to have looked at the American ten year non-forfeiture policies from the surrender point of view. I now propose to do this, and as all numerical results given in the present communication are based upon the Experience rate of mortality and three per cent interest, it will be advisable first to give the following recomputed values, on the same basis, of the numerical illustrations contained in my last letter.

Age at Entry.	Law of Surrender. $p_1=1,\ p_2=p_3=p_4 \ldots\ldots = p_9(=p)$.				Law of Surrender. $p_1=1,\ p_2=p_3=p_4=p_5(=p),\ p_6=p_7=p_8=p_9=1$.			
	$p=0$.	$p=\frac{1}{3}$.	$p=\frac{2}{3}$.	$p=1$.	$p=0$.	$p=\frac{1}{3}$.	$p=\frac{2}{3}$.	$p=1$.
30	4·674	4·690	4·686	4·691	4·674	4·689	4·683	4·691
40	5·636	5·633	5·637	5·630	5·636	5·632	5·635	5·630
50	7·002	7·091	7·080	7·088	7·002	7·088	7·056	7·088

Each of these results denotes the annual premium per cent.

If, now, we call V_n the true cash surrender value of a policy at the end of the nth year, just before the $(n+1)$th premium becomes due, and

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V'_n the corresponding value given by the American plan, we shall have the following formulæ for the computation of V_1, V_2, V_3, &c., the annual premium payable being ϖ, and $\underset{n}{p}$ denoting as before the probability at the time the nth renewal premium becomes due, that it will be paid, supposing the life assured to be then in existence.

$$
\left.\begin{aligned}
V_1 = \frac{1}{D_{x+1}} \Big\{ & \underset{1}{p}(M_{x+1}-M_{x+2}) + (1-\underset{1}{p})D_{x+1}V'_1 \\
& + \underset{1}{p}\underset{2}{p}(M_{x+2}-M_{x+3}) + \underset{1}{p}(1-\underset{2}{p})D_{x+2}V'_2 \\
& + \underset{1}{p}\underset{2}{p}\underset{3}{p}(M_{x+3}-M_{x+4}) + \underset{1}{p}\underset{2}{p}(1-\underset{3}{p})D_{x+3}V'_3 \\
& \quad \vdots \qquad\qquad\qquad\qquad \vdots \\
& + \underset{1}{p}\underset{2}{p}\underset{3}{p}\ldots\underset{8}{p}(M_{x+8}-M_{x+9}) + \underset{1}{p}\underset{2}{p}\underset{3}{p}\ldots\underset{7}{p}(1-\underset{8}{p})D_{x+8}V'_8 \\
& + \underset{1}{p}\underset{2}{p}\underset{3}{p}\ldots\underset{9}{p}M_{x+9} + \underset{1}{p}\underset{2}{p}\underset{3}{p}\ldots\underset{8}{p}(1-\underset{9}{p})D_{x+9}V'_9 \\
& - \varpi\underset{1}{p}(D_{x+1} + \underset{2}{p}D_{x+2} + \underset{2}{p}\underset{3}{p}D_{x+3} \ldots + \underset{2}{p}\underset{3}{p}\ldots\underset{9}{p}D_{x+9}) \Big\} \\
V_2 = \frac{1}{D_{x+2}} \Big\{ & \underset{2}{p}(M_{x+2}-M_{x+3}) + (1-\underset{2}{p})D_{x+2}V'_2 \\
& + \underset{2}{p}\underset{3}{p}(M_{x+3}-M_{x+4}) + \underset{2}{p}(1-\underset{3}{p})D_{x+3}V'_3 \\
& + \underset{2}{p}\underset{3}{p}\underset{4}{p}(M_{x+4}-M_{x+5}) + \underset{2}{p}\underset{3}{p}(1-\underset{4}{p})D_{x+4}V'_4 \\
& \quad \vdots \qquad\qquad\qquad\qquad \vdots \\
& + \underset{2}{p}\underset{3}{p}\ldots\underset{8}{p}(M_{x+8}-M_{x+9}) + \underset{2}{p}\underset{3}{p}\ldots\underset{7}{p}(1-\underset{8}{p})D_{x+8}V'_8 \\
& + \underset{2}{p}\underset{3}{p}\ldots\underset{9}{p}M_{x+9} + \underset{2}{p}\underset{3}{p}\ldots\underset{8}{p}(1-\underset{9}{p})D_{x+9}V'_9 \\
& - \varpi\underset{2}{p}(D_{x+2} + \underset{3}{p}D_{x+3} + \underset{3}{p}\underset{4}{p}D_{x+4} \ldots + \underset{3}{p}\underset{4}{p}\ldots\underset{9}{p}D_{x+9}) \Big\}
\end{aligned}\right\} \text{(A)}
$$

It is not necessary to give the expressions for V_3, V_4, &c., the law of their formation being sufficiently obvious from the above formulæ. The concluding values of the series are

$$
\left.\begin{aligned}
V_8 = \frac{1}{D_{x+8}} \Big\{ & \underset{8}{p}(M_{x+8}-M_{x+9}) + (1-\underset{8}{p})D_{x+8}V'_8 \\
& + \underset{8}{p}\underset{9}{p}M_{x+9} + \underset{8}{p}(1-\underset{9}{p})D_{x+9}V'_9 \\
& - \varpi\underset{8}{p}(D_{x+8} + \underset{9}{p}D_{x+9}) \Big\} \\
V_9 = \frac{1}{D_{x+9}} \Big\{ & \underset{9}{p}M_{x+9} + (1-\underset{9}{p})D_{x+9}V'_9 - \varpi\underset{9}{p}D_{x+9} \Big\}
\end{aligned}\right\} \text{(A) } \textit{continued.}
$$

According to American practice $\underset{1}{p}=1$ and $V'_2 = \frac{2}{10}\frac{M_{x+2}}{D_{x+2}}$,

$V'_3 = \frac{3}{10}\frac{M_{x+3}}{D_{x+3}}$, ... $V'_9 = \frac{9}{10}\frac{M_{x+9}}{D_{x+9}}$. These values being substituted in the above, and the further supposition made that $\underset{2}{p} = \underset{3}{p} = \underset{4}{p} \ldots = \underset{9}{p} (=p)$, we get, after a little reduction,

$$V_1 = \frac{1}{D_{x+1}}\left\{M_{x+1} - (1-p)\left(\frac{8}{10}M_{x+2} + \frac{7}{10}pM_{x+3} + \frac{6}{10}p^2M_{x+4} \ldots + \frac{1}{10}p^7M_{x+9}\right) - \varpi(D_{x+1} + pD_{x+2} + p^2D_{x+3} \ldots + p^8D_{x+9})\right\}$$

$$V_2 = \frac{1}{D_{x+2}}\left\{M_{x+2} - (1-p)\left(\frac{8}{10}M_{x+2} + \frac{7}{10}pM_{x+3} + \frac{6}{10}p^2M_{x+4} \ldots + \frac{1}{10}p^7M_{x+9}\right) - \varpi p(D_{x+2} + pD_{x+3} + p^2D_{x+4} \ldots + p^7D_{x+9})\right\}$$

$$V_3 = \frac{1}{D_{x+3}}\left\{M_{x+3} - (1-p)\left(\frac{7}{10}M_{x+3} + \frac{6}{10}pM_{x+4} + \frac{5}{10}p^2M_{x+5} \ldots + \frac{1}{10}p^6M_{x+9}\right) - \varpi p(D_{x+3} + pD_{x+4} + p^2D_{x+5} \ldots + p^6D_{x+9})\right\}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$V_8 = \frac{1}{D_{x+8}}\left\{M_{x+8} - (1-p)\left(\frac{2}{10}M_{x+8} + \frac{1}{10}pM_{x+9}\right) - \varpi p(D_{x+8} + pD_{x+9})\right\}$$

$$V_9 = \frac{1}{D_{x+9}}\left\{M_{x+9} - (1-p)\frac{1}{10}M_{x+9} - \varpi pD_{x+9}\right\}$$

In writing down the values of V_4, V_5, V_6, V_7, the law indicated by the expressions for V_2 and V_3 must be followed. V_1 is not included in that law owing to the exceptional value of $\underset{1}{p}$ as compared with $\underset{2}{p}$, $\underset{3}{p}$, &c.

If we use the same values of V'_2, V'_3, &c., as above, and suppose $\underset{1}{p} = 1$, $\underset{2}{p} = \underset{3}{p} = \underset{4}{p} = \underset{5}{p} (=p)$ and $\underset{6}{p} = \underset{7}{p} = \underset{8}{p} = \underset{9}{p} = 1$ we shall obtain from the equations (A) the following series of surrender values.

$$V_1 = \frac{1}{D_{x+1}}\left[M_{x+1} - (1-p)\left(\frac{8}{10}M_{x+2} + \frac{7}{10}pM_{x+3} + \frac{6}{10}p^2M_{x+4} + \frac{5}{10}p^3M_{x+5}\right) - \varpi\{D_{x+1} + pD_{x+2} + p^2D_{x+3} + p^3D_{x+4} + p^4(N_{x+4} - N_{x+9})\}\right]$$

$$V_2 = \frac{1}{D_{x+2}}\left[M_{x+2} - (1-p)\left(\frac{8}{10}M_{x+2} + \frac{7}{10}pM_{x+3} + \frac{6}{10}p^2M_{x+4} + \frac{5}{10}p^3M_{x+5}\right) - \varpi p\{D_{x+2} + pD_{x+3} + p^2D_{x+4} + p^3(N_{x+4} - N_{x+9})\}\right]$$

$$V_3 = \frac{1}{D_{x+3}}\left[M_{x+3} - (1-p)\left(\frac{7}{10}M_{x+3} + \frac{6}{10}pM_{x+4} + \frac{5}{10}p^2M_{x+5}\right) - \varpi p\{D_{x+3} + pD_{x+4} + p^2(N_{x+4} - N_{x+9})\}\right]$$

$$V_4 = \frac{1}{D_{x+4}}\left[M_{x+4} - (1-p)\left(\frac{6}{10}M_{x+4} + \frac{5}{10}pM_{x+5}\right) - \varpi p\{D_{x+4} + p(N_{x+4} - N_{x+9})\}\right]$$

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$$V_5=\frac{1}{D_{x+5}}\left\{M_{x+5}-(1-p)\left(\frac{5}{10}M_{x+5}\right)-\varpi p(N_{x+4}-N_{x+9})\right\}$$

$$V_6=\frac{1}{D_{x+6}}\{M_{x+6}-\varpi(N_{x+5}-N_{x+9})\}$$

$$V_7=\frac{1}{D_{x+7}}\{M_{x+7}-\varpi(N_{x+6}-N_{x+9})\}$$

$$V_8=\frac{1}{D_{x+8}}\{M_{x+8}-\varpi(N_{x+7}-N_{x+9})\}$$

$$V_9=\frac{1}{D_{x+9}}\{M_{x+9}-\varpi(N_{x+8}-N_{x+9})\}$$

The numerical values of V_2, V_3, &c., for a policy of £100 deduced from this last set of formulæ on the supposition that $p=\frac{1}{3}$ are set forth in the following table, the age at entry being successively taken at 30, 40, and 50. The values of V_1 are omitted as being unnecessary, the American regulations not allowing a surrender until two annual premiums have been paid. The values of ϖ for ages 30, 40, and 50, as given in the table at the commencement of this letter are ·04689, ·05632, and ·07088 respectively.

Age at Entry.	V_2.	V_3.	V_4.	V_5.	V_6.	V_7.	V_8.	V_9.
30	8·185	12·489	16·938	21·524	26·216	31·179	36·328	41·669
40	9·804	14·983	20·353	25·913	31·657	37·607	43·776	50·180
50	11·782	17·964	24·307	30·727	36·945	44·061	51·486	59·256

These *cash* values are given to enable the reader to convert them into reversionary sums by any table of single premiums he may prefer. For the purpose of illustration I have formed a table of single premiums upon the Experience rate of mortality and 3 per cent interest, with an addition of $7\frac{1}{2}$ per cent throughout,* and by this table the cash sums above given would purchase paid-up policies of the following amounts, P_n being the amount of such policy at the end of the nth year.

Age at Entry.	P_2.	P_3.	P_4.	P_5.	P_6.	P_7.	P_8.	P_9.
30	18·600	27·891	37·171	46·409	55·529	64·871	74·233	83·614
40	18·614	27·921	37·224	46·512	55·769	65·030	74·313	83·636
50	18·604	27·870	37·058	46·046	54·432	63·838	73·373	83·085

It will be seen that the amount is in every case below that given by the American Companies. It is likely however that the tables of single premiums adopted by some London Offices would give larger values for P_7, P_8, P_9, from the same cash values V_7, V_8, V_9, used in forming this table, but in no instance is it probable that the paid-up policy would reach the round numbers held out by the Americans.

* Would it not suffice, considering all the circumstances of the problem, to use the net single premiums, as is virtually the case when all the premiums have been paid?—ED. *J. I. A.*

As appertaining to the general subject in hand it will be well to examine the effect of assuming $V'_1=V_1$, $V'_2=V_2$, ... $V'_9=V_9$ in the formulæ (A). The ten premiums will not now be necessarily equal, therefore we will denote them, in the order in which they are paid, by ϖ_0, ϖ_1, ϖ_2, ϖ_3 .... ϖ_9 respectively. We shall then have

$$\left.\begin{aligned}
&V_9=\underset{9}{p}\frac{M_{x+9}}{D_{x+9}}+(1-\underset{9}{p})V_9-\underset{9}{p}\varpi_9\\
&\therefore\quad V_9=\frac{M_{x+9}}{D_{x+9}}-\varpi_9\\
\text{Also}\qquad &V_8=\underset{8}{p}\frac{M_{x+8}-M_{x+9}}{D_{x+8}}+(1-\underset{8}{p})V_8+\underset{8\,9}{pp}\frac{M_{x+9}}{D_{x+8}}\\
&\quad+\underset{8}{p}(1-\underset{9}{p})\frac{D_{x+9}}{D_{x+8}}V_9-\underset{8}{p}\varpi_8-\underset{8\,9}{pp}\frac{D_{x+9}}{D_{x+8}}\varpi_9
\end{aligned}\right\}\quad\text{(B)}$$

and if we substitute for V_9 its value just found, this equation will reduce to

$$\left.\begin{aligned}
&V_8=\frac{M_{x+8}}{D_{x+8}}-\varpi_8-\frac{D_{x+9}}{D_{x+8}}\varpi_9\\
&\text{Proceeding in the same way we get}\\
&V_7=\frac{M_{x+7}}{D_{x+7}}-\varpi_7-\frac{D_{x+8}}{D_{x+7}}\varpi_8-\frac{D_{x+9}}{D_{x+7}}\varpi_9\\
&\vdots\qquad\qquad\vdots\qquad\qquad\vdots\\
&V_1=\frac{M_{x+1}}{D_{x+1}}-\varpi_1-\frac{D_{x+2}}{D_{x+1}}\varpi_2\ldots\ldots-\frac{D_{x+9}}{D_{x+1}}\varpi_9\\
\text{and}\qquad &V_0=\frac{M_x}{D_x}-\varpi_0-\frac{D_{x+1}}{D_x}\varpi_1\ldots\ldots-\frac{D_{x+9}}{D_x}\varpi_9
\end{aligned}\right\}\quad\text{(B) \textit{continued.}}$$

It will be observed that the quantities $\underset{1}{p}$, $\underset{2}{p}$, ... $\underset{9}{p}$, have now disappeared entirely from the equations, and therefore if we give to V_9, V_8, V_7, &c., any values we please, the ten premiums determined by these ten equations will be true for *all* laws of surrender.

The premiums expressed in terms of V_9, V_8, V_7, &c., are

$$\left.\begin{aligned}
&\varpi_9=\frac{1}{D_{x+9}}(M_{x+9}-D_{x+9}V_9)\\
&\varpi_8=\frac{1}{D_{x+8}}(M_{x+8}-M_{x+9}-D_{x+8}V_8+D_{x+9}V_9)\\
&\varpi_7=\frac{1}{D_{x+7}}(M_{x+7}-M_{x+8}-D_{x+7}V_7+D_{x+8}V_8)\\
&\vdots\qquad\qquad\vdots\qquad\qquad\vdots\\
&\varpi_1=\frac{1}{D_{x+1}}(M_{x+1}-M_{x+2}-D_{x+1}V_1+D_{x+2}V_2)\\
&\varpi_0=\frac{1}{D_x}(M_x-M_{x+1}-D_xV_0+D_{x+1}V_1)
\end{aligned}\right\}\quad\text{(C)}$$

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It will be interesting to see what these premiums are, on the supposition that the various surrender values are those given by the American scheme. It is only necessary to put $V_9=\frac{9}{10}\frac{M_{x+9}}{D_{x+9}}$, $V_8=\frac{8}{10}\frac{M_{x+8}}{D_{x+8}}$, ... $V_2=\frac{2}{10}\frac{M_{x+2}}{D_{x+2}}$, $V_1=0$, and $V_0=0$, in the equations (C), and we shall obtain the following for the true premium values, taking £100 as the amount of the policy.

Age at Entry.	ϖ_0.	ϖ_1.	ϖ_2.	ϖ_3.	ϖ_4.	ϖ_5.	ϖ_6.	ϖ_7.	ϖ_8.	ϖ_9.
30	0·818	8·713	4·688	4·685	4·680	4·675	4·668	4 659	4·649	4·636
40	1·006	10·443	5·640	5·647	5·654	5·657	5·654	5·642	5·618	5·581
50	1·547	12·886	7·111	7·108	7·088	7·049	6·987	6·900	6·784	6·634

These premiums would, as I have already intimated, give the assurance company an exact equivalent for the risk undertaken, whatever were the law according to which surrenders might happen to take place. The supposition $V_1=0$, made above, causes the value of ϖ_0 to express merely the assurance risk of the first year, leaving ϖ_1 to provide for the assurance risk of the second year, and for the whole of the surrender value at the end of that year; but, as no surrender value is allowed the first year, we may equalize the first two premiums without disturbing the accuracy of the table just given.

Let ϖ' be the annual payment for the first and second year equivalent to the premiums ϖ_0 and ϖ_1, then

$$\varpi'\left(1+\frac{D_{x+1}}{D_x}\right)=\varpi_0+\varpi_1\frac{D_{x+1}}{D_x} \quad \therefore \quad \varpi'=\frac{\varpi_0 D_x+\varpi_1 D_{x+1}}{D_x+D_{x+1}}.$$

This expression for ϖ', however, may be simplified for calculation. By adding together the two last of equations (C), remembering that $V_0=0$ and $V_1=0$, we get

$$\varpi_0 D_x+\varpi_1 D_{x+1}=M_x-M_{x+2}+D_{x+2}V_2=M_x-\frac{4}{5}M_{x+2}$$

and this being the numerator of ϖ' we have $\varpi'=\frac{M_x-\frac{4}{5}M_{x+2}}{D_x+D_{x+1}}$. When $x=30$ we shall find $\varpi'=4·691$; therefore, in the place of each of the values 0·818 and 8·713 in the table, we are at liberty to put 4·691, and this substitution brings the whole series of premiums for age 30 within much nearer limits of equality. At age 40 we find $\varpi'=5·630$, and at 50, $\varpi'=7·088$, which may be substituted in the same manner for the tabular values of ϖ_0 and ϖ_1 at those ages.

By examining the equations (C) it appears that the expression for ϖ_0, namely, $\frac{M_x-M_{x+1}}{D_x}+\frac{D_{x+1}}{D_x}V_1$, is composed of the value, at the commencement of the first year, of that year's risk and of the cash surrender, payable at the end of the year. It is therefore evident that whatever be the number of surrenders in the first year (supposing any to be then allowed) the premium ϖ_0, thus calculated, would be sufficient to provide

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for them all. Next take the case of a policy upon which the second year's premium ϖ_1 has just been paid. Here the sum V_1 not having been taken, stands to the credit of the policyholder when he enters upon the second year, and therefore when he pays the premium ϖ_1 the office holds $V_1+\varpi_1$. Now, from the equation expressing the value of ϖ_1 in (C), we find that $V_1+\varpi_1=\frac{M_{x+1}-M_{x+2}}{D_{x+1}}+\frac{D_{x+2}}{D_{x+1}}V_2$, which shows that the sum in the hands of the Society at the commencement of the second year is exactly sufficient to provide for the second year's risk and the surrender V_2 at the end of that year, hence the Society cannot suffer loss however many surrenders take place in the second year. The same reasoning applied to the subsequent years will be found to lead to similar results.

Suppose we now proceed to find what *uniform* annual premium (ϖ') is equivalent to the series of premiums $\varpi_0, \varpi_1, \varpi_2, \varpi_3 \ldots . \varpi_9$ as determined by (C). We must then have an equation satisfied, which, after multiplying both sides by D_x, becomes

$$\varpi'(D_x+\underset{1}{p}D_{x+1}+\underset{1\,2}{pp}D_{x+2}\ldots . +\underset{1\,2}{pp}\ldots .\underset{9}{p}D_{x+9})=\varpi_0 D_x+\underset{1}{p}D_{x+1}\varpi_1$$
$$+\underset{1\,2}{pp}D_{x+2}\varpi_2\ldots . +\underset{1\,2}{pp}\ldots .\underset{9}{p}D_{x+9}\varpi_9$$

and if we substitute for ϖ_0, ϖ_1, &c., their values given by (C), previously putting $V_0=0$, $V_1=\frac{1}{10}\frac{M_{x+1}}{D_{x+1}}$, $V_2=\frac{2}{10}\frac{M_{x+2}}{D_{x+2}}$, &c., the above equation will be found to give for ϖ' precisely the same expression as that which was obtained for ϖ by the formulæ (1) and (2) in my last letter. From this we see that it is solely on account of charging a *uniform* premium that it becomes necessary to introduce the probabilities of surrender, and thus, in the absence of the knowledge of what those probabilities are, to bring a speculative element into the contract.

There is yet one other case to be glanced at. We may suppose the premiums to be all equal, or $\varpi_0=\varpi_1=\varpi_2 \ldots . =\varpi_9(=\varpi)$, the values of V_9, V_8, V_7, &c., then become,

$$V_9=\frac{M_{x+9}}{D_{x+9}}-\varpi$$
$$V_8=\frac{M_{x+8}}{D_{x+8}}-\varpi\frac{N_{x+7}-N_{x+9}}{D_{x+8}}$$
$$V_7=\frac{M_{x+7}}{D_{x+7}}-\varpi\frac{N_{x+6}-N_{x+9}}{D_{x+7}}$$
$$\vdots \qquad \vdots$$
$$V_1=\frac{M_{x+1}}{D_{x+1}}-\varpi\frac{N_{x}-N_{x+9}}{D_{x+1}}$$
$$V_0=\frac{M_{x}}{D_{x}}-\varpi\frac{N_{x-1}-N_{x+9}}{D_{x}}$$

Since $V_0=0$ in practice, the last equation gives $\varpi=\frac{M_x}{N_{x-1}-N_{x+9}}$, which determines the premium, and, substituting this value in each of the other equations, the nine surrender values become known. We have here the familiar case of a whole-life assurance, all the premiums for which are comprised in ten equal annual payments, one at the commencement of each of the first ten years, but where no surrender values of given amounts form any part of the contract, and it is evident from what precedes that this is the only possible instance—for the same description of policy—in which a *uniform* premium will exactly provide (under any law of surrender) for a set of surrender values, the amounts of which *might* be specified beforehand or at the time the assurance was effected.

The actual amounts of the surrenders which might be thus held out to the assurer, without introducing any uncertainty or speculation into the transaction, are the values of V_9, V_8, &c., given by the last set of formulæ, and these are specified for ages 30, 40, and 50, at entry, in the following table. On comparing them with the results previously obtained for V_9, V_8, &c., on another hypothesis, it will be seen that the difference is only in the decimal in each case.

Age at Entry	ϖ.	V_2.	V_3.	V_4.	V_5.	V_6.	V_7.	V_8.	V_9.
30	4·674	8·152	12·445	16·891	21·498	26·273	31·223	36·357	41·684
40	5·636	9·812	14·986	20·345	25·894	31·641	37·594	43·768	50·176
50	7·002	11·600	17·683	23·974	30·494	37·262	44·304	51·653	59·342

The figures here given for ϖ are derived, of course, from $\varpi = \dfrac{M_x}{N_{x-1} - N_{x+9}}$.

Sufficient materials have now probably been given in this and my former letter to enable any one interested in the subject to form an opinion as to the merits of the American system of ten year nonforfeiture policies. Its simplicity of statement is its one recommendation, and no doubt a great and important one, but it is plain that if a Company issued a considerable number of such policies, some care would be necessary at each periodical valuation in determining the reserve required for the risks, in order to attain that degree of exactness and certainty in the results to which most English Actuaries are accustomed.

I am,
Sir,
Your most obedient Servant,
SAMUEL YOUNGER.

17, *Waterloo Place,*
Pall Mall, London,
31*st May,* 1869.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In Mr. Higham's paper on the value of "selection," in vol. i., in discussing the effect of taking the lives in quinquennial groups, he says in a foot-note, page 186, that if the numbers living at ages m, $m+1$, $m+2$, $m+3$, $m+4$, respectively, be represented by 10, 9, 8, 7, 6, then, if the probability of living a year diminish by second differences, the probability for the quinquennial combination is $=$1st term $+\frac{7}{4}d_1+\frac{52}{32}d_2$,

d_1, d_2, being the 1st and 2nd orders of differences of p_m.

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